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Pointwise Convergent Nets of Holomorphic Automorphisms of the Unit Ball of Cartan Factors

J.M. ISIDRO* and L. STACHÓ

ABSTRACT. A classical result due to H. Cartan states that if a sequence (h_n) of holomorphic authomorphisms of the unit disk Δ of $\mathbb{C}$ is pointwise convergent on Δ to a limit h and $(h_n(0))$ is bounded away from the boundary $\partial(\Delta)$, then h is a holomorphic automorphism of Δ. The analogous result for the open unit ball D of a complex Banach space E is not true in general. Here we consider pointwise convergent nets (h_i) of holomorphic automorphisms of the unit balls of those Banach spaces known as special Cartan factors and establish a sufficient condition for the pointwise limit $h = \lim_i h_i$ to be a holomorphic automorphism of D.

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0. INTRODUCTION

Consider the open unit disk Δ of the complex plane $\mathbb{C}$ and denote by G the group of all holomorphic automorphisms of Δ. Each h in G admits a decomposition $h = g_a \circ u_h$ where u_h is a linear isometry of $\mathbb{C}$, $a = h(0)$ and $g_a \in G$ satisfies $g_a(0) = a$. It is a classical result that if a sequence $(h_n : n \in \mathbb{N})$ in G is pointwise convergent on Δ to a limit $h : \Delta \to \mathbb{C}$ and $(h_n(0) : n \in \mathbb{N})$ is bounded away from the boundary $\mathbb{T}$ of Δ, then h belongs to G and h_n tends to h uniformly on compact subsets of Δ. We remark that the restriction on $(h_n(0) : n \in \mathbb{N})$ can not be dropped; however, there is no restriction on the *linear components* u_n *of the* h_n, $n \in \mathbb{N}$, i.e., *once the condition* $\lim_n h_n(0) \notin \mathbb{T}$ *is fulfilled* we have $h \in G$ and h_n tends to h in the topology of G.

A natural generalization of Δ to higher (and even infinite) dimensions are the bounded symmetric domains in their standard realizations. These are precisely the open unit balls D of those complex Banach spaces E for which the group G of all holomorphic automorphisms acts transitively on D. It is therefore reasonable to ask whether the above stated result still holds in the new situation.

Even in infinite dimensions, G is a topological group when endowed with the topology of (local) uniform convergence on D, and every holomorphic automorphism $h \in G$ admits a canonical decomposition $h = g_{h(0)} \circ u_h$ where $g_{h(0)} \in G$, $g_z(0) = z$ $(z \in D)$, the mapping $D \to G$ given by $z \mapsto g_z$ is continuous and u_h is suitable linear E-unitary operator. We shall see (Section 4) that the non-linear part $g_{h(0)}$ can easily be ruled out.

On the other hand, in infinite dimensions even the simplest case provides new features. Consider a Hilbert space H (a factor of type 1) with an orthonormal basis $\{e_1, e_2, \ldots\}$. The sequence $U_1, U_2, \ldots$ of reflections in H acting on the basis as $U_n(e_k) := (-1)^{1+\delta_{nk}} e_k$ $(n, k = 1, 2, \ldots)$ converges pointwise but not (locally) uniformly on H. Moreover there are pointwise convergent sequences of H-unitary operators whose inverses diverge. A typical example is given by $V_n(e_k) := e_{\tau_n(k)}$ $(n, k = 1, 2, \ldots)$ where τ_n denotes the cyclic permutation of the first n indices (i.e. $\tau(k) := k + 1$ for $k < n$, $\tau(n) := 1$ and $\tau(k) := k$ if $k > n$). The pointwise limit of the V_n is a unilateral shift of H, which is not an H-unitary operator.

Our considerations in this work are inspired by these two examples. Infinite dimensional Cartan factors can be represented as spaces of linear operators acting between Hilbert spaces. Their atoms are operators of rank at most two in these representations, and we may control the effect of pointwise convergent nets of linear automorphisms by investigating some attached nets of Hilbert space unitary operators. Hence we achieve relevant information concerning pointwise convergent nets of holomorphic automorphism in Cartan factors and in spaces of Cartan factor-valued continuous functions. In particular, we establish a sufficient condition for the pointwise limit $h = \lim_\alpha h_\alpha$ of a net in G to be a holomorphic automorphism of D.

Notation and background material Let D be a bounded domain in a complex Banach space E. A function $f : D \to E$ is called *holomorphic* if for every $a \in D$ the Fréchet derivative $f'(a) \in \mathcal{L}(E)$ exists. A *holomorphic automorphism* of D is a bijection $h : D \to D$ such that h and h^{-1} are holomorphic, and $G := \mathbf{Aut}(D)$ denotes the group of all holomorphic automorphisms of D endowed with the topology of local uniform convergence. Then D is called *symmetric* if to every $a \in D$ there is an $s = s_a \in G$ with $s^2 = \mathbf{id}$ having a as isolated fixed point. It is known that G acts transitively on D, and that D is biholomorphically equivalent to the open unit ball of E when renormed adequately (see [9] for details).

A complex Banach space E is called a *JB^*-triple* if the open unit ball $D \subset E$ is symmetric, or equivalently if the automorphism group $G = \mathbf{Aut}(D)$ acts transitively on D. Then there exists a uniquely determined continuous ternary operation (called the *Jordan triple product* on E) $(x, y, z) \mapsto \{xyz\}$ from E^3 to E such that, by writing $x \,\square y$ for the linear operator $z \mapsto \{xyz\}$ on E, the following axioms are satisfied

(J_1) $\{xyz\}$ is symmetric bilinear in the outer variables x, z and conjugate linear in the inner variable y

(J_2) $[x \,\square x, y \,\square y] = \{xxy\} \,\square y + y \,\square \{yxx\}$

(J_3) $x \,\square x$ is hermitian and has spectrum ≥ 0

(J_4) $||\{xxx\}|| = ||x||^3$

for all $x, y \in E$ and $[,]$ being the commutator product of linear operators.

On the other hand, every complex Banach space E admitting a continuous mapping $\{,,\}$ with (J_1)-(J_4) is a JB^*-triple. The notion of *triple automorphism* can be introduced in the natural way, and the group $\mathbf{Aut}(E)$ of all triple automorphisms of E coincides with the group of all surjective linear isometries of E. An element $e \in E$ is called a *tripotent* if $\{eee\} = e$ and a tripotent e is called an *atom* in E if $\{eEe\} = \mathbb{C}e$. The set $\mathbf{at}(E)$ of all atoms is closed in E and, except for $e = 0$, is contained in ∂D. The following examples of JB^*-triples are known as *Cartan factors* of type k, $(k = 1,2,3,4)$:

Type 1: Are the spaces $\mathcal{L}(H,K)$ of all bounded linear operators $x : H \to K$ where H and K are complex Hilbert spaces.

