

The semantic method of extension and intension and the four criteria of the conditional described by Sextus Empiricus¹

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Abstract. In this paper an analysis is conducted of the debate about the most suitable way to understand the conditional, which took place in the 4th century B.C. This is done using the extension and intension method provided by Rudolf Carnap to study the meaning of expressions. The results seem to show that, according to Sextus Empiricus, although the debate was about four different criteria to understand the conditional, three of those criteria actually appear to be the same and have a very clear common logical form under Carnap's framework.

Keywords: conditional; meaning; modal logic; semantics; state-description.

[esp.] El método semántico de extensión e intensión y los cuatro criterios del condicional descritos por Sexto Empirico

Resumen. En este artículo se analiza el debate sobre la forma adecuada de entender el condicional producido en el siglo IV a. C. El análisis se lleva a cabo mediante el método proporcionado por Rudolf Carnap para estudiar el significado de las expresiones, es decir, mediante el método de extensión e intensión. Los resultados de ese análisis parecen mostrar que, aunque, según Sexto Empirico, el debate fue sobre cuatro criterios diferentes para comprender el condicional, en realidad tres de esos criterios parecen ser iguales y tienen una forma lógica común muy clara en el marco de Carnap.

Palabras clave: condicional; sentido; lógica modal semántica; descripción de estado.

Sumario: 1. Introduction; 2. The four criteria of the conditional following Sextus Empiricus; 3. Some important aspects of Carnap's method; 4. Are really the criteria described by Sextus Empiricus four?; 5. Conclusions; 6. Bibliographic references.

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1. Introduction

As it is well known, Sextus Empiricus, in books such as *Adversus Mathematicos* (VIII, 111) and *Pyrrhoneae Hypotyposes* (II, 104), speaks about a polemic on the best way to consider the conditional. That polemic happened throughout the 4th century B.C. (e.g., O’Toole and Jennings, 2004). Nevertheless, in this paper, it will be analyzed by means of a method given in the 20th century. That is the method of extension and intension proposed by Rudolf Carnap (Carnap, 1947), a method that, as it is also known, is intended to show the real meanings of the different kinds of linguistic expressions. Thus, the aim is, as shown below, to argue that, if that method is assumed, it is only possible to accept that Sextus refers just to two interpretations of the conditional, since, although he states that the accounts are four, three of them cannot be clearly distinguished.

To achieve that goal, firstly, the four criteria indicated by Sextus will be described. Then, the concepts of Carnap’s method necessary to review them will be commented on. Finally, the analysis of the accounts by means of the method of extension and intension will be presented.

2. The four criteria of the conditional following Sextus Empiricus

As said, the criteria are to be found in the books authored by Sextus Empiricus cited above (*Adversus Mathematicos* VIII, 111; *Pyrrhoneae Hypotyposes* II, 104). Sextus’ real intention is, as known, to argue against the dogmatic philosophers in general and his analysis of the conditional is focused on a particular group of them: the Stoics. In this way, what Sextus Empiricus wants to do is to find arguments showing contradictions or inconsistencies with the main theses of such philosophers. Thus, he tries to make it explicit that, for the specific problem of the conditional, the Stoics do not have an unified position, since they do not agree about when a conditional sentence is clearly true or sound. In his view, there are until four different criteria, which are those that will be dealt with in this paper. However, maybe it is important to indicate that, actually, all of those criteria are only commented on in *Pyrrhoneae Hypotyposes* (II, 104), as in *Adversus Mathematicos* (VIII, 111) only two of them (the one of Philo of Megara and the one of Diodorus Cronus described above) are considered. Nonetheless, in any case, in this section, the description that will be taken into account will be basically the one of O’Toole and Jennings (2004; for further analyses about these issues, see also, e.g., Lukasiewicz, 1967).

As stated, the interpretations are four, and the first one is assigned to Philo of Megara by Sextus. It is known too that this first account is traditionally named ‘the material interpretation of the conditional’ and that usually adopted in classical logic (e.g., Bocheński, 1963; López-Astorga, 2015; Mates, 1953; O’Toole & Jennings, 2004). In this way, it is nowadays generally represented by the symbol ‘ \rightarrow ’ as follows:

[I]: $p \rightarrow q$, where ‘p’ is the if-clause and ‘q’ is the then-clause.

