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On Weak Monomorphisms and Weak Epimorphisms in the Category of Pro-Groups

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ABSTRACT. In this paper we characterize weak monomorphisms and weak epimorphisms in the category of pro-groups. Also we define the notion of weakly exact sequence and we study this notion in the category of pro-groups.

1. INTRODUCTION

If $\mathscr{C}$ is a category with zero-objects, then a morphism $f: A \to B$ of $\mathscr{C}$ is a *weak monomorphism* if $f \circ u = 0$ implies $u = 0$. A morphism $f: A \to B$ is called a *weak epimorphism* if $u \circ f = 0$ implies $u = 0$.

Weakened versions of the categorical notions of monomorphism and epimorphism have proved to be of some interest in pointed homotopy theory. In 1967, T. Ganea [1] utilized extensive homotopy-theoretic calculations to exhibit examples, in the pointed homotopy category, of weak monomorphism which are not monomorphism. In 1986 J. Roitberg [7] used the properties of a remarkable group discovered by G. Higman [3] to exhibit examples, again in the pointed homotopy category, of weak epimorphisms which are not epimorphisms.

In this paper we intend to study weak monomorphisms and weak epimorphisms in the shape theory. In this first part we examine weak monomorphisms and weak epimorphisms in the category of pro-groups. Also we define the notion of *weakly exact sequence* and we study this notion in the category of pro-groups.

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2. WEAK MONOMORPHISMS IN THE CATEGORY OF PRO-GROUPS

We consider the category pro-Grp of which objects are inverse systems of groups with homomorphisms as bonding morphisms. The notions and properties of pro-groups which are used in this paper are those from the book of S. Mardešić and J. Segal [4].

The category pro-Grp is a category with zero-objects. The trivial group 0 is a zero-object of the category Grp of groups and the rudimentary system $\underline{0}$ is a zero-object of the category pro-Grp.

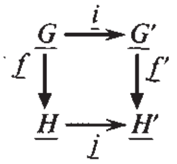
Theorem 1. *A morphism $\underline{f}: \underline{G} \to \underline{H}$ of pro-Grp is a monomorphism if and only if $\underline{f}$ is a weak monomorphism.*

Proof. The necessity is obvious.

To prove the converse implication, first we consider the case when the pro-groups $\underline{G}=(G_\lambda, p_{\lambda\lambda'}, \Lambda)$ and $\underline{H}=(H_\lambda, q_{\lambda\lambda'}, \Lambda)$ are indexed by the same index set and the morphism $\underline{f}$ is given by a level morphism of systems $(f_\lambda): \underline{G} \to \underline{H}$. Hence for every $\lambda \in \Lambda$, f_λ is a morphism $f_\lambda : G_\lambda \to H_\lambda$.

For $\lambda \in \Lambda$, let $N_\lambda = \operatorname{Ker} f_\lambda$. Since $\lambda' \geq \lambda$ implies $f_\lambda p_{\lambda\lambda'} = q_{\lambda\lambda'} f_{\lambda'}$, it follows $p_{\lambda\lambda'}(N_{\lambda'}) \subseteq N_\lambda$, so that $\underline{N}=(N_\lambda, p_{\lambda\lambda'}|N_{\lambda'}, \Lambda)$ is a pro-group. The inclusion maps $i_\lambda : N_\lambda \to G_\lambda$ define a morphism of pro-Grp $\underline{i}: \underline{N} \to \underline{G}$. Let $e_\lambda : N_\lambda \to G_\lambda$, the zero-morphism, for every $\lambda \in \Lambda$. Then we obtain the zero-morphism $\underline{e}=(e_\lambda): \underline{N} \to \underline{G}$ of pro-Grp. For every $\lambda \in \Lambda$, we have $f_\lambda i_\lambda = f_\lambda e_\lambda$, which implies $\underline{f} \circ \underline{i} = \underline{f} \circ \underline{e} = \underline{0}$. Since $\underline{f}$ is a weak monomorphism, the equality $\underline{f} \circ \underline{i} = \underline{0}$ implies $\underline{i} = \underline{0}$. This means that for every $\lambda \in \Lambda$ there exists $\lambda' \geq \lambda$ such that $i_\lambda p_{\lambda\lambda'} = 0$. If $x \in \operatorname{Ker} f_{\lambda'}$, then $f_{\lambda'}(x) = *$ and if we put $p_{\lambda\lambda'}(x) = y \in N_\lambda$ then $* = i_\lambda p_{\lambda\lambda'}(x) = i_\lambda(y) = y$, which implies $x \in \operatorname{Ker} p_{\lambda\lambda'}$. In this way we proved the inclusion $\operatorname{Ker} f_{\lambda'} \subseteq \operatorname{Ker} p_{\lambda\lambda'}$. Applying Theorem 1 of [4, p. 107], we conclude that $\underline{f}$ is a monomorphism.