Types 2,3: Let H be a complex Hilbert space with a conjugation $^-$ and let ^{t} be the induced transposition on $\mathcal{L}(H)$; for $\varepsilon = 1$ and $\varepsilon = -1$ the spaces $\{x \in \mathcal{L}(H) \mid x^t = \varepsilon x\}$ are called Cartan factors of types 2 and 3, respectively.

Type 4: Also called spin factors, are defined as any norm closed selfadjoint complex subspace $\mathcal{U} \subset \mathcal{L}(H)$ such that $\{x^2 \mid x \in \mathcal{U}\} \subset \mathbb{C}1_H$ and $\dim \mathcal{U} > 2$.

In all these cases the triple product is defined by $\{xyz\} := (xy^*z + zy^*x)/2$, where y^* denotes the adjoint of the operator y. If $\mathcal{U}$ is a spin factor, then for every pair $a,b \in \mathcal{U}$ we have $ab^* + b^*a = \langle a|b\rangle 1_H$ for some $\langle a|b\rangle \in \mathbb{C}$, and $\langle .|.\rangle$ is an inner product in $\mathcal{U}$ whose norm $|||\cdot|||$ is equivalent to the operator norm $||\cdot||$. We refer to $\mathcal{H} := (\mathcal{U}, |||\cdot|||)$ as the *Hilbert space associated* to $\mathcal{U}$.

Besides these *special* Cartan factors, there are two *exceptional* Cartan factors which are finite dimensional spaces (see [11]). If Ω is a locally compact Hausdorff space and E is a JB^*-triple, then $C_0(\Omega, E)$, the space of continuous E-valued functions that vanish at infinity with the pointwise triple product and the norm of the supremum, is also a JB^*-triple.

For a special Cartan factor E, we shall need the characterization of its atoms that is given in [4] and the representation of its surjective linear isometries given in [10] for $k = 1$, in [5] for $k = 2,3$ and in [6] for $k = 4$.

The group of surjective linear isometries (or unitary operators) on a Banach space E will be denoted by $\mathcal{U}(E)$. Whenever $(x_i, i \in I)$ is a net in E and $\lim_i x_i = x$ holds for some $x \in E$ with respect to the *norm topology*, we shall say that (x_i) converges in E and write $x_i \to x$ or $x_i = x + o(1)$. We shall also write $x_i \to x$ and $x_i = x + o(1)$ if $\lim_i x_i = x$ holds with respect to some other topology τ on E, but then an explicit reference to τ will be made. In particular, the abbreviations SOT and SSOT stand respectively for the strong operator and the strong star operator topologies on $\mathcal{L}(H, K)$.

1. SOME PRELIMINARY RESULTS

1.1 Lemma. *Let H and K be the Hilbert spaces and suppose that (e_i) and (f_i), $i \in I$, are nets of unit vectors in K and H, respectively, such that the operators $e_i \otimes f_i^* := \langle \cdot, f_i \rangle e_i$ converge in the SOT of $\mathcal{L}(H, K)$. Then (f_i) converges in H if and only if (e_i) converges in K.*

Proof. If (f_i) converges in H then we have

$$||e_i - e_j|| = ||(e_i - e_j) \otimes f_i^*|| = ||e_i \otimes f_i^* - e_j \otimes f_j^* + e_j \otimes (f_j - f_i)^*||$$

$$\leq ||e_i \otimes f_i^* - e_j \otimes f_j^*|| + ||f_j - f_i|| \to 0.$$

Thus (e_i) is a Cauchy net. The converse argument is similar. ■

1.2 Proposition. *Let H and K be Hilbert spaces and let (e_i) and (f_i), $i \in I$, be nets of unit vectors in K and H, respectively, such that the operators $(e_i \otimes f_i^*)$ converge in $\mathcal{L}(H, K)$. Then there is a net $(\alpha_i : i \in I)$ in $\mathbb{T}$ such that $(\alpha_i e_i)$ and $(\alpha_i f_i)$ converge in K and H respectively.*

Proof. Since $\mathbf{at}[\mathcal{L}(H, K)]$ is closed we can find unit vectors $e \in K$ and $f \in H$ such that

$$e_i \otimes f_i^* \to e \otimes f^*$$

holds in $\mathcal{L}(H, K)$. By writting $\langle \cdot, \cdot \rangle$ for the scalar product both in H and K we have

$$\langle e_i, e \rangle \langle f, f_i \rangle = \langle (e_i \otimes f_i^*) f, e \rangle \to$$

$$\langle (e \otimes f^*) f, e \rangle = \langle e, e \rangle \langle f, f \rangle = 1$$

Therefore the definition $\alpha_i := \langle e, e_i\rangle / |\langle e, e_i\rangle|$ makes sense for $i \in I$, $i \geq i_0$, and

$$||\alpha_i e_i - e||^2 = ||\alpha_i e_i||^2 + ||e||^2 - 2\mathcal{R}e\langle \alpha_i e_i, e\rangle$$

$$= 2\Big(1 - \mathcal{R}e\,\frac{\langle e, e_i\rangle\langle e_i, e\rangle}{|\langle e, e_i\rangle|}\Big) = 2(1 - |\langle e, e_i\rangle|) \to 0.$$

Thus $(\alpha_i e_i)$ converges in K. Since $e_i \otimes f_i^* = (\alpha_i e_i) \otimes (\alpha_i f_i)^*$ for $i \in I$, by (1.1) the net $(\alpha_i f_i)$ converges in H. ∎

1.3 Corollary. *Let H be a Hilbert space and $^-$ a conjugation on H. If $(e_i :\ i \in I)$ is a net of unit vectors in H such that the operators $e_i \otimes \bar{e}_i^*$ converge in $\mathcal{L}(H)$, then there is a net $(\varepsilon_i :\ i \in I)$ in $\{-1, 1\}$ such that $(\varepsilon_i e_i)$ converges in H.*