And its truth table is not unknown either. It provides that a formula such as [I] can only be false in one hypothetical scenario: in the case in which p is true and q, on the contrary, is false. In all of the other three possibilities (both p and q being true, p being false and q being true, and both p and q being false), [I] is true.

Nonetheless, the second account, which is explicitly attributed to Diodorus Cronus by Sextus Empiricus, is different. According to Diodorus, the contents of *p* and *q* can be incidental and hence, if Philo's criterion is adopted, it is always possible that [I] is false. So, the conditional should fulfill another requirement as well: it has to have a thematic content such that it is not possible that the sentence in entirety is false (e.g., López-Astorga, 2015; Mates, 1953; O'Toole and Jennings, 2004). Obviously, these two accounts are undoubtedly different, and with the help of a contemporary psychological approach, the mental models theory (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015; Khemlani, Hinterecker, & Johnson-Laird, 2017), that has been shown (López-Astorga, 2015). So, this point will not be challenged here. The idea of this paper is that this last interpretation, the one of Diodorus Cronus, is, under Carnap's framework, essentially the same as the two remaining accounts.

And one of those two remaining criteria is the one that O'Toole and Jennings (2004) call 'connexivist'. In this case, Sextus Empiricus gives no name of a writer, but *De Fato* by Cicero seems to prove that the philosopher supporting this account should have been Chrysippus of Soli (e.g., Kneale & Kneale, 1962; López-Astorga, 2016; O'Toole and Jennings, 2004; see also, e.g., Barnes, Bobzien, & Mignucci, 2008). This criterion is also clear: a conditional can be accepted if and only if the negation of its then-clause is in contradiction (again, by virtue of their thematic contents) with the if-clause.

Finally, the last interpretation claims that a correct conditional is that whose if-clause includes the then-clause in a potential way. Sextus Empiricus does not offer the name of a proponent for this criterion either, which has led to think that perhaps this account is just a version of the one of Chrysippus expressed in a stricter manner (e.g., López-Astorga, 2016; Mates, 1953; O'Toole and Jennings, 2004). However, what this paper tries to show is not only that this interpretation is related to the connexivist one, but that, excluding that of Philo of Megara, the other three accounts cannot be differentiated from each other if the method of extension and intension is used. The next section is devoted to a description of the elements of this method necessary to make that evident.

3. Some important aspects of Carnap's method

What Carnap (1947) actually wishes to raise is a new method. A new method that is basically semantic and allows revealing the real meaning of several kinds of linguistic expressions that are to be found in different communicative situations. In this manner, concepts such as those of 'class' or 'sentence' have particular emphasis and are reinterpreted under his approach. So, the method proposed by Carnap is, of course, a framework much more developed than what can be indicated in a paper such as this one, and, for this reason, as pointed out, this section only describes the aspects of it needed to achieve the goal indicated above. Nevertheless, for that description, in a manner similar to Carnap's book, a semantic system *S* will be assumed here as well.

But, in this paper, *S*, in addition to all of the rules, symbols, and requirements of classical logic, consists of rules of designation for predicates referring to all of the definitions and meanings that belong to English language. Thus, taking *S* as the 'object language', two first definitions can be important:

[II]: Falsity: a is false (in S) =_{df} $\neg a$ is true (in S), where ‘ \neg ’ stands for negation.

(With not exactly the same symbols, [II] is basically definition 1-7 in Carnap, 1947, p. 6).

[III]: Equivalence: a is equivalent to b (in S) =_{df} $a \leftrightarrow b$ is true (in S), where ‘ \leftrightarrow ’ denotes biconditional relationship.

(With not exactly the same symbols, [III] is basically definition 1-8 in Carnap, 1947, p. 6).