Suppose now that $\underline{f}$ is a morphism of pro-Grp between arbitrary pro-groups: $\underline{G}=(G_\lambda, p_{\lambda\lambda'}, \Lambda)$, $\underline{H}=(H_\mu, q_{\mu\mu'}, M)$, $\underline{f}=(f_\mu, \varphi): \underline{G} \to \underline{H}$, with $\varphi : M \to \Lambda$, and with the morphisms of groups $f_\mu : G_{\varphi(\mu)} \to H_\mu$. By the reindexing theorem [4, p. 12] there exist pro-groups $\underline{G}'=(G'_\nu, p'_{\nu\nu'}, N)$, $\underline{H}'=(H'_\nu, q'_{\nu\nu'}, N)$ over the same index set N, and a morphism of pro-Grp, $\underline{f}': \underline{G}' \to \underline{H}'$, given by a level morphism of systems (f'_ν): $\underline{G}' \to \underline{H}'$ and there exist isomorphisms of pro-groups $\underline{i}: \underline{G} \to \underline{G}'$, $\underline{j}: \underline{H} \to \underline{H}'$, such that the following diagram commutes:



Suppose that $\underline{f}'\circ \underline{u}=\underline{0}$. Then, if $\underline{i}':\underline{G}'\to\underline{G}$ is the inverse of $\underline{i}$, we have $\underline{j}\circ\underline{f}\circ\underline{i}'\circ\underline{u}=\underline{0}$ and since $\underline{j}$ is an isomorphism, we deduce that $\underline{f}\circ\underline{i}'\circ\underline{u}=\underline{0}$ and, by hypothesis, $\underline{i}'\circ\underline{u}=\underline{0}$, which implies $\underline{u}=\underline{0}$. This proves that $\underline{f}'$ is a weak monomorphism. By the first part of this proof, we conclude that $\underline{f}'$ is a monomorphism of pro-Grp. For this reason and by using the above commutative diagram, we deduce that $\underline{f}$ is a monomorphism of pro-Grp. This completes the proof of Theorem 1.

3. WEAK EPIMORPHISMS IN THE CATEGORY OF PRO-GROUPS

The following intrinsic characterization of weak epimorphism in the category Grp is stated in [6] (see also [5], [7]):

Lemma 1. [6] *In the category Grp, a morphism $f:G\to G'$ is a weak epimorphism of groups if and only if the normal subgroup of G' generated by $Im\ f$ is all of G'.*

Proof. Suppose that f is a weak epimorphism and let $(Imf)^N_{G'}$ be the normal subgroup of G' generated by $Im\ f$.

Let H' be a normal subgroup of G' which Imf.

Consider the canonical epimorphism $u: G'\to G'/H'$. Then $u\circ f=0$ and, by hypothesis, it follows $u=0$, what means $H'=G'$, so that $(Imf)^N_{G'}=G'$.

Conversely, suppose that $(Imf)^N_{G'}=G'$ and let $u: G'\to G''$ be a morphism for which $u\circ f=0$. It follows that $Imf\subseteq \mathrm{Ker}\ u$ and because $\mathrm{Ker}\ u$ is a normal subgroup of G', we deduce that $\mathrm{Ker}\ u=G'$, so that $u=0$ and therefore f is a weak epimorphism.

If $\underline{f}:\underline{G}\to\underline{H}$ is an epimorphism of pro-groups, then it is evident that $\underline{f}$ is a weak epimorphism. But, since a weak epimorphism of groups is not necessarily an epimorphism, it is obvious that there exist morphisms of pro-groups which are weak epimorphism but which are not epimorphism.