Proof. By (1.2) we can find a net (α_i) in $\mathbb{T}$ such that $(\alpha_i e_i)$ and $(\alpha_i \bar{e}_i)$ converge in H. By conjugation also $(\overline{\alpha_i e_i})$ converges in H, and so $e_i \otimes \bar{e}_i^*$ and $\alpha_i e_i \otimes \overline{\alpha_i e_i^*} = \alpha_i^2 e_i \otimes \bar{e}_i^*$ converge in $\mathcal{L}(H)$. Therefore $\alpha_i^2 \to \alpha$ for some $\alpha \in \mathbb{T}$. Fix a determination of $\sqrt{\alpha}$. Then $\varepsilon_i := \operatorname{sgn}(\mathcal{R}e(\alpha_i/\sqrt{\alpha}))$, which is well-defined for $i \geq i_0$, suits our requirements. ∎

1.4 Proposition. *Let H be a Hilbert space and $^-$ a conjugation on H. Let (e_i) and (f_i) be nets of unit vectors in H such that $e_i \perp f_i$, $i \in I$, and the operators $e_i \otimes \bar{f}_i^* - f_i \otimes \bar{e}_i^*$ converge in $\mathcal{L}(H)$. Then there are nets (α_i) and (β_i) in $\mathbb{C}$ such that $|\alpha_i|^2 + |\beta_i|^2 = 1$, $i \in I$, and both $\alpha_i e_i + \beta_i f_i$ and $-\bar{\beta}_i e_i + \bar{\alpha}_i f_i$ converge in H.*

Proof. Since $\mathbf{at}[\mathcal{L}(H)]$ is closed in $\mathcal{L}(H)$, there exists an orthonormal frame $\{e, f\}$ in H with

$$e_i \otimes \bar{f}_i^* - f_i \otimes \bar{e}_i^* \to e \otimes \bar{f}^* - f \otimes \bar{e}^* \qquad \text{in } \mathcal{L}(H).$$

Set $\lambda_i := \langle e, e_i\rangle$, $\mu_i := \langle e, f_i\rangle$ and $g_i := \lambda_i e_i + \mu_i f_i$, $h_i := -\bar{\mu}_i e_i + \bar{\lambda}_i f_i$ for all $i \in I$. Then

$$g_i \otimes \bar{h}_i^* - h_i \otimes \bar{g}_i^* = (|\lambda_i|^2 + |\mu_i|^2)(e_i \otimes \bar{f}_i^* - f_i \otimes \bar{e}_i^*).$$

By the reflexivity of H we can choose a subnet $(i_k : k \in \mathcal{K})$ and two vectors $e_0, f_0 \in H$ such that

$$e_{i_k} \to e_0 , \quad f_{i_k} \to f_0 \quad \text{(weak convergence in } H)$$

$$\lambda_{i_k} \to \lambda , \quad \mu_{i_k} \to \mu \quad \text{where } \lambda := \langle e, e_0 \rangle,\ \mu := \langle e, f_0 \rangle .$$

Since for all $x, y \in H$ we have

$$\langle (e_{ik} \otimes \overline{f_{ik}^*})x, y \rangle = \langle e_{ik}, y \rangle \langle x, f_{ik} \rangle \to$$

$$\langle e_0, y \rangle \langle x, f_0 \rangle = \langle (e_0 \otimes \overline{f_0^*})x, y \rangle ,$$

it follows that

$$\begin{aligned} e_0 \otimes \bar{f}_0^* - f_0 \otimes \bar{e}_0^* &= e \otimes \bar{f}^* - f \otimes \bar{e}^* \\ g_0 \otimes \bar{h}_0^* - h_0 \otimes \bar{g}_0^* &= (|\lambda|^2 + |\mu|^2)(e \otimes \bar{f}^* - f \otimes \bar{e}^*) \end{aligned} \tag{1}$$

where

$$g_0 := \lambda e_0 + \mu f_0 , \quad h_0 := -\bar{\mu} e_0 + \bar{\lambda} f_0 .$$

A comparison of the ranges in (1) gives the existence of $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ with

$$e_0 = \alpha e + \beta f , \quad f_0 = \gamma e + \delta f .$$

Hence

$$e \otimes \bar{f}^* - f \otimes \bar{e}^* = e_0 \otimes \bar{f}_0^* - f_0 \otimes \bar{e}_0^* = (\alpha\delta - \beta\gamma)(e \otimes \bar{f}^* - f \otimes \bar{e}^*) .$$

Thus

$$\alpha\delta - \beta\gamma = 1 \text{ and } ||e_0||^2 = |\alpha|^2 + |\beta|^2 \leq 1,\ ||f_0||^2 = |\gamma|^2 + |\delta|^2 \leq 1 .$$

By the Schwarz inequality, the relation $1 = \alpha\delta - \beta\gamma = \langle(\alpha,\beta),(\bar{\delta},-\bar{\gamma})\rangle$ may hold only if $\alpha = \bar{\delta}$ and $\beta = -\bar{\gamma}$. Therefore

$$e_0 = \alpha e + \beta f \quad f_0 = -\bar{\beta} e + \bar{\alpha} f$$

$$\alpha = \langle e_0, e\rangle = \bar{\lambda}, \ -\bar{\beta} = \langle f_0, e\rangle = \bar{\mu}, \ \gamma = -\bar{\beta} = \bar{\mu}, \ \delta = \bar{\alpha} = \lambda$$

$$1 = \alpha\delta - \beta\gamma = |\lambda|^2 + |\mu|^2$$

$$g_0 = \lambda e_0 + \mu f_0 = (|\lambda|^2 + |\mu|^2)e = e \ , \ h_0 = (|\lambda|^2 + |\mu|^2)f = f.$$

From the above considerations we can conclude that

$$\lambda_{i_k} e_{i_k} + \mu_{i_k} f_{i_k} \to e, \ -\bar{\mu}_{i_k} e_{i_k} + \bar{\lambda}_{i_k} f_{i_k} \to f \text{ (weak convergence in } H)$$

$$|\langle e, e_{i_k}\rangle|^2 + |\langle e, f_{i_k}\rangle|^2 = |\lambda_{i_k}|^2 + |\mu_{i_k}|^2 \to 1.$$

Notice that these relations are valid for arbitrary weakly convergent subnets $(e_{ik}),(f_{ik})$. Since the closed unit ball of H is weakly compact, the same statement holds for the whole nets, i.e.