As far as [III] is concerned, maybe is relevant to add that, according to Carnap, “Two sentences are equivalent if and only if both have the same truth-value, that is to say, both are true or both are false” (Carnap, 1947, p. 6; Result 1-9). Nonetheless, a much more important concept in his system is evidently the one of ‘L-truth’. This concept refers to the situation in which a sentence is true exclusively by virtue of semantic rules, without the need to check empirical facts. As Carnap (1947) acknowledges, it is akin to Leibniz’s concept of ‘necessary truth’ or Kant’s concept of ‘analytic truth’. However, he explains it by means of another concept: ‘state-description’, which, as Carnap (1947) also explicitly claims, is similar to the one of ‘possible worlds’ provided by Leibniz or the one of ‘possible state of affairs’ given by Wittgenstein as well. Obviously, as its name indicates, a state-description “gives a complete description of a possible state of the universe of individuals with respect to all properties and relations expressed by predicates of the system” (Carnap, 1947, p. 9). But perhaps what is more interesting now is the final definition of L-truth:

[IV]: L-truth: a is L-true (in S) =_{df} a holds in all the state-descriptions (in S).

(With not exactly the same symbols and wording, [IV] is basically definition 2-2 in Carnap, 1947, p. 10).

And, from here, other definitions are not hard to understand:

[V]: L-falsity: a is L-false (in S) =_{df} $\neg a$ is L-true (in S).

(With not exactly the same symbols, [V] is basically definition 2-3-a in Carnap, 1947, p. 11).

[VI]: L-implication: a L-implies b (in S) =_{df} the sentence $a \rightarrow b$ is L-true (in S).

(With not exactly the same symbols, [VI] is basically definition 2-3-b in Carnap, 1947, p. 11).

[VII]: L-equivalence: a is L-equivalent to b (in S) =_{df} the sentence $a \leftrightarrow b$ is L-true (in S).

(With not exactly the same symbols, [VII] is basically definition 2-3-c in Carnap, 1947, p. 11).

So, it is also clear that

[VIII]: a is L-false if and only if (from now on, IFF) a holds in no state description.

(With not exactly the same symbols and wording, [VIII] is basically result 2-4 in Carnap, 1947, p. 11).

[IX]: a L-implies b IFF b holds in the same state-descriptions in which a also holds.

(With not exactly the same symbols and wording, [IX] is basically result 2-5 in Carnap, 1947, p. 11).

[X]: a is L-equivalent to b IFF b holds in the same state-descriptions in which a also holds, and a holds in the same state-descriptions in which b also holds.

(With not exactly the same symbols and wording, [X] is basically result 2-6 in Carnap, 1947, p. 11).

One more important point can be the concept of ‘factual truth’, which, following Carnap, corresponds to the one of ‘synthetic judgment’ such as Kant understands it. Clearly, a sentence is only factually true, or ‘F-true’, when it is true by virtue of facts, and not by virtue of just semantic rules of S. Thus, it can be thought that a sentence that is F-true is not necessarily true in all of the state-descriptions (i.e., it is not L-true). And, based on this, the definitions of F-falsity, F-implication, and F-equivalence can be derived, in an analogous way, with not difficulty. In fact, to indicate which those definitions are exactly can be even trivial (in any case, they are to be found in Carnap, 1947, p. 12).

Lastly, it only seems to be necessary to remind that, as to all the linguistic expressions, Carnap (1947) assigns an extension and an intension to sentences. The former is their truth-value and the latter is the proposition to which the particular sentence refers. Nevertheless, and this is very relevant for the analysis below, Carnap also clearly states that to be L-equivalent is to share the same intension (see definition 5-2 in Carnap, 1947, p. 23). This last idea will be a key element in the argumentation of the next section, which will be, evidently, supported by definitions and results [II] to [X] too. But that idea is especially important because what is actually intended to do below is to mainly show that the three last criteria described by Sextus Empiricus are the same because they are L-equivalent and hence their intension is the same.