Theorem 2. *Let $\underline{G}=(G_\lambda, p_{\lambda\lambda'}, \Lambda)$ and $\underline{H}=(H_\lambda, q_{\lambda\lambda'}, \Lambda)$ pro-groups indexed by the same index set and let $\underline{f}:\underline{G}\to\underline{H}$ be a morphism given by a level morphism of systems $\underline{f}=(f_\lambda)$. If $\underline{f}$ is a weak epimorphism then the following condition is satisfied:*

(WE) *For every $\lambda\in\Lambda$, there is a $\lambda'\geq\lambda$, such that $Im(q_{\lambda\lambda'})\subseteq(Im f_\lambda)^N_{H_\lambda}$*

where $(Im f_\lambda)^N_{H_\lambda}$ *denotes the normal subgroup of* H_λ *generated by* $Im f_\lambda$.

Conversely, if the morphism $\underline{f}$ *satisfies the condition*

(WE) *and if, supplementary, for every pair* $(\lambda, \lambda') \in \Lambda \times \Lambda$ *with* $\lambda' \geq \lambda$, *the bonding morphism* $p_{\lambda\lambda'}$ *is a weak epimorphism of groups then* $\underline{f}$ *is a weak epimorphism of pro-Grp.*

Proof. Suppose that $\underline{f}$ is a weak epimorphism. For $\lambda \in \Lambda$ let N_λ be a normal subgroup of the group H_λ containing $Im f_\lambda$. We consider the rudimentary system $\underline{K} = (H_\lambda / N_\lambda)$ and the morphism of pro-groups $\underline{u}: \underline{H} \to \underline{K}$ defined by the inclusion $\varphi: \{\lambda\} \to \Lambda$ and by the canonical epimorphism $u: H_\lambda \to H_\lambda / N_\lambda$. Then we have $\underline{u} \circ \underline{f} = (uf_\lambda)$, with $uf_\lambda : G_\lambda \to H_\lambda \to H_\lambda / N_\lambda$. Since $Im f_\lambda \subseteq N_\lambda$, it follows $uf_\lambda = 0$ and therefore $\underline{u} \circ \underline{f} = \underline{0}$. From this and because $\underline{f}$ is a weak epimorphism, it follows $\underline{u} = \underline{0}$. This means that there exists $\lambda' \in \Lambda$, $\lambda' \geq \lambda$, such that $uq_{\lambda\lambda'} = 0$, which proves the inclusion $Im\, q_{\lambda\lambda'} \subseteq$ Ker u, that is the inclusion $Im\, q_{\lambda\lambda'} \subseteq N_\lambda$. Since N_λ is arbitrary, it follows $Im\, q_{\lambda\lambda'} \subseteq (Im f_\lambda)^N_{H_\lambda}$.

Conversely, suppose that $p_{\lambda\lambda'}: G_{\lambda'} \to G_\lambda$ are weak epimorphisms of groups, for any $\lambda, \lambda' \in \Lambda$, with $\lambda' \geq \lambda$, and that the condition (WE) is satisfied also.

Let $\underline{u}: \underline{H} \to \underline{K}$ be a morphism of pro-groups, for $\underline{K} = (K_{\nu\nu'}, r_{\nu\nu'}, N)$, with $\underline{u} = (u_\gamma, \psi)$, for a function $\psi: N \to \Lambda$ and for some morphisms of groups $u_\nu : H_{\psi(\nu)} \to K_\nu$, such that $\underline{u} \circ \underline{f} = \underline{0}$.

This means that for every $\nu \in N$ there is $\lambda \geq \psi(\nu)$ such that:

$$u_\nu\, f_{\psi(\nu)}\, p_{\psi(\nu)\lambda} = 0 \tag{1}$$

This relation implies $Im(f_{\psi(\nu)}\, p_{\psi(\nu)\lambda}) \subseteq$ Ker u_ν, that is $f_{\psi(\nu)}\,(Im\, p_{\psi(\nu)\lambda}) \subseteq$ Ker u_ν. Then, since $p_{\psi(\nu)\lambda}: G_\lambda \to G_{\psi(\nu)}$ is a weak epimorphism, we have $(Im\, p_{\psi(\nu)\lambda})^N_{G_{\psi(\nu)}} = G_{\psi(\nu)}$ (Lemma 1). On the other hand, since Ker u_ν is a normal subgroup of $H_{\psi(\nu)}$ and since $Im\, p_{\psi(\nu)\lambda} \subseteq f^{-1}_{\psi(\nu)}$ (Ker u_ν), we deduce the inclusion $(Im\, p_{\psi(\nu)\nu})^N_{G_{\psi(\nu)}} \subseteq f^{-1}_{\psi(\nu)}$ (Ker u_ν) and this induces $G_{\psi(\nu)} \subseteq f^{-1}_{\psi(\nu)}$ (Ker u_ν), which can be written

