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A-realcompact spaces.

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Abstract

Relations between homomorphisms on a real function algebra and different properties (such as being inverse-closed and closed under bounded inversion) are studied.

1 Introduction and notation

By a function algebra A on X we mean a family of real-valued functions on X such that: 1) A is a linear algebra with unit under operations defined pointwise, 2) A separates points on X and 3) A is closed under bounded inversion, that is, if $f \in A$ and $f \geq 1$, then $\frac{1}{f} \in A$. We denote by $Hom(A)$ the family of all A-homomorphisms, that is, non null multiplicative real linear functionals on A, endowed with the Gelfand topology.

$Hom(A)$ has been intensively studied when X is a completely regular Hausdorff space and A is $C(X)$ (see [12]). In recent years different papers have been devoted to study homomorphisms on some subalgebras of $C(X)$, for example algebras of differentiable functions have been considered in [1]-[5], [14] and [15]. As can be seen in the quoted papers, in studying function algebras frequently one needs results asserting that a homomorphism is the evaluation at some point of the supporting space. This paper is devoted to elaborate a general theory related with this subject.

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2 Single-set evaluating algebras and A-realcompactness

2.1.- Let X be a completely regular Hausdorff space, $Y \subset X$ and $f : Y \to \mathbb{R}$ a continuous map. If f has a continuous extension to $p \in X \setminus Y$, this extension will be denoted by $\hat{f}(p)$. For $f : X \to \mathbb{R}$, $Z(f) = \{x \in X : f(x) = 0\}$. A set $S \subset Y$ is a zero set if there exists $g \in C(Y)$ such that $S = Z(g)$ and $\overline{S}^X$ is the closure of S in X. As usual βX denotes the Stone-Čech compactification of X.

2.2.- The elements of any function algebra can be considered as uniformly continuous functions on X in the following sense. Denote by A_b the subalgebra of all bounded functions in A. Let U_A be the uniformity generated on X by A_b, that is U_A is defined by the pseudometrics

$$d_f(x,y) = | f(x) - f(y) |; \quad f \in A_b, x, y \in X.$$

Let τ_A denote the topology induced by U_A on X. Since A separates points in X , (X, τ_A) is a completely regular Hausdorff space. All topological notions on X are assumed in the τ_A topology.

Denote by X_A the completion of the uniform space (X, U_A), then X_A is a compact Hausdorff space and X can be considered as a dense subspace of X_A. It is known that each $f \in A_b$ has a unique continuous extension $\hat{f}$ to X_A. Set $\hat{A} = \{\hat{f} : f \in A_b\}$. $\hat{A}$ separates points in X_A ([7]) then, by the Stone-Weierstrass theorem, $\hat{A}$ is a dense subspace of $C(X_A)$ in the uniform norm.

2.3.- The following result from [7] will be used in the sequel:

Theorem. *Let A be a function algebra on X, then*

(a) $\varphi \in Hom(A_b)$ if and only if there exists a (unique) $p \in X_A$ such that $\varphi(f) = \hat{f}(p)$ for every $f \in A$. Moreover X_A is (homeomorphic to) the maximal ideal space of A_b;

(b) $\varphi \in Hom(A)$ if and only if there exists a (unique) point $p \in X_A$ such that, every $f \in A$ has a finite continuous extension $\hat{f}(p)$ to p and $\varphi(f) = \hat{f}(p)$. The set $I(A)$ of all such p, with the topology induced by X_A, is (homeomorphic to) the maximal ideal space of A.

2.4.- In what follows we associate to a given function algebra A the spaces X_A and $I(A)$ defined above. Moreover, we identify *Hom(A)* with $I(A)$ and X with a (dense) subset of X_A. Thus we have the inclusions,

$$X \subset I(A) \subset X_A.$$

In studying properties of homomorphisms it is important to have conditions to recognize points in $I(A) \setminus X$. It is easy to verify that for a point $p \in X_A \setminus X$ the following assertions are equivalents:

(a) $p \in I(A)$;

(b) for every $f \in A$, there exists a net $\{x_\lambda\}$ in X such that $x_\lambda \to p$ and $f(x_\lambda)$ is bounded;

(c) for every $f \in A$, there exists a neighbourhood V of p in X_A such that $f(V \bigcap X)$ is bounded.