4. Are really the criteria described by Sextus Empiricus four?

By applying the method of extension and intension, it is clear that the material interpretation of the conditional, that is to say, the account attributed to Philo of Megara, is different from the other three criteria. If, according to that interpretation, the conditional can be true in three cases (i.e., in the cases in which [I] can be true following its truth-table in classical propositional logic: a) p is true and q is true, b) p is untrue and q is true, c) p is untrue and q is untrue), it is obvious that there are at least three different state-descriptions in which, with different combinations of truth-

values for p and q , [I] can be true. However, this circumstance is not, in principle, what differentiates this criterion from the others. If only those three cases were possible, by [IV], [I] would be L-true. The problem is that the complaint raised by Diodorus Cronus seems to reveal that, in this interpretation, there is also a possibility that, in another state-description, p is true and q is untrue, which, given that, in that state-description, the negation of [I] would be true, by [II], would cause [I] to be false. So, it can be said that, under Philo's criterion, when [I] is true, it is really F-true, since there can be state-descriptions in which it is false, and that, when it is false, it is really F-false, since there can be state-descriptions in which it is true.

Nevertheless, the situation completely changes with Diodorus' account. He claims that [I] is always true, which means that a formula such as the following is always false:

[XI]: $p \wedge \neg q$, where ' \wedge ' is conjunction.

But, if [XI] is always false, then, by [II], this formula is always true:

[XII]: $\neg(p \wedge \neg q)$

And, if [XII] is always true, by [IV], [XII] is L-true, and, by [V] and [VIII], [XI] is L-false. Thus, given a formula such as this one:

[XIII]: $p \rightarrow_1 q$, where ' \rightarrow_1 ' indicates that the conditional linking p and q fulfills the criterion provided by Diodorus Cronus,

It is necessary to state that, since [XI] is L-false and, therefore, [XII] is L-true, by [VI], p L-implies q , and that, by [IX], q holds in the same state-descriptions in which p also holds.

However, it is still possible even to further advance. When speaking about modal logic, Carnap says that

[XIV]: $\Box a$ is L-true if a is L-true; if a is not L-true, $\Box a$ is L-false, where ' \Box ' is 'the box', i.e., the modal symbol for necessity.

(With not exactly the same symbols and wording, [XIV] is basically convention 39-3 in Carnap, 1947, p. 175).

And, by [III], [XIV] leads to the equivalence between [XIII] and

[XV]: $\Box(p \rightarrow q)$

Since, in [XIII], as said, p L-implies q and hence, by [VI], its conditional relationship is L-true, and, accordingly, it can be assumed that

[XVI]: $[XIII] \leftrightarrow [XV]$

Nevertheless, in fact, the results of this are even greater, as what can be truly

assumed is not just an equivalence between [XIII] and [XV], but, by virtue of [X] and [VII], that they are L-equivalent and, therefore, that [XVI] is L-true.

There is no doubt that arguments such as these ones remind theoretical positions such as, for example, the one of Lewis (1918), which propose strict implications that can be transformed into formulae such as the following:

[XVII]: $\neg\Diamond(p \wedge \neg q)$, where ‘ \Diamond ’ is ‘the diamond’, that is, the symbol in modal logic for possibility.

Which in turn leads to formulae such as [XV]. However, although such relationships can be more or less clear, especially, in the case of the next criterion, that is, the one of Chrysippus of Soli, which has been explicitly compared to Lewis’ strict implication and it has been analyzed to what extent those notions are actually similar (e.g., Gould, 1970), a study of them is beyond the aim indicated above. That aim is just to show that, if Carnap’s method is adopted, three of the four criteria of the conditional mentioned by Sextus Empiricus are the same. So, it appears that what is important now is to continue with Chrysippus’ interpretation.

As commented on, this last thinker speaks about a conditional in which, if its then-clause is negated, that clause is incompatible with the thematic content of the antecedent. In this way, given that, as also said, all the English words and expressions are considered by the rules of designation for predicates in S, it can be claimed that, if [I] satisfies Chrysippus’ account, there is no a state-description in which p is true and q is false, as, in that case, there would be a contradiction. However, that means that, again, by [VIII], [XI] is L-false. Likewise, by [IV] and [V], [XII] is L-true here too, and, by [VI] and [IX], this formula also stands for that p L-implies q:

[XVIII]: $p \rightarrow_2 q$, where ‘ \rightarrow_2 ’ indicates that the conditional linking p and q fulfills the criterion provided by Chrysippus of Soli.

