REVISTA MATEMÁTICA de la
Universidad Complutense de Madrid
Volumen 8, número 2: 1995
http://dx.doi.org/10.5209/rev_REMA.1995.v8.n2.17653

On the WM Points of Orlicz Function Spaces Endowed with Luxemburg Norm*

Tingfu WANG, Cuixia HAO and Minli LI

ABSTRACT. The concept of WM point is introduced and the criterion of WM property in Orlicz function spaces endowed with Luxemburg norm is given.

It is well known that WM property is an important property in Geometry of Orlicz Spaces. The criteria of WM property have been discussed. [1-3] In this paper, we introduce the concept of WM point and give a criterion of WM point in Orlicz function spaces L_M endowed with Luxemburg norm. Hence, we get easily a sufficient and necessary condition that L_M has WM property.

Let X be a Banach Space, $B(X)$, $S(X)$ be the unit ball and unit sphere of X, respectively. $x \in S(X)$ is called WM point provided $x_n \in B(X)$ and $\|x_n + x\| \to 2$ imply that there exists a supporting functional

1991 Mathematics Subject Classification: 46B30
Servicio publicaciones Univ. Complutense. Madrid, 1995.
*The subject supported by NSFC and NSFH.

f at x such that $f(x_n) \to 1$. If every $x \in S(X)$ is WM point, then we say that X has WM property.

$M(u)$, $N(v)$ denote a pair of complemented N-functions, $p_-(u)$ and $p(u)$ denote the left and right derivatives of $M(u)$, respectively. S_M denote a set of strictly convex points. $[a,b]$ is called a structural affine interval of $M(u)$, if $M(u)$ is linear on $[a,b]$, and for any $\varepsilon > 0$, $M(u)$ is neither linear on $[a-\varepsilon, b]$ nor on $[a, b+\varepsilon]$. We say $M \in \Delta_2$, if $M(2u) \leq KM(u)$ for all large u, where $K > 2$ is a previously constant. $M(u) \in \nabla_2$ means that $N(v) \in \Delta_2$. Suppose that (G, Σ, μ) is a nonatomic finite measure space. For each Σ-measurable function $x(t)$ on G, we denote its mordular by

$$\rho_M(x) = \int_G M(x(t))d\mu.$$

The set

$$L_M = \{x(t):\ \rho_M(\lambda x) < \infty \quad \text{for some } \lambda > 0\}$$

endowed with Luxemburg norm

$$||x|| = \inf\{\lambda > 0,\ \rho_M(x/\lambda) \leq 1\}$$

and Orlicz norm

$$||x||^o = \inf\{k^{-1}[1 + \rho_M(kx)]:\ k > 0\}$$

we denote by L_M, $L_M{}^o$ respectively, and we call them Orlicz Spaces.

It is known that for all $x \neq 0$, there exists $k_x > 0$ such that

$$||x||^o = k_x^{-1}(1 + \rho_M(k_x x)).$$

First we give some auxiliary Lemmas.

Lemma 1. *Any $f \in L_M$ has the unique decomposition*

$$f = v + \phi \quad (v \in L_N{}^o,\ \phi \text{ is a singular functional}),\ i.e.$$

$$f(x) = \int_G x(t)v(t)dt + \phi(x) \qquad (x \in L_M).$$

Proof. See [4].

Lemma 2. *$f = v + \phi$ is a support functional at $x \in L_M \setminus \{0\}$ iff*

(1) $\rho_M(\frac{x}{||x||}) = 1$,

(2) $||\phi|| = \phi(\frac{x}{||x||})$,

(3) $x(t)v(t) \geq 0$ and for any (equivalently for some) $k \in K(v)$,

$$p_-\left(\frac{|x(t)|}{||x||}\right) \leq k|v(t)| \leq p\left(\frac{|x(t)|}{||x||}\right)$$

Proof. See [4].

Lemma 3. *If $M \in \nabla_2$, $[a,b]$ is an affine interval of $M(u)$, and $M(u)$ is neither linear on $[a-\varepsilon, a]$, nor on $[b, b+\varepsilon]$ for any $\varepsilon > 0$, then for any $\varepsilon > 0$, there exists $\delta > 0$ such that for any $v \in [a,b]$,*

$$\frac{M(u)+M(v)}{2} - M\left(\frac{u+v}{2}\right) < \delta \text{ implies } u \in [a-\varepsilon, b+\varepsilon].$$

Proof. See [1].