$$\lambda_i e_i + \mu_i f_i \to e, \ -\bar{\mu}_i e_i + \bar{\lambda}_i f_i \to f \text{ (weak convergence in } H)$$

$$|\langle e, e_i\rangle|^2 + |\langle e, f_i\rangle|^2 \to 1.$$

A weakly convergent net of unit vectors whose limit is a unit vector converges in norm, hence

$$\alpha_i := \frac{\lambda_i}{\sqrt{|\lambda_i|^2 + |\mu_i|^2}} = \frac{\langle e, e_i\rangle}{\sqrt{|\langle e, e_i\rangle|^2 + |\langle e, f_i\rangle|^2}}$$

$$\beta_i := \frac{\mu_i}{\sqrt{|\lambda_i|^2 + |\mu_i|^2}} = \frac{\langle e, f_i\rangle}{\sqrt{|\langle e, e_i\rangle|^2 + |\langle e, f_i\rangle|^2}}$$

suits our requirements. ■

1.5 Lemma. *Let $(u_i : i \in I)$ be a net of unitary operators in a Hilbert space H. Let e, f be orthogonal unit vectors in H and $(\alpha_i), (\beta_i)$, nets in $\mathbb{C}$ such that $|\alpha_i|^2 + |\beta_i|^2 = 1$, $i \in I$, and*

$$\alpha_i u_i(e) + \beta_i u_i(f) \to e_0 \ , \quad -\bar{\beta}_i u_i(e) + \bar{\alpha}_i u_i(f) \to f_0$$

for some orthonormal couple $e_0, f_0 \in H$. Then for each orthonormal basis x_0, y_0 of $\mathbb{C}e_0 + \mathbb{C}f_0$ there exists a pair $(\lambda_i), (\mu_i)$ of nets in $\mathbb{C}$ such that

$$|\lambda_i|^2 + |\mu_i|^2 = 1, \ u_i(e) = \lambda_i x_0 + \mu_i y_0 + o(1), \ u_i(f) = -\bar{\mu}_i x_0 + \bar{\lambda}_i y_0 + o(1).$$

Proof. There exist $\gamma, \delta \in \mathbb{C}$ with

$$x_0 = \gamma e_0 + \delta f_0 \ , \quad y_0 = -\bar{\delta} e_0 + \bar{\gamma} f_0 \ , \quad |\gamma|^2 + |\delta|^2 = 1 \ .$$

By assumption $\alpha_i u_i(e) + \beta_i u_i(f) = e_0 + o(1)$, $-\bar{\beta}_i u_i(e) + \bar{\alpha}_i u_i(f) = f_0 + o(1)$ whence

$$\begin{pmatrix} u_i(e) \\ u_i(f) \end{pmatrix} = \begin{pmatrix} \bar{\alpha}_i & -\beta_i \\ \bar{\beta}_i & \alpha_i \end{pmatrix} \begin{pmatrix} e_0 \\ f_0 \end{pmatrix} + o(1)$$

$$= \begin{pmatrix} \bar{\alpha}_i & -\beta_i \\ \bar{\beta}_i & \alpha_i \end{pmatrix} \begin{pmatrix} \bar{\gamma} & -\delta \\ \bar{\delta} & \gamma \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + o(1)$$

$$= \begin{pmatrix} \lambda_i & \mu_i \\ -\bar{\mu}_i & \bar{\lambda} \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + o(1)$$

where $\lambda_i := \overline{\alpha_i \gamma} - \beta_i \bar{\delta}$, $\mu_i := -\bar{\alpha}_i \delta - \beta_i \gamma$. Thus $|\lambda_i|^2 + |\mu_i|^2 = 1$, $i \in I$, holds. ■

1.6 Lemma. *Let $(u_i : i \in I)$ be a net of unitary operators in a Hilbert space H. Suppose that for every $e \in H$ there exists a net*

$(\alpha_i^e : i \in I)$ *in* $\mathbb{T}$ *such that* $(\alpha_i^e u_i(e))$ *converges in* H. *Then there exists a net* $(\alpha_i : i \in I)$ *such that the operators* $(\alpha_i u_i)$ *converge in the SOT of* $\mathcal{L}(H)$.

Proof. Let us fix any unit vector $e \in H$ and write $e_0 :=$ $\lim_i \alpha_i^e u_i(e)$. Consider any $f \in H$ lying orthogonally to e. It suffices to show that $\alpha_i^e u_i(f)$ is norm-convergent.

We may assume $\|f\| = 1$. Set $f_0 := \lim_i \alpha_i^f u_i(f)$. Since unitary operators preserve the scalar product, $\alpha_i^e u_i(e) \perp \alpha_i^f u_i(f)$, $i \in I$, and hence $e_0 \perp f_0$. Moreover there is $g_0 \in H$ such that

$$\alpha_i^{e+f} u_i(e+f) \to g_0 \ , \quad \|g_0\| = \|e+f\| = \sqrt{2} \ .$$

We have

$$u_i(e) = \overline{\alpha_i^e} e_0 + o(1), \ u_i(f) = \overline{\alpha_i^f} f_0 + o(1),$$

$$u_i(e) + u_i(f) = u_i(e+f) = \overline{\alpha_i^{e+f}} g_0 + o(1).$$

Thus

$$g_0 = \alpha_i^{e+f} \overline{\alpha_i^e} e_0 + \alpha_i^{e+f} \overline{\alpha_i^f} f_0 + o(1) \ .$$

Therefore there exist $\sigma, \tau \in \mathbb{T}$ with

$$\alpha_i^{e+f} \overline{\alpha_i^e} \to \sigma \ , \ \alpha_i^{e+f} \overline{\alpha_i^f} \to \tau \ , \ g_0 = \sigma e_0 + \tau f_0 \ .$$

It follows $\alpha_i^f / \alpha_i^e \to \sigma/\tau$ that is $\alpha_i^e = \tau \alpha_i^f / \sigma + o(1)$ and $\alpha_i^e u_i(f) = \left(\frac{\tau}{\sigma} \alpha_i^f + o(1)\right) u_i(f) = \frac{\tau}{\sigma} \alpha_i^f u_i(f) + o(1) \to \frac{\tau}{\sigma} f_0$. ∎

2. NETS OF ISOMETRIES OF $\mathcal{C}(\Omega, E)$

In this section E denotes an arbitrary Cartan factor, Ω is a compact Hausdorff space and $\mathcal{U} := \mathcal{C}(\Omega, E)$ is the JB^*-triple of continuous functions $f : \Omega \to E$ with the norm of the supremum. We recall that ([1] p. 142) a Banach space X has the *strong Banach-Stone property* if whenever M and N are locally compact topological spaces and $l : \mathcal{C}_0(M, X) \to \mathcal{C}_0(N, X)$ is a surjective linear isometry, then l can be represented in the form

$$(lf)(\omega) = u(\omega) f[\tau(\omega)], \quad \omega \in N \quad f \in \mathcal{C}_0(M, X) \tag{2}$$

for some homeomorphism $\tau : N \to M$ and some continuous function $u : N \to \mathcal{U}(X)$, where $\mathcal{U}(X) \subset \mathcal{L}(X)$ is the group of surjective linear isometries of X with the strong operator topology. If l admits the representation (2), then we write $l := (u, \tau)$