So, [XVIII] is L-true as well, and, accordingly, by [X], [XIII] and [XVIII] are L-equivalent, that is, by [VII], the following is also a L-true formula:

[XIX]: [XIII] \leftrightarrow [XVIII]

And, if what is expressed in [XVI] is taken into account, an obvious deduction from it and [XIX] is:

[XX]: [XIII] \leftrightarrow [XVIII] \leftrightarrow [XV]

What reveals that, on the one hand, both Diodorus’ criterion ([XIII]) and Chrysippus’ interpretation ([XVIII]) refer to the same modal logical form ([XV]), and, on the other hand, they speak about kinds of conditionals with exactly the same intension, and, therefore, expressing the same proposition (as mentioned, according to Carnap, if two expressions are L-equivalent, their intension cannot be different).

In this way, it appears to be clear that the method of extension and intension implies that the accounts given by Diodorus Cronus and Chrysippus of Soli are really the same. Nevertheless, the same thing can also be stated in connection to the

last criterion. As mentioned, some writers have already noted the relationship that there can be between this last interpretation and the one of Chrysippus. Nonetheless, Carnap's method allows making that relationship evident as well. If, following the last account, the then-clause is already included in the if-clause, situations of a true if-clause with an untrue then-clause are not possible in this case either. Thus, once again, this leads, by [VIII], to that [XI] is a L-false formula, and, by [IV] and [V], to that, on the contrary, [XII] is L-true. Therefore, as in the previous cases, by [VI] and [IX], it can be stated that the L-implication between p and q occurs in a formula such as the following as well:

[XXI]: $p \rightarrow_3 q$, where ' \rightarrow_3 ' indicates that the conditional linking p and q fulfills the fourth criterion described by Sextus Empiricus.

But this also means here that [XXI] is L-true, that, by [X], [XIII], [XVIII], [XXI], and [XV] are L-equivalent, and that, by [VII], this is another L-true formula:

[XXII]: $[XIII] \leftrightarrow [XVIII] \leftrightarrow [XXI] \leftrightarrow [XV]$

Therefore, it is clear that, if the method of extension and intension is applied, the three last criteria explained by Sextus Empiricus are the same. And this is so, at least, in the sense that they all have the same logical form ([XV]) and, for being L-equivalent, they are interchangeable, are true in the same state-descriptions (in all of them), refer to the same proposition, and hence have the same intension.

5. Conclusions

Accordingly, as indicated at the beginning of this paper, although Sextus Empiricus tries to account for four different criteria of the conditional, the analysis with a method such as the one of extension and intension seems to reveal that his description only truly refers to two interpretations of that connective. As shown, the arguments in favor of this idea is that three of those accounts are L-true and, therefore, L-equivalent, and hence do not have a different intension.

Undoubtedly, this is a very interesting fact, since it enables to think about the possibility of continuing the analysis of the four criteria with more elements of Carnap's method in order to check whether or not more significant conclusions are obtained. Obviously, a task in this way would not imply to assume all of the theses and philosophical positions supported by Carnap. Probably, it would be necessary only to follow the analytical aspects of his method required for the issue that the researcher wanted to review, in a similar manner as in this paper.

Furthermore, it is also evident that the method can be applied, in the same way, to the study of other logical or philosophical theories given in the past. As said, it is not necessary to accept the theoretical framework provided by Carnap in entirety, but just tools appropriate for a particular point.

Clearly, it is worth at least trying to do a task of this type in order to see the results that can be achieved. A psychological theory, the mental models theory cited above, has already been used with a similar intention (e.g., López-Astorga, 2015),

that is, with the aim to clarify some aspects of the theories offered by certain ancient thinkers. So, one might ask about the reasons why it would not be advisable to work in a similar manner with a method such as the one of Carnap, which, in addition, directly comes from philosophy and the philosophical tradition, and was built with the goal to improve language and remove ambiguities in the daily use of certain words and expressions.

Thus, while the aims do not have to be as broad as those of Carnap, the validity that some aspects of his method can still have for the study of the history of philosophy in general, and of the history of the ancient philosophy in particular, have become clear throughout this paper. Accordingly, Carnap's methodological legacy can keep being useful, whether or not his general view of the world and philosophy is accepted.

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