Lemma 4. *Assume $x \in S(L_M)$. If $\theta(x) = \inf\{c > 0 : \rho_M(x/c) < \infty\} < 1$, then all support functionals of x are in $L_N{}^{\circ}$.*

Proof. See Lemma 2 in [5].

Theorem. *$x \in S(X)$ is WM point if and only if*

(1) there exists $\tau > 0$ such that $\rho_M((1+\tau)x) < \infty$.

(2) $\mu\{t \in G : |x(t)| \in (a,b]\} = 0$ or $M \in \nabla_2$, where $[a,b]$ is an arbitrary structural affine interval of $M(u)$.

(3) For any common end-point of two neighbour affine intervals $c, \mu\{t \in G: \ |x(t)| = c\} = 0$.

Proof. Necessity. Without loss of generality, we assume $x(t) \geq 0$. If (1) is not true, then for any $\varepsilon > 0$, $\rho_M((1+\varepsilon)x) = \infty$. Take $0 < c < d$ such that $\mu E = \mu\{t: \ c \leq x(t) \leq d\} > 0$. Put

$$y = -x|_E + x|_{G\setminus E}$$

Obviously $\|y\| = \|x\| = 1$. For any $\varepsilon > 0$, since $\rho_M\left(\frac{x+y}{2}\right) = \rho_M(x|_{G\setminus E}) < 1$ and $\rho_M\left((1+\varepsilon)\ \frac{x+y}{2}\right) = \rho_M((1+\varepsilon)x|_{G\setminus E}) = \infty$, we deduce $\left\|\frac{x+y}{2}\right\| = 1$ i.e. $\|x+y\| = 2$.

Denote $G_n = \{t \in G: \ |x(t)| \leq n\}$, $x_n(t) = x(t)|_{G_n}$. Clearly, $\|x_n\| \leq 1$, $\|x_n + x\| \to \|2x\| = 2$. We consider the sequence

$$x_1,\ y,\ x_2,\ y,\ x_3,\ y, \cdots.$$

For any support functional $f = v + \phi$ at x, if $\phi(x) \neq 0$, then

$$f(x_n) = \int_G x_n(t)v(t)dt \to \int_G x(t)v(t)dt = f(x) - \phi(x) \neq f(x) = 1$$

and if $\phi(x) = 0$, then by Lemma 2,

$$f(x-y) = \int_E 2x(t)v(t)dt = \frac{1}{k_v}\int_E 2x(t)k_v v(t)dt \geq \frac{1}{k_v}2cp_-(c)\mu E > 0$$

so $f(y) \neq f(x) = 1$ and denoting the sequence by (Z_n), we have $Z_n \in B(L_M)$, $\|x + Z_n\| \to 2$ and $f(Z_n) \not\to 1$ for any support functional at x, i.e. x is not WM point.

If (2) does not hold, then there exists an affine interval $[a,b]$ of $M(u)$, such that $\mu\{t \in G: \ x(t) \in (a,b]\} > 0$ and there is a sequence

(u_n) such that $u_n \uparrow \infty$, and

$$M\left(\frac{u_n}{2}\right) > \left(1 - \frac{1}{n}\right) \frac{M(u_n)}{2} .$$

Take $\varepsilon > 0$ such that $\mu E = \mu \{t \in G : x(t) \in [a + \varepsilon, b]\} > 0$ and measurable sets $E_n \subset E$ satisfying

$$\rho_M(x|_{G \setminus E}) + \int_E M(x(t) - \varepsilon)dt + M(u_n)\mu(E \setminus E_n) = 1 .$$

Then $\mu(E \setminus E_n) \to 0$ and $\mu E_n \to \mu E$. Let

$$y_n = x|_{G \setminus E} + (x(t) - \varepsilon)|_{E_n} + u_n|_{E \setminus E_n} .$$

Then $\|y_n\| = 1$ and

$$\begin{aligned}
\rho_M\left(\frac{x + y_n}{2}\right) &= \rho_M(x|_{G \setminus E}) + \rho_M\left(\frac{x + x - \varepsilon}{2}\, |_{E_n}\right) \\
&\quad + M\left(\frac{u_n}{2}\right)\mu(E \setminus E_n) \\
&> \rho_M(x|_{G \setminus E}) + \frac{1}{2}\rho_M(x|_{E_n}) + \frac{1}{2}\rho_M((x - \varepsilon)|_{E_n}) \\
&\quad + \left(1 - \frac{1}{n}\right) \frac{M(u_n)}{2}\, \mu(E \setminus E_n) \\
&> \frac{1}{2}\left(1 - \frac{1}{n}\right)(\rho_M(y_n) + \rho_M(x|_{G \setminus (E \setminus E_n)})) \to 1
\end{aligned}$$

whence $\|x + y_n\| \to 2$.