2.1 Lemma. *In the above conditions, let $l_1 = (u, \tau)$ and $l_2 := (v, \sigma)$ be two isometries of $\mathcal{U}$ with $\tau \neq \sigma$. Then there exists $f \in \mathcal{U}$ with $||f|| = 1$ such that $||l_1(f) - l_2(f)|| \geq 1$.*

Proof. Fix any $\omega_0 \in \Omega$ with $\tau(\omega_0) \neq \sigma(\omega_0)$ and let $K \subset \Omega$ be a neighbourhood of ω_0 such that $\tau(K) \cap \sigma(K) = \emptyset$. Take any $\varphi \in \mathcal{C}(\Omega)$ with values in $[0, 1]$ such that

$$\varphi_{|\tau(K)} \equiv 1 \quad \text{and} \quad \varphi_{|\sigma(K)} \equiv 0$$

Fix any $a \in E$ with $||a|| = 1$. Then $f := \varphi \otimes a$ satisfies the requirements. ■

2.2 Lemma. *Let $\mathcal{U} := \mathcal{C}(\Omega, E)$ for an arbitrary Cartan factor E. Then E has the strong Banach-Stone property, and for every $l \in$* **Aut**$(\mathcal{U})$ *the representation (2) is unique.*

Proof. By ([2], cor. 2.11) the centralizer $Z(E)$ of E is a one-dimensional space, hence by ([1], th. 8.11) E has the strong Banach-Stone property.

The function $1 \otimes a$ is in $\mathcal{C}(\Omega, E)$ for every $a \in E$, hence $l(1 \otimes a)$ is well defined and

$$l(1 \otimes a)(\omega) = u(\omega)[(1 \otimes a)(\tau(\omega))] = u(\omega)a$$

which shows that u is unique and is given by $u(\omega)a = l(1 \otimes a)(\omega)$ for all $\omega \in \Omega$. Now the uniqueness of (2) is an immediate consequence of (2.1). ■

2.3 Theorem. *Let $\mathcal{U} := \mathcal{C}(\Omega, E)$ for an arbitrary Cartan factor E and let $(l_i;\ i \in I)$ be a net in* **Aut**$(\mathcal{U})$ *such that for every $f \in \mathcal{U}$, $l_i(f)$ and $l_i^{-1}(f)$ converge in $\mathcal{U}$. Then the mapping $l : f \mapsto \lim_i l_i(f)$ is surjective and for every $f \in \mathcal{U}$ we have $\lim_i l_i^{-1}(f) = l^{-1}(f)$ in $\mathcal{U}$.*

Proof. We have

$$l_i(f)(\omega) = u_i(\omega)f[\tau_i(\omega)] \to l(f)(\omega), \quad (\omega \in \Omega,\ f \in \mathcal{U}) \tag{3}$$

By (2.1) this implies $\tau_i = \tau$ for some homeomorphism of Ω and all $i \geq i_0$. Applying (3) to $f := 1 \otimes a$ for $a \in E$, we get

$$l_i(1 \otimes a)(\omega) = u_i(\omega)a \to l(1 \otimes a)(\omega)$$

uniformly for $\omega \in \Omega$. Hence to every $\omega \in \Omega$ and every $a \in E$, the net $(u_i(\omega)a,\ i \in I)$ converges in E. Thus $u(\omega):\ a \mapsto \lim_i u_i(\omega)a$ is in an isometry of E and $u \in \mathcal{C}(\Omega, \mathcal{L}(E))$. We claim that $u(\omega)$ is surjective for every $\omega \in \Omega$. Indeed, by (2.1) and (2.2) we have

$$(l_i^{-1} f)(\omega) = v_i(\omega)f[\sigma_i(\omega)]$$

where $v_i(\omega) = u_i(\omega)^{-1}$, $\sigma_i = \tau_i^{-1}$ for all $\omega \in \Omega$, $i \in I$, and $\sigma_i = \sigma$ for some homeomorphism $\sigma :\ \Omega \to \Omega$ and $i \geq i_0$. Reasoning as we did before, $(v_i(\omega))$ tends to $v(\omega)$ for some continuous function $v:\ \Omega \to \mathcal{L}(E)$. Moreover, we have

$$u(\omega) \circ v(\omega) = id_E = v(\omega) \circ u(\omega), \quad (\omega \in \Omega)$$

since otherwise we would have

$$u(\omega_0)v(\omega_0)\ a = b \neq a$$

for some $\omega_0 \in \Omega$ and $a \in E$. But then

$$\begin{aligned} ||a - b|| &= ||v_i(\omega_0)u_i(\omega_0)a - v(\omega_0)u(\omega_0)a|| \\ &\leq ||v_i(\omega_0)[u_i(\omega_0)a - u(\omega_0)a]|| + ||[v_i(\omega_0) - v(\omega_0)]u(\omega_0)a|| \\ &\leq ||u_i(\omega_0)a - u(\omega_0)a|| + ||[v_i(\omega_0) - v(\omega_0)]u(\omega_0)a|| \to 0 \end{aligned}$$

which contradicts $a \neq b$. Thus u is surjective and so $u \in \mathbf{Aut}(E)$. Clearly

$$l(g)(\omega) := \lim_i l_i g(\omega) = u(\omega) g[\tau(\omega)]$$

holds for all $\omega \in \Omega$ and $g \in \mathcal{U}$, i.e., we have $l = (u, \tau)$ which shows that l is surjective. ■

3. NETS OF ISOMETRIES OF SPECIAL CARTAN FACTORS

If Ω is a single point and E is special, then the above results can be improved.