2.5.- We need some definitions: a function algebra A on X is called *single-set evaluating* if, for every $\varphi \in A$ and each $f \in A$, there exists $x \in X$ such that $\varphi(f) = f(x)$. A is called *inverse-closed* if for every $f \in A$ such that $Z(f) = \emptyset$, $\frac{1}{f} \in A$. It is easy to prove that inverse-closed algebras are single-set evaluating. There exist single-set evaluating algebras which are not inverse-closed [6].

2.6.- Given a nonempty set X, (A, B) is called a *subordinated pair* [7] on X if: i) A and B are function algebras on X; ii) $B \subset A$; iii) every homomorphism on B has an extension to a homomorphism on A.

2.7.- **Theorem.** *For a function algebra A on X the following conditions are equivalent:*

(a) A is single-set evaluating;

(b) For all $p \in I(A) \setminus X$, if $f \in A$ and $0 < f \leq 1$, then $\hat{f}(p) \neq 0$;

(c) (RA,A) is a sudordinated pair, where RA the smallest inverse-closed algebra on X containing A.

Proof.

i) Suppose that (a) holds but (b) does not. Fix $p \in I(A) \setminus X$ and $h \in A$ such that $0 < h \leq 1$ and $\hat{h}(p) = 0$. Since evaluation at p is a homomorphism on A, A is not single-set evaluating.

ii) Suppose that (b) holds and A is not single-set evaluating. Take $\varphi \in Hom(A)$, $p \in I(A)$ and $k \in A$ such that $\varphi(g) = \hat{g}(p)$ for every $g \in A$ and $\varphi(k) \neq k(x)$ for all $x \in X$. Set $h(x) = (k(x) - \varphi(k))^2$ and $f(x) = \frac{h(x)}{1+h(x)}$. Then $\hat{f}(p) = \varphi(f) = 0$ and $0 < f(x) \leq 1$. This contradicts (b).

iii) For (a) implies (c) see lemma 16 of [6].

iv) Since RA is inverse-closed it is single-set evaluating. If (RA, A) is a subordinated pair, then A is single-set evaluating.

■

2.8.- Recall that a completely regular Hausdorff space Y is realcompact [12] if every $C(Y)$-homomorphism is the evaluation at some point p in Y. This concept can be generalized in the following way: if A is a function algebra on X, X is said to be *A-realcompact* if every A-homomorphism is the evaluation at some point p of X. A similar notion was used in [8], [16] and [17].

2.9.- **Remarks.**

1) If $A_b = A$, then X is A-realcompact if and only if X is compact (in the τ_A topology). When $X_A \setminus X \neq \emptyset$ we can obtain A-realcompactness only when A contains an unbounded function. In particular if (X, τ) is a pseudocompact noncompact, completely regular Hausdorff space and $A = C(X)$, then X is not A-realcompact.

2) Notice that if A and B are function algebras on X, $B \subset A$, with X A-realcompact, then X is B-realcompact if and only if (A, B) is a subordinated pair.

2.10.- **Proposition.** *Let A and B be function algebras on X with B uniformly dense in A. Then (A, B) is a subordinated pair.*

Proof. Since B_b is uniformly dense in A_b, the spaces $C(X_A)$ and $C(X_B)$ are isomorphic, thus by the Banach-Stone theorem (see [12]) X_A and X_B are homeomorphic. We may identify X_A and X_B. Fix a homomorphism φ on B and a point $p \in X_A$ such that for every $f \in B$, $\varphi(f) = \hat{f}(p)$. We will finish our proof by showing that every $g \in A$ has a (unique) continuous finite extension to p. Fix $g \in A$ and $f \in B$ such that $\sup_{x\in X} | f(x) - g(x) | \leq 1$. There exist a neighbourhood V of p in X_A and a positive constant M such that for every $y \in V \bigcap X$, $| f(y) | \leq M$. Then for every $y \in V \bigcap X$, $| g(y) | \leq M + 1$, now the assertion follows from 2.4.