Take any support functional $f = v+\phi$ at x. Since $\rho_M((1+\tau)x) < \infty$ for some $\tau > 0$, we have $\theta(x) \le \frac{1}{1+\tau} < 1$. By Lemma 4, we get that all support functionals at x are in $L_N{}^o$. Therefore

$$f(x-y_n) = < x - y_n, v > = \int_{E_n} \varepsilon v(t)dt - \int_{E\setminus E_n} u_n v(t)dt$$

$$+ \int_{E\setminus E_n} x(t)v(t)dt$$

$$\ge \int_{E_n} \varepsilon v(t)dt - \int_{E\setminus E_n} u_n v(t)dt$$

$$\ge \frac{1}{k_v}(\varepsilon p(a)\mu E_n - ||y_n|| p(b) ||\chi_{E\setminus E_n}||_N{}^o)$$

$$\to \frac{1}{k_v}\varepsilon p(a)\mu E > 0.$$

This implies $f(y_n) \ne f(x) = 1$ which gives that x is not a WM point. This is a contradiction.

The proof of (3) is trivial, it is omited.

Sufficiency. Let $x_n \in B(X)$ and $||x_n + x|| \to 2$.

First we will prove that

$$\rho_M(x_n) \to 1,\ \rho_M\left(\frac{x_n + x}{2}\right) \to 1\ .$$

For any $\varepsilon > 0$ and n large enough we have $\left|\left|(1+\varepsilon)\frac{x_n+x}{2}\right|\right| > 1$. Then

$$1 < \rho_M\left((1+\varepsilon)\frac{x_n + x}{2}\right) = \rho_M\left(\frac{1+\varepsilon}{2}x_n + \frac{1-\varepsilon}{2}\frac{1+\varepsilon}{1-\varepsilon}x\right)$$

$$\leq \frac{1+\varepsilon}{2}\rho_M(x_n) + \frac{1-\varepsilon}{2}\rho_M\left(\frac{1+\varepsilon}{1-\varepsilon}x\right)$$

Take ε small enough that $\frac{1+\varepsilon}{1-\varepsilon} < 1+\tau$. By (1), we have

$$1 \leq \frac{1+\varepsilon}{2}\rho_M(x_n) + \frac{1-\varepsilon}{2}(\rho_M(x) + o(\varepsilon)) .$$

Since ε is arbitrary, we get $\rho_M(x_n) \to 1$ immediately.

Similarly, by $\left\|\frac{x_n+x}{2} + x\right\| \to 2$, we can deduce that $\rho_M\left(\frac{x_n+x}{2}\right) \to 1$. We will discuss two cases.

I. For any affine interval $(a,b]$ of $M(u)$, $\mu\{t: \ x(t) \in (a,b]\} = 0$.

First we will prove that $x_n - x \xrightarrow{\mu} 0$.

Denote by $\{a_i\}$ the left endpoints of all structural affine intervals. $G_a = \{t \in G: \ x(t) \in \{a_i\}_{i=1}^{\infty}\}$. If $x_n - x \xrightarrow{\mu} 0$ on $G \setminus G_a$ is not true, then there exists ε, $\sigma > 0$ such that

$$\mu\{t \in G \setminus G_a: \ |x_n(t) - x(t)| \geq \varepsilon\} \geq \sigma .$$

By $1 \leftarrow \rho_M(x_n) \geq M(D)\mu\{t: \ |x_n(t)| > D\}$, we can take D large enough such that

$$\mu\{t: \ |x_n(t)| > D\} < \frac{\sigma}{4},\ \mu\{t: \ |x(t)| > D\} < \frac{\sigma}{4} .$$