$$Im\, f_{\psi(\nu)} \subseteq \text{Ker } u_\nu \tag{2}$$

But, since Ker u_ν is a normal subgroup of $H_{\psi(\nu)}$, the inclusion (2) implies

$$(Im\, f_{\psi(\nu)})^N_{H_{\psi(\nu)}} \subseteq \text{Ker } u_\nu . \tag{3}$$

Then, by hypothesis, it follows that there exists $\lambda' \geq \psi(\nu)$ such that

(4) $$Im\, q_{\psi(\nu)\nu'} \subseteq (Im\, f_{\psi(\nu)})^N_{H_{\psi(\nu)}} \subseteq \text{Ker}\, u_\nu.$$

By this, we have

(5) $$u_\nu\, q_{\psi(\nu)\nu'} = 0, \text{ for } \lambda' \geq \psi(\nu),$$

and this means that $\underline{u} = 0$ what proves that $\underline{f}$ is a weak epimorphism.

Example 1. If the pro-group $\underline{G}$ has the Mittag-Leffler property [4, p. 165] and if for the morphism $\underline{f}: \underline{G} \to \underline{H}$ of pro-groups, given by a level system (f_λ), the morphisms $f_\lambda: G_\lambda \to H_\lambda$ are weak epimorphisms, then $\underline{f}$ is a weak epimorphism of pro-groups. This results by Theorem 2 using Theorem 7 of [4, p. 166].

Theorem 3. *Let* $\underline{G} = (G_\lambda, p_{\lambda\lambda'}, \Lambda)$ *and* $\underline{H} = (H_\mu, q_{\mu\mu'}, M)$ *two arbitrary pro-groups and* $\underline{f}: \underline{G} \to \underline{H}$ *a morphism of systems* $(f_\mu, \varphi): \underline{G} \to \underline{H}$.

If $\underline{f}$ *is a weak epimorphism of pro-groups then the following condition is satisfied:*

(WE)′ *For any admissible pair* $(\lambda, \mu) \in \Lambda \times M$ *for* (f_μ, φ), *i.e. having the property* $\lambda \geq \varphi(\mu)$, *there exists an admissible pair* (λ', μ') *for* (f_μ, φ), *with* $(\lambda', \mu') \geq (\lambda, \mu)$, *i.e.* $\lambda' \geq \lambda$ *and* $\mu' \geq \mu$, *such that* $Im\,(q_{\mu\mu'}) \subseteq (Im\,(f_\mu\, p_{\varphi(\mu)\lambda}))^N_{H_\mu}$

Conversely, if the morphism $\underline{f}$ *satisfies the condition* (WE)′ *and if, supplementary, for every pair* $(\lambda, \lambda') \in \Lambda \times M$, *with* $\lambda' \geq \lambda$, *the bonding morphism* $p_{\lambda\lambda'}: G_{\lambda'} \to G_\lambda$ *is a weak epimorphism, then* $\underline{f}$ *is a weak epimorphism.*

Proof. Suppose that $\underline{f}$ is a weak epimorphism and we refer to the pro-groups $\underline{G}'$, $\underline{H}'$ and the morphism $\underline{f}': \underline{G}' \to \underline{H}'$ considered in the proof of Theorem 1. By hypothesis, it follows that $\underline{f}'$ is a weak epimorphism and therefore for this morphism the condition (WE) is satisfied. Then, by the proof of the reindexing theorem [4, p. 12] we have $p'_{\nu\nu'} = p_{\lambda\lambda'}$, if $\nu = (\lambda, \mu) \leq \leq (\lambda', \mu') = \nu'$. So, $p_{\lambda\lambda'}$ is a weak epimorphism for any pair $(\lambda, \lambda') \in \Lambda \times \Lambda$, with $\lambda' \geq \lambda$, if and only if $p'_{\nu\nu'}$ is a weak epimorphism for any pair $(\nu, \nu') \in N \times N'$, with $\nu' \geq \nu$.

