REVISTA MATEMATICA de la
Universidad Complutense de Madrid
Volumen 3, número 1; 1990.
http://dx.doi.org/10.5209/rev_REMA.1990.v3.n1.18034

P—Adic Ascoli theorems

J. MARTÍNEZ-MAURICA and S. NAVARRO

ABSTRACT. The aim of this paper is the study of a certain class of compact-like sets within some spaces of continuous functions over non-archimedean ground fields. As a result, some p—adic Ascoli theorems are obtained.

INTRODUCTION

In recent years, there has been a renewed interest in the study of analysis over the field $\mathbb{Q}_p$ of p—adic numbers (or more in general over any complete non-archimedean valued field $\mathbb{K}$) in view of its new applications in some parts of modern physics (see for instance [4], [5] and [15]).

The aim of this paper is to give some p-adic Ascoli theorems; this is, we will explore the relationships between a certain kind of compact-like sets and equicontinuous sets within some subspaces of the space $C(X)$ of all continuous functions $f: X \to \mathbb{K}$ where X is a given separated topological space. In order to ensure the existence of enough elements in $C(X)$ we shall assume in addition that X is zerodimensional. Also, the valuation over $\mathbb{K}$ is supposed to be non trivial.

The first difference with its archimedean analog is the class of compact-like sets we are going to consider. For that it is worth mentioning here that (pre) compactness is not very interesting in p-adic analysis; in fact, there is no compact convex subset of a locally convex space over $\mathbb{K}$ with more than one point unless $\mathbb{K}$ is locally compact. Altough $\mathbb{Q}_p$ is locally compact, in many occasions it is certainly useful to consider some other valued fields apart from $\mathbb{Q}_p$ (for instance, the non locally compact field $\mathbb{C}_p$ defined as the completion of the algebraic closure of $\mathbb{Q}_p$).

1980 Mathematics Subject Classification (1985 revision): 46P05
Editorial de la Universidad Complutense. Madrid, 1990.

Quite a number of different variants of (pre)compact sets have been studied in p-adic analysis (see [19]), and it seems for many reasons that the most succesful ones are compactoids defined in [6] as follows: a subset A of a locally convex space E is said to be compactoid if for every neighborhood of zero U there exists a finite set $Y \subset E$ such that $A \subset U + c_o(Y)$, where $c_o(Y)$ denotes the absolutely convex hull of Y.

So, we shall study the relationships between compactoids and equicontinuous sets in some different spaces of continuous functions.

1. THE CASE OF THE TOPOLOGY OF UNIFORM CONVERGENCE

Following [16], we are going to indicate by $PC(X)$ the space of all continuous functions $f \in C(X)$ such that $f(X)$ is a precompact subset of $\mathbb{K}$, endowed with the topology of uniform convergence: this is the topology defined by the norm $\|f\| = \|f\|_\infty = \sup_{x \in X} |f(x)|$. If X is also locally compact, $C_\infty(X)$ will indicate the subspace of $PC(X)$ consisting of all continuous functions which vanish at infinity.

Given a subset $\mathscr{F}$ of $\mathbb{K}$-valued functions defined on X, we define $\mathscr{F}(x) = \{f(x) : f \in \mathscr{F}\}$. Also $B_\varepsilon(0)$ will indicate the closed ball in $PC(X)$ with center 0 and radius ε.

Theorem 1. *A subset $\mathscr{F} \subset PC(X)$ is compactoid if and only if the following properties are satisfied:*

(a) $\mathscr{F}(x)$ is bounded in $\mathbb{K}$ for every $x \in X$.

and (b) For every $\varepsilon > 0$, there exists a finite partition $X_1, \ldots, X_n$ of X consisting of clopen sets such that $x, y \in X_i \Longrightarrow |f(x) - f(y)| \leq \varepsilon$ for all $f \in \mathscr{F}$ $(i = 1, \ldots, n)$.

Proof: First we assume that $\mathscr{F}$ is compactoid. Given $x \in X$, the map $H_x : PC(X) \to \mathbb{K}$ defined by $H_x(f) = f(x)$ is linear and continuous. Hence $\mathscr{F}(x) = H_x(\mathscr{F})$ is compactoid in $\mathbb{K}$.

Also, given $\varepsilon > 0$, there exists $Y = \{\varphi_1, \ldots, \varphi_m\} \subset PC(X)$ such that $\mathscr{F} \subset B_\varepsilon(0) + c_o(Y)$. Now for every $j \in \{1, \ldots, m\}$ we consider the equivalence relation R_j in X defined by.

$$x R_j y \text{ if } |\varphi_j(x) - \varphi_j(y)| \leq \varepsilon \quad (x, y \in X)$$

It is well known that for each $j \in \{1, \ldots, m\}$ there is only a finite number of equivalence classes and that these classes are clopen sets in X.