Hence

$$\mu\{t \in G \setminus G_a: \ |x_n(t) - x(t)| \geq \varepsilon,\ |x(t)|,\ |x_n(t)| \leq D\} \geq \frac{\sigma}{2} .$$

Let $r_1, r_2, \ldots, r_i, \ldots$, be endpoints of all structural affine intervals of $M(u)$, then $\mu\{t \in G \setminus G_n : x(t) = r_i\} = 0$. So there exists a neighbourhood V_i of r_i small enough such that $\mu\{t \in G \setminus G_a : x(t) \in V_i\} < \frac{\sigma}{4 \cdot 2^i}$. Therefore

$$\mu\{t : x(t) \in \bigcup_{i=1}^{\infty} V_i\} < \frac{\sigma}{4} .$$

Thus $\mu G_n > \frac{\sigma}{4}$ where

$$G_n = \{t \in G \setminus G_a : |x_n(t) - x(t)| \geq \varepsilon, \, |x(t)|, \, |x_n(t)| \leq D,$$

$$x(t) \in S_M \setminus \bigcup_{i=1}^{\infty} V_i\} .$$

It is easy to know that there exists $\delta, 0 < \delta < 1$, such that for any $t \in G_n$

$$M\Big(\frac{x_n(t) + x(t)}{2}\Big) \leq (1 - \delta)\Big(\frac{M(x_n(t)) + M(x(t))}{2}\Big)$$

We have a contradiction

$$0 \leftarrow \Big(\frac{\rho_M(x_n) + \rho_M(x)}{2} - \rho_M\Big(\frac{x_n(t) + x(t)}{2}\Big)$$

$$= \int_G \Big(\frac{M(x_n(t)) + M(x(t))}{2} - M\Big(\frac{x_n(t) + x(t)}{2}\Big)\Big)dt$$

$$\geq \int_{G_n} \Big(\frac{M(x_n(t)) + M(x(t))}{2} - M\Big(\frac{x_n(t) + x(t)}{2}\Big)\Big)dt$$

$$\geq \delta \int_{G_n} \frac{M(x_n(t)) + M(x(t))}{2} dt$$

$$\geq \frac{\delta}{2} M\left(\frac{\varepsilon}{2}\right)\frac{\sigma}{4} .$$

Using the same method as above, we get for any $\varepsilon > 0$,

$$\mu\{t \in G_a : \ x_n(t) \leq x(t) - \varepsilon\} \to 0 .$$

Combining this with $x_n - x \xrightarrow{\mu} 0$ on $G \setminus G_a$, we deduce that for any $E \subset G$.

$$\lim_{n\to\infty} \inf \rho_M(x_n|_E) \geq \rho_M(x|_E) . \tag{1}$$

If $x_n - x \xrightarrow{\mu} 0$ on G_a is not true, then there exists positive numbers a, ε and σ such that $\mu E_a = \mu\{t \in G_a : \ x_n(t) - x(t) = x_n(t) - a \geq \varepsilon\} \geq \sigma$. Thus we can deduce easily

$$\lim_{n\to\infty} \inf \rho_M(x_n|_{E_a}) > \rho_M(x|_{E_a}) .$$

Combining this with (1), we get a contradiction

$$1 = \lim_{n\to\infty} \rho_M(x_n) > \rho_M(x) = 1 .$$

This finishes the proof that $x_n - x \xrightarrow{\mu} 0$.

Take any support functional f at x. By the assumption (1), we have $f = v \in S(L_N{}^o)$. Now we will prove $< x_n, v > \to 0$ or $< x_n - x, v > \to 0$.

For any $\varepsilon > 0$, there exists $\delta > 0$ such that for any $e \subset G$, $\mu e < \delta$ implies $\rho_N(v|_e) < \varepsilon$ and $\rho_M(x|_e) < \varepsilon$. Since $x_n - x \xrightarrow{\mu} 0$, there exists $F \subset G$ with $\mu(G \setminus F) < \delta$ such that $x_n - x \to 0$ uniformly on F.