Similarly, $q'_{\nu\nu'} = q_{\mu\mu'}$ and $f'_\nu = f_\mu\, p_{\varphi(\mu)\lambda}$. Using these notations, the condition (WE) for $\underline{f}'$ is even the condition (WE)′ for $\underline{f}$, what finishes the proof of Theorem 3.

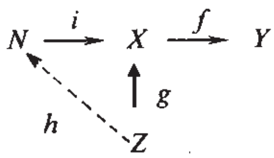
Example 2. With the notations of Theorem 3, if f_μ are weak epimorphisms of groups and if the pro-group $\underline{G}$ has the Mittag-Leffler property, then $\underline{f}$ is a weak epimorphism of pro-groups.

4. WEAKLY EXACT SEQUENCES OF PRO-GROUPS

First, we recall that if $\mathscr{C}$ is a category with zero-objects then a kernel of a morphism $f: X \to Y$ is defined as a morphism $i: N \to X$, with the following properties:

(i) $f \circ i = 0$,

(ii) whenever $g: Z \to X$ is a morphism with $fg = 0$, then there is a unique morphism $h: Z \to N$ such that $ih = g$



The kernels of f are unique up to natural isomorphism.

It is proved [4, p. 117] that the category of pro-Grp is a category with kernels.

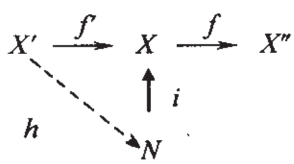
Definition. *Let $\mathscr{C}$ be a category with zero-objects and kernels and let $X' \xrightarrow{f'} X \xrightarrow{f} X''$ be a sequence consisting of two morphisms in $\mathscr{C}$.*

This sequence is said to be weakly exact (at *X*) *provided the following holds:*

(i) $ff' = 0$,

(ii) *Let $i: N \to X$ be the kernel of f and let $h: X' \to N$ be the unique morphism with $ih = f'$.*

Then h is a weak epimorphism.



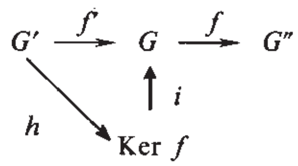
Example 3. A sequence $X' \xrightarrow{f'} X \to 0$ is weakly exact if and only if f' is a weak epimorphism.

Example 4. Let K be the fundamental group of the Klein bottle, that is the group with two generators a, b, satisfying the relation $bab = a$. Consider the following subgroups of K: $K_{1,1,2} = (ab, b^2)$ and $K_{1,1} = (ab)$. Then $K_{1,1,2}$ is a normal subgroup of K and $(K_{1,1})_K^N = K_{1,1,2}$. If $i: K_{1,1} \to K$ is the inclusion morphism and if $\pi: K \to K/K_{1,1,2}$ is the canonical epimorphism, then the sequence $K_{1,1} \xrightarrow{i} K \xrightarrow{\pi} K/K_{1,1,2}$ is weakly exact.

Lemma 2. *If the sequence of groups* $G' \xrightarrow{f'} G \xrightarrow{f} G''$ *is weakly exact, then* $(\mathrm{Im} f')_G^N = \mathrm{Ker}\, f$.

Conversely, if $(\mathrm{Im} f')_G^N = \mathrm{Ker}\, f$ *and if any chain normal groups* $H \subseteq \mathrm{Ker}\, f \subseteq G$ *of* G *is invariant, then the sequence* $G' \xrightarrow{f'} G \xrightarrow{f} G''$ *is weakly exact.*

Proof. Suppose that the sequence $G' \xrightarrow{f'} G \xrightarrow{f} G''$ is weakly exact. The equality $ff' = 0$ implies the inclusion $Im f' \subseteq \mathrm{Ker}\, f$. Then, if we consider the diagram



we have $(Im\, h)_{\mathrm{Ker} f}^N = \mathrm{Ker} f$ (by the definition of weakly exact sequences and by Lemma 1). If H is a normal subgroup of G containing $Im f'$, then by the equality $f' = ih$ and since i is an inclusion, we obtain $Im\, h \subseteq H$ and by this $Im\, f \subseteq H \cap \mathrm{Ker}\, f$ so that $H \cap \mathrm{Ker}\, f = \mathrm{Ker}\, f$ and finally $\mathrm{Ker}\, f \subseteq H$. As H is arbitrary, we conclude that $(Im f')_G^N = \mathrm{Ker}\, f$.