3.1 Theorem. *Let $\mathcal{U}$ be a special Cartan factor and let $(l_i;\ i \in I)$ be a net in $\mathbf{Aut}(\mathcal{U})$ such that $(l_i(a))$ and $(l_i^{-1}(a))$ converge in $\mathcal{U}$ for every $a \in \mathbf{at}(\mathcal{U})$. Then there are a subnet $(i_k :\ k \in \mathcal{K})$ and an $l \in \mathbf{Aut}(\mathcal{U})$ such that $l_{i_k}(x) \to l(x)$ holds in the SOT for every $x \in \mathcal{U}$.*

Proof. We make a type by type discussion. Suppose $k = 1$. By ([10], Satz 4) every l_i admits one of these two representations:

$$\text{(a) } l_i(x) = u_i \circ x \circ v_i \qquad \text{(b) } l_i(x) = u_i \circ x^t \circ v_i \qquad (x \in \mathcal{U})$$

for some $u_i \in \mathcal{U}(K)$, $v_i \in \mathcal{U}(H)$ or, respectively, for some surjective isometries $u_i, v_i :\ H \to K$. We claim that for large enough indices (say $i \geq i_0$) either all l_i have the form (a) or all l_i have the form (b). Indeed, fix a non zero atom $a \in \mathbf{at}(\mathcal{U})$; then $(l_i(a))$ is a Cauchy net in $\mathcal{U}$ and the claim follows easily. Clearly it suffices to consider the possibility (a) since (b) is quite similar. Fix two unit vectors $e \in K$ and $f \in H$ arbitrarily. Then the operator $e \otimes f^*$ is an atom of $\mathcal{U}$ and

$$u_i \circ (e \otimes f^*) \circ v_i = u_i(e) \otimes v_i^*(f)^*.$$

By (1.2) there exists a net $(\alpha_i :\ i \in I)$ in $\mathbb{T}$ such that $(\alpha_i u_i(e))$ and $(\alpha_i v_i^*(f))$ converge in K and H respectively. We claim that (α_i) does not depend on the pair e, f. Indeed, consider any couple of unit vectors $g \in K,\ h \in H$. By assumption, the operators

$$u_i \circ (g \otimes f^*) \circ v_i^* = u_i(g) \otimes v_i^*(f)^*$$

converge in $\mathcal{U}$. Hence we have

$$\|\alpha_i u_i(g) - \alpha_j u_j(g)\|$$

$$= \|[\alpha_i u_i(g)] \otimes [\alpha_i v_i^*(f)]^* - [\alpha_j u_j(g)] \otimes [\alpha_i v_i^*(f)]^*\|$$

$$\leq \|[\alpha_i u_i(g)] \otimes [\alpha_i v_i^*(f)]^* - [\alpha_j u_j(g)] \otimes [\alpha_j v_j^*(f)]^*$$

$$+ [\alpha_j u_j(g)] \otimes [\alpha_j v_j^*(f) - \alpha_i v_i^*(f)]^*\|$$

$$\leq \|[\alpha_i u_i(g)] \otimes [\alpha_i v_i^*(f)]^* - [\alpha_j u_j(g)] \otimes [\alpha_j v_j^*(f)]^*\|$$

$$+ \|\alpha_j v_j^*(f) - \alpha_i v_i^*(f)\| \to 0$$

Thus $(\alpha_i u_i(g))$ is a Cauchy net in K. To deduce the convergence of $(\alpha_i v_i^*(h))$, we consider the adjoint space $\mathcal{L}(K, H)$ and apply the same argument to the convergent net of operators

$$[\alpha_i v_i^*(h)] \otimes [\alpha_i u_i(g)]^* = [u_i(g) \otimes v_i^*(h)^*]^*.$$

Therefore

$$\alpha_i u_i \to u \qquad \text{and} \qquad \alpha_i v_i^* \to v^* \tag{4}$$

in the SOT of $\mathcal{U}$ for some partial isometries $u \in \mathcal{L}(K)$, $v^* \in \mathcal{L}(H)$. But $(l_i^{-1}(a))$ also converges in $\mathcal{U}$ for every atom $a \in \mathbf{at}(\mathcal{U})$. Since

$$l_i^{-1}(x) = v_i^* \circ x \circ v_i^*, \qquad (x \in \mathcal{U},\ i \in I),$$

by the same argument there exists a net (β_i) in $\mathbb{T}$ such that $\beta_i u_i^* \to r^*$ and $\beta_i v_i \to s$ in the SOT of $\mathcal{U}$ for some partial isometries $r^* \in \mathcal{L}(K)$, $s \in \mathcal{L}(H)$. By setting $\gamma_i := \alpha_i \beta_i$, $i \in I$, we have $\gamma_i 1_H = \gamma_i u_i \circ u_i^* \to u \circ r^*$. By compactness of $\mathbb{T}$, there is a subnet (γ_{i_k}) and some $\gamma_0 \in \mathbb{T}$ such that $\gamma_{i_k} \to \gamma_0$ and so $\gamma_{i_k} 1_H \to \gamma_0 1_H$. Therefore, $\gamma_0 1_H = u \circ r^*$ and u, r^* are surjective. Similarly v^*, s are surjective. By ([12] Remark 4.10, p. 84), on the unitary group $\mathcal{U}(H)$ the SOT coincides with the SSOT. As the adjoint operation $x \mapsto x^*$ is SSOT continuous, (4) yields $\bar{\alpha}_i v_i \to v$. Multiplication in $\mathcal{U}$ restricted to bounded sets is jointly continuous with respect to the SOT, hence

$$l_i(x) = (\alpha_i u_i) \circ x \circ (\bar{\alpha}_i v_i) \to l(x) := u \circ x \circ v$$

in the SOT and $l \in \mathbf{Aut}(\mathcal{U})$.

Suppose $k = 2$. By [5] we have

$$l_i(x) = u_i \circ x \circ u_i^t, \quad (x \in \mathcal{U},\ i \in I)$$

for some unitary operators $u_i \in \mathcal{U}(H)$. Fix any atom $e \otimes \bar{e}^*$ in $\mathcal{U}$. By assumption, the operators

$$u_i \circ (e \otimes \bar{e}^*) \circ u_i^t = u_i(e) \otimes \overline{u_i(e)^*}$$

converge in $\mathcal{U}$. By (1.3) applied to $e_i := u_i(e)$, for any unit vector $e \in H$ there exists a net $(\varepsilon_i^e : i \in I)$ in $\{-1,1\}$ such that $\varepsilon_i^e u_i(e)$ converge. Fix a unit vector $e \in H$ arbitrarily. We claim that $\varepsilon_i^e u_i(f)$ converges in H whenever f is a unit vector lying orthogonally to e. Write $e_0 := \lim_i \varepsilon_i^e u_i(e)$, $f_0 := \lim_i \varepsilon_i^f u_i(f)$, $g_0 := \lim_i \varepsilon_i^g u_i(g)$ where $g := \frac{1}{\sqrt{2}}(e+f)$. Thus

$$\varepsilon_i^x u_i(x) = x_0 + o(1)$$

for $x = e, f, g$ and $x_0 = e_0, f_0, g_0$ respectively. Since $\varepsilon_i^x \in \{-1,1\}$ and so $(\varepsilon_i^x)^2 = 1$, we have

$$u_i(x) = \varepsilon_i^x x_0 + o(1)$$

whence

$$\varepsilon_i^e e_0 + \varepsilon_i^f f_0 = u_i(e) + u_i(f) + o(1) = \sqrt{2} u_i(g) + o(1) = \sqrt{2} \varepsilon_i^g g_0 + o(1)$$