So there exists n_0, $|x_n(t) - x(t)| < \varepsilon$ $(t \in F)$, and

$$\left| \int_F (M(x_n(t)) - M(x(t)))dt \right| < \varepsilon \qquad \text{as } n \geq n_0 .$$

Noticing $\rho_M(x_n) \to 1 = \rho_M(x)$, we have $\left|\int_{G\setminus F}(M(x_n(t)-M(x(t))dt\right| <$ ε for n large enough. So

$$\int_{G\setminus F} M(x_n(t))dt < \int_{G\setminus F} M(x(t))dt + \varepsilon < 2\varepsilon \text{ for } n \text{ large enough.}$$

Thus

$$< x_n - x, v > = \int_F (x_n(t) - x(t))v(t)dt + \int_{G\setminus F}(x_n(t) - x(t))v(t)dt$$

$$\leq \varepsilon||1|| + \rho_M(x_n|_{G\setminus F}) + \rho_M(x|_{G\setminus F}) + 2\rho_N(v|_{G\setminus F})$$

$$= 0(\varepsilon) .$$

By the arbitrariness of ε we get $< x_n - x, v >\to 0$

II. $M \in \nabla_2$, First we will prove that

$$\lim_{\mu e\to 0} \sup_n \rho_M(x_n|_e) = 0 . \tag{2}$$

If it is not true, then there exists $\varepsilon > 0$ and e_n, $\mu e_n \downarrow 0$ with $\rho_M(x_n|e_n) \geq \varepsilon$.

Without loss of generality, we may assume $|x_n(t)| \geq u_0 > 0$ $(t \in e_n)$.

Let $1 + \tau = \frac{1}{1-\tau'}$ $(\tau' > 0)$. Since $M \in \nabla_2$, there exists $\delta > 0$ such that $M\left(\frac{u}{1+\tau'}\right) \leq (1 - \delta)\frac{M(u)}{1+\tau'}$ $(u \geq u_0)$.

By $\rho_M\left(\frac{x}{1-\tau'}\right) < \infty$, we have $\rho_M\left(\frac{x}{1-\tau'}, |_{en}\right) < \frac{\delta\varepsilon}{2}$ as n is large enough. Hence

$$\int_{e_n} M\left(\frac{x_n(t)+x(t)}{2}\right)dt = \int_{e_n} M\left(\frac{1+\tau'}{2}\frac{x_n(t)}{1+\tau'} + \frac{1-\tau'}{2}\frac{x(t)}{1-\tau'}\right)dt$$

$$\leq \frac{1+\tau'}{2}\int_{e_n} M\left(\frac{x_n(t)}{1+\tau'}\right)dt + \frac{1-\tau'}{2}\int_{e_n} M\left(\frac{x(t)}{1-\tau'}\right)dt$$

$$\leq \frac{1+\tau'}{2}(1-\delta)\frac{1}{1+\tau'}\int_{e_n} M(x_n(t))dt + \frac{\delta\varepsilon}{4}$$

$$\leq \frac{1}{2}\int_{e_n} M(x_n(t)dt - \frac{\delta\varepsilon}{2} + \frac{\delta\varepsilon}{4}$$

$$= \frac{1}{2}\int_{e_n} M(x_n(t))dt - \frac{\delta\varepsilon}{4}.$$

Therefore, we get a contradiction

$$1 \leftarrow \rho_M\left(\frac{x_n+x}{2}\right) = \rho_M\left(\frac{x_n+x}{2}|_{G\setminus e_n}\right) + \rho_M\left(\frac{x_n+x}{2}|_{e_n}\right)$$

$$\leq \frac{\rho_M(x_n|_{G\setminus e_n}) + \rho_M(x|_{G\setminus e_n})}{2} + \frac{1}{2}\rho_M(x_n|_{e_n}) - \frac{\delta\varepsilon}{4}$$

$$\to 1 - \frac{\delta\varepsilon}{4}\ .$$

Denote by $\{b_i\}$ the right endpoints of the structural affine intervals. Put $v(t) = \frac{1}{k}u(t)$, where

$$u(t) = \begin{cases} p_-(x(t)) & x(t) \in \{b_i\}_{i=1}^{\infty} \\ p(x(t)) & \text{otherwise} \end{cases} \text{ and } k = \|u\|_N^\circ$$

It is easy to prove $k_v = k$. By Lemma 2, we know $v \in S(L_{N^o})$ is a support functional of x.

Now we prove that $< x_n, v > \to 1$.

Let $\{[a_i, b_i]\}_i$ be all structural affine intervals of $M(u)$, and $E_i = \{t \in G : x(t) \in [a_i, b_i]\}$.