Conversely, suppose $(Im f')_G^N = \mathrm{Ker} f$. Then, the inclusion $Im\, f' \subseteq \mathrm{Ker}\, f$ implies $ff' = 0$. Further, if H is a normal subgroup of $\mathrm{Ker}\, f$ such that $Im\, h \subseteq H$ then, by the hypothesis, H is a normal subgroup of G. By this and since $Im\, f' \subseteq H$ $(Im\ f' = Im\ h)$ we conclude that $H = \mathrm{Ker} f$. Hence

$(Im\ h)^N_{\mathrm{Ker} f} = \mathrm{Ker} f$, that is h is a weak epimorphism and therefore the given sequence is weakly exact.

Theorem 4. *Let* $\underline{G}' = (G'_\lambda, p'_{\lambda\lambda'}, \Lambda)$, $\underline{G} = (G_\lambda, p_{\lambda\lambda'}, \Lambda)$ *and* $\underline{G}'' = (G''_\lambda, p''_{\lambda\lambda'}, \Lambda)$, *be pro-groups over the same index set* Λ *and let* $\underline{f}': \underline{G}' \to \underline{G}$, $\underline{f}: \underline{G} \to \underline{G}''$ *be morphisms of pro-groups given by level morphisms of systems* $(f'_\lambda): \underline{G}' \to \underline{G}$ *and* $(f_\lambda): \underline{G} \to \underline{G}''$ *respectively. If the sequence of groups*

$$G'_\lambda \xrightarrow{f'_\lambda} G_\lambda \xrightarrow{f_\lambda} G''_\lambda \tag{6}$$

is weakly exact for every $\lambda \in \Lambda$ *and if for every pair* $(\lambda, \lambda') \in \Lambda \times \Lambda$, *with* $\lambda' \geq \lambda$, *the bonding morphism* $p_{\lambda\lambda'}$ *is a weak epimorphism of groups, then the sequence of pro-groups.*

$$\underline{G}' \xrightarrow{\underline{f}'} \underline{G} \xrightarrow{\underline{f}} \underline{G}'' \tag{7}$$

is also weakly exact.

Proof. By (6), $f_\lambda f'_\lambda = 0$ for $\lambda \in \Lambda$, which implies $\underline{f}\underline{f}' = \underline{0}$. If $N_\lambda = f_\lambda^{-1}(*)$, $n_{\lambda\lambda'} = p_{\lambda\lambda'}|N_{\lambda'}: N_{\lambda'} \to N_\lambda$, and $i_\lambda: N_\lambda \to G_\lambda$ is the inclusion homomorphism, then $\underline{i}: \underline{N} = (N_\lambda, n_{\lambda\lambda'}, \Lambda) \to \underline{G}$, given by $(i_\lambda, 1_\Lambda)$, is the kernel of $\underline{f}$[4, p. 177]. There are unique morphisms $h_\lambda: G'_\lambda \to N_\lambda$ such that $f'_\lambda = i_\lambda h_\lambda$ and they determine a unique morphism $\underline{h}: \underline{G}' \to \underline{N}$ such that $\underline{i} \circ \underline{h} = \underline{f}'$. Since, by (6), every h_λ is a weak epimorphism, it follows the equality $(Im\, h_\lambda)^N_{N_\lambda} = N_\lambda$ and therefore $Im\,(n_{\lambda\lambda'}) \subseteq N_\lambda = (Im\, h_\lambda)^N_{N_\lambda}$. This inclusion and the condition about the bonding morphisms $p_{\lambda\lambda'}$ of $\underline{G}'$ show that we can apply Theorem 2 and we can conclude that $\underline{h}: \underline{G} \to \underline{N}$ is a weak epimorphism. So we have satisfied the conditions which shows that (7) is a weakly exact sequence of pro-groups.

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