■

In [10] (proposition 1.8) was proved the following fact: if X is a realcompact space and $A \subset C(X)$ is a subalgebra with unit, closed under bounded inversion, uniformly dense in $C(X)$, then $Hom(A) = X$. Our next result, as an application of proposition 2.10 (see remark 2.9.2), provides a natural extension.

2.11.- **Corollary.** *Let A and B be function algebras on X, $B \subset A$. If B is uniformly dense in A and X is A-realcompact, then X is B-realcompact.*

2.12.- **Theorem.** *Let A be a single-set evaluating algebra on X. Then X is A-realcompact if and only X is RA-realcompact (see (c) in 2.7). Moreover if A is inverse-closed, then X is A-realcompact if and only if for every $p \in X_A \setminus X$, there exists*

$$f \in A_b, \quad 0 < f \leq 1, \qquad \textit{such that } \hat{f}(p) = 0. \tag{1}$$

Proof. The first part follows from theorem 2.7, the remark 2) in 2.9 and the construction of RA.

For the second part suppose first that X is A-realcompact. Suppose that $p \in X_A \setminus X$. Taking into account that $p \notin I(A) = X$, there exists $f \in A \setminus A_b$ such that for every net $\{x_\lambda\}$ in X, with $x_\lambda \to p$, $f(x_\lambda)$ is unbounded (see the last assertion in 2.4). Then $\hat{h}(p) = 0$ and $0 < h(x) \leq 1$ for $x \in X$, where $h(x) = \frac{1+f^2(x)}{1+f^4(x)}$.

Suppose now that for all $p \in X_A \setminus X$ there exists $f \in A$ such that $0 < f \leq 1$ and $\hat{f}(p) = 0$. By defining g(x) $= \frac{1}{f(x)}$, we have that $g \in A$

and for every net $\{x_\lambda\}$ in X, $x_\lambda \to p$, $\{g(x_\lambda)\}$ is not bounded. This completes the proof.

■

2.13.- **Remark.** In general condition (1) does not imply A-realcompactness. For example, let X be the real interval $(0,1]$ and A the restriction of continuous functions in $[0,1]$ to $(0,1]$. In this case the condition holds but X is not A-realcompact (notice that $X_A = [0,1]$).

2.14.- **Theorem.** *Let A be a function algebra. Then X_A is the Stone-Čech compactification of X if and only if for any disjoint zero sets S and T in X, there exists $f \in A$, such that*

$$0 \leq f \leq 1, \quad f(S) = \{0\} \text{ and } \quad f(T) = \{1\}. \tag{2}$$

Proof. If A satisfies (2) by theorem 11 of [11], A_b is uniformly dense in the space $C_b(X)$ of all real continuous bounded functions on X, then $\beta X = X_A$.

On the other hand if $\beta X = X_A$, A_b is dense in $C_b(X)$ and the result follows again from theorem 11 of [11].

■

From theorems 2.12 and 2.14 we obtain a proof of the following result due to S. Mrówka (proposition 3.11.10 in [9]).

2.15.- **Corollary.** *Let X be a completely regular Hausdorff space. Then X is realcompact if and only if for every $p \in \beta X \setminus X$, there exists $f \in C(X)$ such that $0 < f(x) \leq 1$, $x \in X$, and $\hat{f}(p) = 0$.*

The next result extends Theorem 2 of [15]. Jaramillo presented in [15] different examples of functions algebras for which Theorem 2.16 may be applied.

2.16.- **Theorem.** *Let us suppose that a function algebra A on X satisfies the following conditions:*

(a) for every $f, g \in A$ and $\rho, \epsilon > 0$, if the sets

$$P_\epsilon(f) = \{x : \mid f(x) \mid \leq \epsilon\} \text{ and } Q_\rho(g) = \{x : \mid g(x) \mid \geq \rho\}$$

are not empty and disjoint, there exists $h \in A$, $0 \leq h \leq 1$, such that

$$h(P_\epsilon(f)) = \{0\} \text{ and } h(Q_\rho(g)) = \{1\};$$

(b) given an open (in the τ_A topology) cover $\{H_n\}$ of X , such that $\overline{H_n} \subset H_{n+1}$, and $f : X \to \mathbf{R}$, if there exists a sequence f_n in A such that $f_n |_{H_n} = f |_{H_n}$, then $f \in A$;

(c) for every $p \in X_A \setminus X$ there exists $g \in C(X_A)$ which satisfies (1).