Since $e_0 \perp f_0$, it follows

$$\varepsilon_i^e = \sqrt{2} \langle g_0, e_0 \rangle \varepsilon_i^g + o(1) , \quad \varepsilon_i^f = \sqrt{2} \langle g_0, f_0 \rangle \varepsilon_i^g + o(1) .$$

Hence $\varepsilon_i^e \varepsilon_i^f$ = constant + $o(1)$. Therefore $\varepsilon_i^e u_i(e)$ and $\varepsilon_i^e u_i(f)$ converge simultaneously, and we can define a net $(\varepsilon_i,\ i \in I)$ not depending on $e \in H$ so that $\varepsilon_i u_i \to u$ in the SOT for some partial isometry $u \in \mathcal{L}(H)$. The same argument applied to

$$l_i^{-1}(x) = u_i^* \circ x \circ u_i^{*t} \quad (x \in \mathcal{U},\ i \in I)$$

yields the existence of a net $(\eta_i,\ i \in I)$ in $\{1, -1\}$ such that $\eta_i u_i^* \to v^*$ in the SOT for some partial isometry $v^* \in \mathcal{L}(H)$. But then

$$\varepsilon_i \eta_i 1_H = \varepsilon_i \eta_i u_i \circ u_i^* \to u \circ v^*$$

in the SOT. By taking a subnet $(i_k : k \in \mathcal{K})$ we may assume that $\rho_{i_k} := \varepsilon_{i_k} \eta_{i_k} \to \rho_0$ for some $\rho_0 \in \mathbb{T}$, hence $\rho_{i_k} 1_H \to \rho_0 1_H$ and so $\rho_0 1_H = u \circ v^*$ which shows that $u, v^* \in \mathcal{U}(H)$. Thus $l(x) := u \circ x \circ u^t$ defines an $l \in \mathbf{Aut}(\mathcal{U})$ and $l_i(x) \to l(x)$ in the SOT for every $x \in \mathcal{U}$.

Suppose $k = 3$. By [5], we have

$$l_i(x) = u_i \circ x \circ u_i^t\,, \quad (x \in \mathcal{U},\ i \in I)$$

for some $u_i \in \mathcal{U}(H)$. Fix any atom $e \otimes \bar{f}^* - f \otimes \bar{e}^*$ in $\mathcal{U}$. By assumption, the operators

$$u_i \circ (e \otimes \bar{f}^* - f \otimes \bar{e}^*) \circ u_i^t = u_i(e) \otimes \overline{u_i(f)}^* - u_i(f) \otimes \overline{u_i(e)}^*$$

converge in $\mathcal{U}$, whence by (1.4) there are nets (α_i) and (β_i) in $\bar{\Delta}$ such that

$$\alpha_i u_i(e) + \beta_i u_i(f) \to e_0 \quad \text{and} \quad -\bar{\beta}_i u_i(f) + \bar{\alpha}_i u_i(e) \to f_0$$

hold in H for some orthogonal unit vectors $e_0, f_0 \in H$. By (1.5) if we fix a basis x_0, y_0 in $\mathbb{C}e_0 + \mathbb{C}f_0$, then there are nets (λ_i), (μ_i), in $\bar{\Delta}$ such that

$$u_i(e) = \lambda_i x_0 + \mu_i y_0 + o(1) \quad \text{and} \quad u_i(f) = -\bar{\mu}_i x_0 + \bar{\lambda}_i y_0 + o(1) \qquad (5)$$

By compactness of $\bar{\Delta}$, there is a subnet $(i_k;\ k \in \mathcal{K})$ such that $\lambda_{i_k} \to \lambda_0$, and $\mu_{i_k} \to \mu_0$. From (5) we get in particular that $(u_{i_k}(e))$ converges in H

$$u_{i_k}(e) \to \lambda_0 x_0 + \mu_0 y_0$$

By (1.6) the subnet $(i_k;\ k \in \mathcal{K})$ does not depend on the pair of vectors e, f. Thus $(u_{i_k} :\ k \in \mathcal{K})$ is well defined and $u_{i_k} \to u$ in the SOT for some partial isometry $u \in \mathcal{L}(H)$. A similar argument applied to $(l_{i_k}^{-1})$ gives the existence of a subnet (still denoted by $(i_k :\ k \in \mathcal{K})$) such that $u_{i_k}^* \to v$ in the SOT for some partial isometry $v \in \mathcal{L}(H)$ and a standard reasoning gives that $u \in \mathcal{U}(H)$. Therefore $l(x) := u \circ x \circ u^t$ for $x \in \mathcal{U}$ satisfies the requierements. Notice that, contrary to the other types of Cartan factors, the consideration of a subnet of (l_i) has now been necesary.

Suppose $k = 4$. Let $\mathcal{H}$ be the Hilbert space associated to $\mathcal{U}$. Thus non zero atoms of $\mathcal{U}$ are the same as unit vectors of $\mathcal{H}$. By [6] we have

$$l_i(x) = \lambda_i u_i(x), \qquad (i \in I,\ x \in \mathcal{U}) \tag{6}$$

for some $\lambda_i \in \mathbb{T}$ and some unitary operators $u_i \in \mathcal{U}(\mathcal{H})$ such that $u_i(x)^* = u_i(x^*)$ for $x \in \mathcal{U}$. Suppose that, for every $x \in \mathcal{H}$, the net

$$l_i(x) = \lambda_i u_i(x), \qquad (i \in I) \tag{7}$$

converges in $\mathcal{U}$, (hence also in $\mathcal{H}$). By taking a subnet we may assume that $\lambda_i \to \lambda_0$ for some $\lambda_0 \in \mathbb{T}$. By (7), (u_i) converges pointwise to a partial isometry u of $\mathcal{H}$. A similar argument with (l_i^{-1}) yields $u_i^* \to v^*$ pointwise on $\mathcal{H}$ and one easily sees that $u \circ v^* = 1_H = v^* \circ u$. Thus $u \in \mathcal{U}(\mathcal{H})$. Since the star operation is norm continuous in $\mathcal{U}$, we have

$$[u(x)]^* = [\lim_{i\to\infty} u_i(x)]^* = \lim_{i\to\infty} [u_i(x)^*] = \lim_{i\to\infty} [u_i^*(x)] = u^*(x).$$

Then $l(x) := \lambda_0 u(x)$ for $x \in \mathcal{U}$ satisfies the requirements. ■

4. NETS OF HOLOMORPHIC AUTOMORPHISMS IN CARTAN FACTORS

If $\mathcal{U}$ is an arbitrary Cartan factor with open unit ball D and $g \in G := \mathbf{Aut}(D)$, then by [9] we have $g = g_a \circ \lambda = \lambda \circ g_{\lambda^{-1}(a)}$ for some $\lambda \in \mathbf{Aut}(\mathcal{U})$ and $g_a \in G$ with $a := g(0) = g_a(0)$ and $g_a^{-1} = g_{-a}$. By [7] every $g \in G$ extends to a uniformly continuous holomorphic function a neighbourhood of $\bar{D}$ and $g(\partial D) \subset \partial D$.