By assumption (3) $E_i \cap E_j = \phi$ $(i \neq j)$.

For any $\varepsilon > 0$, there exists d, such that $e \subset G$, $\mu e < d$ imply

$$\rho_M(x|_e) < \varepsilon, \ \rho_N(kv|_e) < \varepsilon \quad \text{and } \rho_M(x_n|_e) < \varepsilon \ .$$

Recall that the last inequality holds uniformly with respect to n by $M \in \nabla_2$. Since $\sum_{i=1}^{\infty} \mu E_i \leq \mu G < \infty$, there exists m such that $\mu(\bigcup_{i>m} E_i) < d$.
For each $i \leq m$ and all $u \in [a_i, b_i]$, we have

$$up(a_i) = M(u) + N(p(a_i)) \ .$$

Therefore, we can find $\beta > 0$ such that if $u \in [a_i - \beta, b_i + \beta]$, then

$$up(a_i) > M(u) + N(p(a_i)) - \varepsilon \quad (i = 1, 2, \ldots, m) \tag{3}$$

By Lemma 3, there exists $\delta > 0$ such that

$$\frac{M(u) + M(w)}{2} - \frac{M(u+w)}{2} < \delta \quad \text{and } w \in [a_i, b_i]$$

imply

$$u \in [a_i - \beta, b_i + \beta] \quad (i = 1, 2, \ldots, m) \tag{4}$$

Since $\int_G \left(\frac{M(x_n(t)) + M(x(t))}{2} - M\left(\frac{x_n(t)+x(t)}{2}\right)\right) dt \to 0$, we get

$$\frac{M(x_n(t)) + M(x(t))}{2} - M\left(\frac{x_n(t) + x(t)}{2}\right) \xrightarrow{\mu} 0 \tag{5}$$

Denote $F_n = \left\{t \in \bigcup_{i=1}^{m} E_i : \frac{M(x_n(t))+M(x(t))}{2} - M\left(\frac{x_n(t)+x(t)}{2}\right) \geq \delta\right\}$, by (5), we have $\mu F_n < d$ for n large enough. For $t \in \bigcup_{i=1}^{m} E_i \setminus F_n$, we have $x(t) \in \bigcup_{i=1}^{m} [a_i, b_i]$ and

$$\frac{M(x_n(t)) + M(x(t))}{2} - M\left(\frac{x_n(t) + x(t)}{2}\right) < \delta .$$

By (4) $x_n(t) \in [a_i - \beta, b_i + \beta]$, and by (3) we get

$$x_n(t)p(a_i) > M(x_n(t)) + N(p(a_i)) - \varepsilon .$$

Notice that $kv(t) = u(t) = p(a_i)$, so

$$x_n(t)kv(t) > M(x_n(t)) + N(kv(t)) - \varepsilon \quad \left(t \in \bigcup_{i=1}^{m} E_i \setminus F_n\right) \tag{6}$$

Denote $E_0 = G \setminus \bigcup_{i=1}^{\infty} E_i$. Using the same method as in the case I, we get $x_n - x \xrightarrow{\mu} 0$ on E_0. Thus, there exists $F_0 \subset E_0$, $\mu F_0 < d$ such that

$$|x_n(t) - x(t)| < \varepsilon,\ |M(x_n(t)) - M(x(t))| < \varepsilon \text{ uniformly on } E_0 \setminus F_0$$

for n large enough. Hence

$$\begin{aligned} x_n(t)kv(t) &> (x(t) - \varepsilon)kv(t) = M(x(t)) + N(kv(t)) - \varepsilon kv(t) \\ &> M(x_n(t)) + N(kv(t)) - \varepsilon - \varepsilon kv(t) \quad (t \in E_0 \setminus F_0) \end{aligned} \tag{7}$$