Then X is A-realcompact.

Proof. Let φ be a homomorphism on A. There exists $p \in X_A$ such that $\varphi(f) = \hat{f}(p)$ for every $f \in A$. We will show that $p \in X$.

Suppose that $p \in X_A \setminus X$, take $g \in C(X_A)$ such that $0 < g \leq 1$ and $\hat{g}(p) = 0$. Set

$$E_n = \{x \in X_A : g(x) > \frac{1}{2^n}\}, \quad n = 1, 2, \ldots$$

We may suppose that each E_n is not empty. Since $\hat{A}$ is dense in $C(X_A)$, there exists a sequence $\{f_n\}$ in A_b such that

$$\| \hat{f}_n - g \|_\infty \leq \frac{1}{2^{n+3}} \text{ and } \| \hat{f}_n - \hat{f}_{n+1} \|_\infty \leq \frac{1}{2^{n+3}},$$

where $\| \cdot \|_\infty$ denotes the sup norm in $C(X_A)$. Set

$$F_n = \{x \in X_A : | \hat{f}_n(x) | \geq \frac{1}{2^n}\}.$$

It is easy to prove that for $n \geq 2$, $E_{n-1} \subset F_n \subset E_{n+1}$.

Now we have that $(X \bigcap \bigcup_{n \in I\!N} E_n) = \bigcap X \bigcup_{n \in I\!N} F_n$, thus $\{F_{2n} \bigcap X\}$ is an increasing open cover of X. For each $n \geq 2$ take $g_n \in A$, $0 \leq g_n \leq 1$, such that

$$g_n(F^c_{2n+2} \bigcap X) = \{1\} \text{ and } g_n(\overline{F_{2n}} \bigcap X) = \{0\}.$$

Notice that $\hat{g}_n(p) = 1$, thus $\varphi(\hat{g}_n) = 1$. The function $f(x) = \sum_{n=2}^{\infty} g_n(x)$, $x \in X$ is well defined. Set $k_n(x) = \sum_{j=2}^{n} g_j(x)$. Since $k_n \in A$, $f \in A$.

It is easy to see that for every $x \in X$ and each n, $k_n(x) \leq f(x)$, then $\varphi(f) \geq \varphi(k_n) = \sum_{j=1}^{n} \varphi(g_j) = n$ (see 1.4 of [13]), this says that $\varphi(f) = \infty$, a contradiction.

■

2.17.- Theorem 2.3 gives a representation of the real maximal ideal of A but, as the following result will prove, we can not expect to obtain a one to one relation between z-ultrafilters and maximal ideals. The notion on z-filter is used as in [12]. An ideal in A is a proper ideal. For an ideal I, $Z(I) = \{Z(f) : f \in I\}$. If J is a z-filter $J_A^{-1} = \{f \in A : Z(f) \in J\}$.

2.18.- **Theorem.** *Let A be a function algebra which satisfies (2). The following assertion are equivalent:*

(a) for each maximal ideal I in A, there exists $p \in \beta X$ such that

$$I = \{f \in A : p \in \overline{Z(f)}^{\beta X}\}.$$

(b) for each maximal ideal I in A, there exists a maximal ideal J in $C(X)$ such that $I \subset J$;

(c) for each maximal ideal I in A, $Z(I)$ is a z-ultrafilter;

(d) A is inverse-closed.

Proof. Since A satisfies (2), for every zero set P in X there exists $f \in A$ such that $Z(f) = P$.

The assertions (a) implies (b) and (b) implies (a) follow directly from the Gelfand-Kolmogorov theorem ([12], 7.3).

(b) implies (c) Fix maximal ideals I and J in A and $C(X)$ respectively, with $I \subset J$. $Z_A^{-1}(Z(J))$ is an ideal in A. Therefore, $I = Z_A^{-1}(Z(J))$. Since $Z(I) = Z(J)$, $Z(I)$ is a z-ultrafilter.