4.1 Theorem. *Let $\mathcal{U}$ be a special Cartan factor and $(h_i : i \in I)$ a net in G such that:*

(i) For every $x \in D$, the net $(h_i(x))$ is norm convergent to a limit $h(x) \in \mathcal{U}$.

(ii) $(h_i(0))$ is bounded away from ∂D.

(iii) $(h_i^{-1}(e))$ is norm convergent for every $e \in \mathbf{at}(\mathcal{U})$.

Then h belongs to G, and if the set I is countable, then (h_i) converges to h in $\mathcal{U}$ uniformly on every compact subset of D.

Proof. Let h_i have the representation $h_i = g_{a_i} \circ \lambda_i$ where $a_i := h_i(0)$ for $i \in I$. Notice that $0 \in \mathbf{at}(\mathcal{U})$. By (i) and (ii) we have $a_i \to a \in D$ in the norm topology of $\mathcal{U}$. Thus by [8], g_{a_i} tends to g_a uniformly on a neighbourhood of $\bar{D}$. By the uniform continuity of g_a on $\bar{D}$ we have for every $x \in D$

$$\begin{aligned}||\lambda_i(x) - \lambda_j(x)|| &= ||g_{-a_i}h_i(x) - g_{-a_i}h_j(x)|| \\ &\leq ||g_{-a_i}h_i(x) - g_{-a_i}h_i(x)|| + ||g_{-a}h_i(x) - g_{-a}h_j(x)|| \\ &+ ||g_{-a}h_j(x) - g_{-a_j}h_j(x)|| \to 0\end{aligned}$$

which shows that $(\lambda_i(x))$ converges in $\mathcal{U}$ for every $x \in D$, hence also for every $y \in \mathcal{U}$ (and in particular for every atom $e \in \mathbf{at}(\mathcal{U})$) by the linearity of the λ_i.

We now consider the net $(h_i^{-1} : i \in I)$, where $h_i^{-1} = \lambda_i^{-1} \circ g_{-a_i} = g_{b_i} \circ \mu_i$ with $\mu_i := \lambda_i^{-1}$ and $b_i := -\mu_i(a_i)$ for $i \in I$. Then (b_i) is bounded away from ∂D because so is (a_i) and $||\mu_i|| = 1$, hence by (iii) $b_i \to b$ in $\mathcal{U}$

for some $b \in D$. Thus g_{b_i} tends to g_b uniformly on a neighbourhood of $\bar{D}$. If in the above inequalities we replace λ_i, g_{-a_i}, h_i and x by μ_i, g_{b_i}, h_i^{-1} and e respectively and use (iii), then we get that $(\mu_i(e))$ is a Cauchy net in $\mathcal{U}$ for every atom $e \in \mathbf{at}(\mathcal{U})$.

Therefore (3.1) applies to our case, and there exists a subnet $(i_k : k \in \mathcal{K})$ and a $\lambda \in \mathbf{Aut}(\mathcal{U})$ such that $\lambda_{i_k}(x) \to \lambda(x)$ in the SOT for every $x \in \mathcal{U}$. However, as seen before, the whole net $(\lambda_i(x))$ converges in $\mathcal{U}$; therefore we must have $\lambda_{i_k}(x) \to \lambda(x)$ in the norm of $\mathcal{U}$. Then $g_a \circ \lambda$ lies in G because λ is surjective, and $h_i(x) \to g_a \circ \lambda(x)$ in $\mathcal{U}$ for every $x \in D$, hence $h = g_a \circ \lambda$. The remainder of the proof is an easy consequence of the Banach-Steinhauss theorem. ■

References

[1] Behrends, E.: *M-structure and the Banach-Stone theorem.* Lecture Notes in Mathematics Vol. 736. Berlin-Heidelberg-New York: Springer 1979.

[2] Dineen, S. and Timony, R.M.: *The centroid of a JB*-triple system.* Math. Scand. **62**, (1988), 327-342.

[3] Harris, L.A.: *Bounded symmetric homogeneous domains in infinite dimensional spaces.* Lecture Notes in Mathematics Vol. 364. Berlin-Heidelberg-New York: Springer 1973.

[4] Harris, L.A.: *Analytic invariants and the Schwarz-Pick inequality.* Israel J. Maths. **34**, (1979), 177-197.

[5] Hervés, J.: *Linear isometries of Cartan factors in infinite dimensions.* Ann. di Mat. Pura ed Appl. (IV) vol. **CXLII**, (1985), 371-379.

[6] Hervés, J. and Isidro, J.M.: *Isometries and automorphisms of the spaces of spinors.* Rev. Mat. Univ. Compl. Madrid **5**, (1992), 193-200.

[7] Isidro, J.M. and Kaup, W.: *Weak continuity of holomorphic automorphisms in JB*-triples.* Math. Z. **210**, (1992), 277-288.

[8] Isidro, J.M. and Kaup, W.: *Determining boundary sets of bounded symmetric domains.* Manuscripta Math. **81** (1993) 149-159.

[9] Kaup, W.: *A Riemann mapping theorem for bounded symmetric domains in complex Banach spaces.* Math. Z. **183**, (1983), 503-529.

[10] Kaup, W.: *Über die Automorphismen Graßmannscher Mannigfaltigkeiten unendlicher Dimension.* Math. Z. 144, (1975), 75-96.

[11] Loos, O.: *Bounded symmetric domains and Jordan pairs.* Mathematical Lectures, University of California at Irvine 1977.

[12] Takesaki, MP.: *Theory of operator algebras I.* Springer-Verlag New York 1979.

Departamento de Análisis Matemático
Facultad de Matemáticas
15706 Santiago de Compostela
Spain
jmisidro@zmat.usc.es

Bolyai Intézete
Aradi Vértanuk tere 1
6720 Szeged
Hungary

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