It follows from (6) and (7) that

$$< x_n, kv > = \int_{\bigcup\limits_{i=1}^{m} E_i \setminus F_n} + \int_{E_0 \setminus F_0} + \int_{F_n} + \int_{\bigcup\limits_{i>m} E_i} + \int_{F_0} x_n(t)kv(t)dt$$

$$> \int_{\bigcup\limits_{i=1}^{m} E_i \setminus F_n} (M(x_n(t)) + N(kv(t) - \varepsilon))dt$$

$$+ \int_{E_0 \setminus F_0} (M(x_n(t)) + N(kv(t)) - \varepsilon - \varepsilon kv(t))dt$$

$$- \int_{F_n} - \int_{\bigcup\limits_{i>m} E_i} - \int_{F_0} (M(x_n(t)) + N(kv(t)))dt$$

$$> \int_{(\bigcup\limits_{i=1}^{m} E_i \setminus F_n) \cup E_0 \setminus F_0} (M(x_n(t)) + N(kv(t)))dt$$

$$- \varepsilon\mu G - \varepsilon k||1|| - 6\varepsilon$$

$$= \int_G (M(x_n(t)) + N(kv(t))dt$$

$$- \int_{\bigcup\limits_{i>m} E_i \cup F_n \cup F_0} (M(x_n(t)) + N(kv(t))dt$$

$$- \varepsilon\mu G - \varepsilon k||1|| - 6\varepsilon$$

$$\to 1 + \rho_N(kv) - o(\varepsilon) = k||v||_{N^o} - o(\varepsilon) = k - o(\varepsilon).$$

This shows that $< x_n, v > \to 1$.

Corollary L_M has WM property if and only if

(1) $M \in \Delta_2$

(2) $M \in SC$ or $M \in \nabla_2$

(3) There are not two neighbour affine intervals of $M(u)$.

Proof. Sufficiency. Take any $x \in S(L_M)$. Since $M \in \Delta_2$, $L_M = E_M$. So for any $\tau > 0$, $\rho_M((1+\tau)x) < \infty$.

$M \in SC$ implies $M(u)$ has no any affine interval i.e.

$\{t \in G: \ x(t) \in (a,b]\} = \Phi$, then $\mu\{t \in G: \ x(t) \in (a,b]\} = 0$

By Theorem, we get x is WM point.

If $M \in \nabla_2$ combining $\rho_M((1+\tau)x) < \infty$ and by Theorem, we get that x is WM point. So L_M has WM property.

Necessity. If $M \notin \Delta_2$, we may construct $x \in S(L_M)$ with $\rho_M((1+\tau)x) = \infty$, for any $\tau > 0$. By Theorem, x is not WM point.

If $M \notin SC$, and $M \notin \nabla_2$. Then $M(u)$ has an affine interval $[a,b]$. Take d large enough, and measurable $E, F \subset G$, $E \cap F = \Phi$ such that

$$M(b)\mu E + M(d)\mu F = 1,$$

Put $x = b|_E + d|_F$. Then $x \in S(L_M)$ but $\mu\{t \in G: \ x(t) \in (a,b]\} \geq \mu E > 0$.

By Theorem, x is not WM point.

If $[a,c]$, $[c,b]$ are the two neighbour structural affine intervals of $M(u)$, take d large enough, $E, F \subset G$, $E \cap F = \Phi$ satisfying

$$M(c)\mu E + M(d)\mu F = 1 \ .$$

Let $x = c|_E + d|_F$. Then x is not WM point.

References

[1] Chen Shutao and Duan Yanzheng, *WM property of Orlicz spaces*, Northeastern Math. J. 8(4), (1992), 498-502.

[2] Chen Shutao, Lu Yanming and Wang Baoxiang, *WM property and LKR of Orlicz sequence spaces*, Fasciculi, Math. 4(1991), 19-25.

[3] Duan Yanzheng, *Several Structural Problems of Orlicz spaces,* Dissertation of Ph.D, HIT. 1990.

[4] Wu C. and Wang T., *Orlicz spaces and its applications.* Heilongjiang Sci & Tech. press, 1983, Harbin.

[5] Grzaslewicz, R. and Hudzik, H., *Smooth points of Orlicz spaces equipped with Luxemburg norm.* Math. Nachr. 155(1992), 31-45.

[6] Shutao Chen, Huizik, H. and Anna Kaminska, *Support functionals and smooth points in Orlicz function spaces equipped with the Orlicz norm*, Math. Japonica. 39(2)(1994), 271-279.

Wang Tingfu, Hao Cuixia, *Li Minli*
Harbin. Univ. Sci. Tech. Box 610.
Harbin 150080, P.R. China.

Recibido: 11 de Mayo de 1994
Revisado:9 de Enero de 1995