(c) implies (b) Fix a maximal ideal I in A, since $Z(I)$ is a z-ultrafilter $J = \{f \in C(X) : Z(f) \in Z(I)\}$ is a maximal ideal in $C(X)$ containing I.

(c) implies (d) Take $f \in A$ such that $Z(f) = \emptyset$ and set $I = \{gf : g \in A\}$. Since $f \in I$, I can not be an ideal, therefore $I = A$.

(d) implies (c) Fix an ideal I in A. Since A is inverse closed $\emptyset \notin Z(I)$. On the other hand, if $f, g \in I$ and $h \in A$, $Z(f^2 + g^2) = Z(f) \bigcap Z(g)$ and $Z(f) \subset Z(fg) = Z(g)$.

■

3 The sequentially evaluating property

3.1.- A function algebra A on X is called ***sequentially evaluating*** if, for every $\varphi \in Hom(A)$ and each sequence $\{f_n\}$ in A , there exists $x \in X$ such that $\varphi(f_n) = f_n(x)$, for $n = 1, 2, ...$ This property has been intensively studied in [2]. As far as we know the use of this property goes back to S. Mazur (see the note to statement A of [8]). If a function algebra A on X has the sequentially evaluating property, then every homomorphism on A is sequentially continuous on A_p, where A_p is the algebra A endowed with the pointwise convergence topology. This fact was noticed for some particular algebras in [2] and [6].

3.2.- Denote by $[A \bigcup C(X_A)]$ the closed under bounded inversion algebra on X generated by A and $C(X_A)$. By setting

$$A_1 := \{\sum_{k=1}^{n} f_k g_k : f_k \in A, g_k \in C(X_A), n \in \mathbf{N}\},$$

we have that $[A \bigcup C(X_A)] = \{h_1/h_2 : h_1, h_2 \in A_1, h_2 \geq 1\}$.

3.3.- **Theorem.** *Let A be a single-set evaluating algebra on X. The following conditions are equivalent:*

(a) A has the sequentially evaluating property.

(b) Each zero set in $X_A \setminus X$ does not meet $I(A)$.

(c) $[A \bigcup C(X_A)]$ is single-set evaluating.

Proof. Suppose that (a) holds and (b) fails, then there exists a zero set $P \subset X_A \setminus X$ such that $P \bigcap I(A) \neq \emptyset$. Fix $q \in P \bigcap I(A)$ and let φ be the evaluation at q. Since P is a zero set, there exists $f \in C(X_A)$ such that $P = Z(f)$. Since $\hat{A}$ is dense in $C(X_A)$ for the uniform norm, there exists $\{f_n\}$ in A_b, with $\hat{f}_n \to f$ uniformly on X_A. We have that $\varphi(f_n) = \hat{f}_n(q) \to f(q) = 0$. Set $g_n = f_n - \varphi(f_n) \in A_b$. According to the above arguments $\hat{g}_n \to f$ uniformly on X_A and $\varphi(g_n) = 0$. By the sequentially evaluating property there exists $x_0 \in X$ such that $\varphi(g_n) = g_n(x_0) = 0$. This says that $\lim_n g_n(x_0) = f(x_0) = 0$ and we have a contradiction.

(b) implies (c) Suppose that (b) holds and let φ be a homomorphism on $[A \bigcup C(X_A)]$. We will prove that for each $h \in [A \bigcup C(X_A)]$,

$Z(h - \varphi(h)) \neq \emptyset$. Since φ is a homomorphism on A $(C(X_A))$, there exists $p \in I(A)$ $(q \in C(X_A))$ such that, for each $f \in A$ $(g \in C(X_A))$ $\varphi(f) = \hat{f}(p)$ $(\varphi(g) = \hat{g}(q))$. Since $A_b \subset A \bigcap C(X_A)$, for each $f \in A_b$, $\hat{f}(p) = \hat{f}(q)$. Taking into account that $\hat{A}$ separates points in X_A, we have that $p = q$. Now if $f \in (A \bigcup C(X_A))$, set $g_f = f - \varphi(f)$. If $Z(g) \bigcap X = \emptyset$, then $Z(g) \bigcap I(A) = \emptyset$ and this is not posible $(p \in Z(g) \bigcap I(A))$.

Since for every $f \in A$, $\frac{(f-\varphi(f))^2}{1+(f-\varphi(f))^2}$ has a continuous extension to X_A, we have that for any $h \in A_1$ (see 3.2), $Z(h - \varphi(h)) \neq \emptyset$. In fact, if $f_1, ..., f_n \in A$ and $g_1, ..., g_n \in C(X_A)$,

$$\emptyset \neq Z(\sum_{k=1}^{n} \frac{(f_k - \varphi(f_k))^2}{1 + (f_k - \varphi(f_k))^2} + (g_k - \varphi(g_k))^2)$$

$$\subset Z(\sum_{k=1}^{n}(f_k - \varphi(f_k))g_k + \varphi(f_k)(g_k - \varphi(g_k)))$$

$$= Z(\sum_{k=1}^{n} f_k g_k - \varphi(\sum_{k=1}^{n} f_k g_k))$$

Now if $h_1, h_2 \in A_1$ with $h_2 \geq 1$, then

$$Z(\frac{h_1}{h_2} - \varphi(\frac{h_1}{h_2})) = Z(\varphi(h_2)h_1 - \varphi(h_1)h_2)$$

$$= Z(\varphi(h_2)h_1 - \varphi(h_1)h_2 - \varphi(\varphi(h_2)h_1 - \varphi(h_1)h_2)) \neq \emptyset.$$

(c) implies (a) Suppose that $[A \bigcup C(X_A)]$ is single-set evaluating. Fix $\psi \in Hom(A)$. There exists $p \in I(A)$ such that, for each $f \in A$, $\psi(f) = \hat{f}(p)$. Let us prove that ψ may be extended to a homomorphism φ on $[A \bigcup C(X_A)]$. It is sufficient to prove that every function $h \in [A \bigcup C(X_A)]$ has a (unique) continuous extension to p.

Suppose first that $h = \sum_{k=1}^{n} f_k g_k$, with $f_k \in A$ and $g_k \in C(X_A)$, for $k = 1, 2, ..., n$. Set $\hat{h}(p) = \sum_{k=1}^{n} \hat{f}_k(p)\hat{g}_k(p)$. We have that, for any net $\{x_\lambda\}_{\lambda \in \Lambda}$ in X, such that $x_\lambda \to p$ in X_A,

$$\lim_{\lambda} h(x_\lambda) = \sum_{k=1}^{n} \lim_{\lambda} f_k(x_\lambda) \lim_{\lambda} g_k(x_\lambda) = \sum_{k=1}^{n} \hat{f}_k(p)\hat{g}_k(p) = \hat{h}(p).$$

Finally, if $h = \frac{h_1}{h_2} \in [A \bigcup C(X_A)]$. with $h_1, h_2 \in A_1$ ($h_2 \geq 1$), set $\hat{h}(p) = \frac{\hat{h}_1(p)}{\hat{h}_2(p)}$. Then, by defining $\varphi(h) = \hat{h}(p)$ for $h \in [A \bigcup C(X_A)]$, we have that $\varphi \in Hom([A \bigcup C(X_A)])$ and $\varphi(f) = \psi(f)$ for $f \in A$.

Now, fix a sequence $\{f_n\}$ in A. Set $g_n(x) = \frac{1}{2^n} \frac{(f_n(x)-\varphi(f_n))^2}{1+(f_n(x)-\varphi(f_n))^2}$ and $g = \sum_{n=1}^{\infty} g_n$. We have that $\hat{g} \in C(X_A)$. Let us prove that $\varphi(g) = 0$. In fact, notice that the sequence $\{\sum_{k=1}^{n} g_k\}$ converges uniformly to g and $\sum_{k=1}^{n} g_k \leq g$. Then, given $\epsilon > 0$ and n such that $\| \sum_{k=1}^{n} g_k - g \|_\infty < \epsilon$, it follows that

$$0 = \varphi(\sum_{k=1}^{n} g_k) \leq \varphi(g) = \varphi(g - \sum_{k=1}^{n} g_k) \leq \epsilon\varphi(1) = \epsilon.$$

Taking into account that $[A \bigcup C(X_A)]$ is single-set evaluating, there exist $x_0 \in X$ such that $0 = \varphi(g) = g(x_0)$. Therefore $\varphi(f_n) = f_n(x_0)$ for each n.

■

3.4.- **Remark.** If A is an inverse-closed algebra on X closed under the uniform convergence, then $[A \bigcup C(X_A)] = A$, and A has the sequential evaluating property. This assertion can be obtained from the result of S. Mazur quoted in [8] and gives a proof of following fact: X need not be A-realcompact when A is a sequentially evaluating algebra on X. For certain class of algebras the sequentially evaluating property implies A-realcompactness (for example if X is a Lindelöf space in the τ_A topology), this just was the main reason for studying this property in [2].

The last proposition in this section can be proved as theorem 2.16.

3.5.- **Proposition.** *If a function algebra A satisfies conditions (a) and (b) in theorem 2.16 then A has the sequentially evaluating property.*

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References

[1] J. Arias de Reyna, A real valued homomorphism on algebras of differentiable functions, *Proc. Amer. Math. Soc.*, 104 (1988) 1054-1058.

[2] P. Biström, S. Bjon and M. Lindström , Function algebras on which homomorphisms are point evaluation on sequences, *Manuscripta Math.*, 73 (1991) 179-185.

[3] P. Biström, S. Bjon and M. Lindström, Remarks on homomorphisms on certain subalgebras of $C(X)$, *Math. Japonica*, 37 (1992) 105-109.

[4] P. Biström, S. Bjon and M. Lindström, Homomorphisms on some functions algebras. *Monasth. Math.* 111 (1991) 93-97.

[5] P. Biström and M. Lindström, Homomorphisms on C^{∞}(E) and C^{∞}-bounding sets, *Monatsh. Math.*, 115 (1993) 257-266.

[6] P. Biström, J. Jaramillo and M. Lindström, Algebras of real analytic functions; Homomorphisms and bounding sets, *Studia Math.* 115 (1995) 23-37.

[7] J. Bustamante G and R. Escobedo C, Maximal ideal space of algebras of functions, J. Austral. Math. Soc. (Series A) 63 (1997), 78-90.

[8] Kim-Peu Chew and M. Mrowka, Structure of continuous functions XI, *Bull. de l'Acad. Polo. des Sci., Serie de sci. math. astr. et phy.*, Vol. XIX. No. 1 (1971) 1023-1026.

[9] R. Engelking, General Topology, Monograf. Math. Warsaw, 1977.

[10] M.I. Garrido, J. Gomez Gil and J.A. Jaramillo, Homomorphisms on functions algebras, *Can. J. Math.*, 46 (1994) 734-745 (see also *Extracta Math.*, 7 (1992) 46-52).

[11] M. I. Garrido and F. Montalvo , On some generalization of the Kakutani-Stone and Stone-Weierstrass theorems, *Acta Math. Hung.*, 62, 3-4 (1993) 199-208.

[12] L. Gillman and M. Jerison, Rings of Continuous Functions, Van Nostrand, New Jersey, 1960.

[13] J. R. Isbell, Algebras of uniformly continuous functions, *Annals of Math.*, Vol. 68, No. 1 (1958) 96-125.

[14] J. A. Jaramillo and J. G. Llavona, On the spectrum of $C_b^1(E)$, *Math. Ann.*, 287 (1990) 531-538.

[15] J. A. Jaramillo Multiplicative functionals on algebras of differentiable functions, *Archiv Math.*, Vol. 58 (1992) 384-387.

[16] A. Kriegl, P. Michor and W. Schachermayer, Characters on algebras of smooths functions, *Ann. Global Anal. Geom.*, Vol. 7, No. 2 (1989) 85-92.

[17] A. Kriegl and P. W. Michor, More smoothly real compact spaces, *Proc. Amer. Math. Soc.* 117, No. 2 (1993) 467-471.

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