Let us consider for every $x \in X$ and $j \in \{1, ..., m\}$ the class P_j^x which contains x and let $P_x = \bigcap_j P_j^x$. Since $\{P_x : x \in X\}$ is finite, we obtain a finite partition $X_1, ..., X_n$ of X consisting of clopen sets such that

$$x, y \in X_i \Rightarrow |\varphi_j(x) - \varphi_j(y)| \leq \varepsilon \text{ for all } i \in \{1, ..., n\} \text{ and } j \in \{1, ..., m\}$$

Now if $f \in \mathscr{F}$, there are $\lambda_1, ..., \lambda_m \in \mathbb{K}$ with $|\lambda_j| \leq 1$ for all $j \in \{1, ..., m\}$ such that $\|f - \sum_j \lambda_j \varphi_j\| \leq \varepsilon$. It follows that for $x, y \in X_i$.

$$|f(x) - f(y)| \leq \max\{ |f(x) - \sum_j \lambda_j \varphi_j(x)|, |\sum_j \lambda_j (\varphi_j(x) - \varphi_j(y))|, |\sum_j \lambda_j \varphi_j(y) - f(y)| \} \leq \varepsilon.$$

Conversely, take $\varepsilon > 0$ and let $X_1, ..., X_n$ be clopen subsets of X satisfying (b). Pick, for each $i \in \{1, ..., n\}$, x_i within X_i. Since $\bigcup_i \mathscr{F}(x_i)$ is compactoid, there are $\upsilon_1, ..., \upsilon_m$ in $\mathbb{K}$ such that

$$\bigcup \mathscr{F}(x_i) \subset \{\lambda \in \mathbb{K} : |\lambda| \leq \varepsilon\} + C_0\{\upsilon_1, ..., \upsilon_m\}$$

Let us define for $i \in \{1, ..., n\}$ and $j \in \{1, ..., m\}$ $\varphi_{ij}: X \to \mathbb{K}$ by $\varphi_{ij} = \upsilon_j \xi_{X_i}$ where ξ_{X_i} stands for the characteristic function of X_i.

It is obvious that $\varphi_{ij} \in PC(X)$. Also, if $f \in \mathscr{F}$ there are for each $i \in \{1, ..., n\}$ $\lambda_{ij} \in \mathbb{K}$ ($j \in \{1, ..., m\}$) such that $|\lambda_{ij}| \leq 1$ and $|f(x_i) - \sum_j \lambda_{ij} \upsilon_j| \leq \varepsilon$. Hence, given $x \in X$, we have

$$|f(x) - \sum_{i,j} \lambda_{ij} \varphi_{ij}(x)| = |f(x) - \sum_j \lambda_{i_0 j} \upsilon_j| \leq \max\{ |f(x) - f(x_{i_0})|, |f(x_{i_0}) - \sum_j \lambda_{i_0 j} \upsilon_j| \} \leq \varepsilon$$

if $x \in X_{i_0}$, which finally implies that $\mathscr{F} \subset B_\varepsilon(0) + c_0(\{\varphi_{ij}\})$.

Remarks: (1) Condition (b) in the above theorem implies equicontinuity of $\mathscr{F}$. Also, if X is compact both properties coincide.

(2) Our theorem 1 is a generalization of a previous one of N. De Grande-de Kimpe [2, theorem 1.8] in which she characterizes compactoids in the space $C(X)$ where X is a compact subset of a nonarchimedean valued field $\mathbb{K}$.

Corollary 2: *A subset $\mathscr{F}$ of $C_\infty(X)$ is compactoid if and only if*

(a) $\mathscr{F}(x)$ is bounded in $\mathbb{K}$ for every $x \in X$.

and (b) For every $\varepsilon > o$, there exists a finite number of pairwise disjoint clopen compact sets $P_1, ..., P_n$ in X such that $x, y \in P_i \Rightarrow |f(x) - f(y)| \leq \varepsilon$ for all $f \in \mathscr{F}$, $i \in \{1, ..., n\}$ and $|f(x)| < \varepsilon$ for every $x \in X - (\bigcup_i P_i)$, $f \in \mathscr{F}$.

Proof: First we assume that $\mathscr{F}$ *is compactoid in* $C_\infty(X)$ (which is the same as compactoid in *PC(X)*, see [8] theorem 4.1). Then property (a) is satisfied and there is, for a given $\varepsilon>0$, a finite number of clopen sets $X_1,\ldots,X_n$ verifying condition (b) of theorem 1. On the other hand let $\varphi_1,\ldots,\varphi_m$ be in $C_\infty(X)$ such that $\mathscr{F}\subset B_\varepsilon(0)+c_o(\{\varphi_1,\ldots,\varphi_m\})$ and let K be a compact clopen set in X such that $|\varphi_j(x)|<\varepsilon$ for all $x\in X-K$ and $j\in\{1,\ldots,m\}$. Now we define $P_i=X_i\cap K$ for every $i\in\{1,\ldots,n\}$; it is easy to check that $P_1,\ldots,P_n$ satisfy property (b).

In order to prove the converse, it is enough to take $X_i=P_i$ for $i=1,\ldots,n$ and $X_{n+1}=X-(\bigcup_{i=1}^{n} P_i)$ and then apply theorem 1.

Another characterization of compactoids in $C_\infty(X)$ is contained in the following corollary which is an easy consequence of the above results.

Corollary 3: *A subset* $\mathscr{F}$ *of* $C_\infty(X)$ *is compactoid if and only if*

(a) $\mathscr{F}(x)$ *is bounded in* $\mathbb{K}$ *for every* $x\in X$.

(b) $\mathscr{F}$ *is equicontinuous.*

and (c) For every $\varepsilon>0$, *there exists a compact set* K *in* X *such that* $|f(x)|<\epsilon$ *for every* $f\in\mathscr{F}$ *and every* $x\in X-K$.

2. ULTRA K-SPACES

A topological space X is called a k-space when a subset $A\subset X$ is open if $A\cap K$ is open in K for every compact set K in X. More generally X is called a k_Y-space (for a given topogical space Y) if $f: X\to Y$ is continuous when f/K is continuous for each compact $K\subset X$.

Definition 4: *A zerodimensional space X is called an ultra k-space (or a* k_o*-space, see* [16] p. 273) *if it is a* $k_{\{0,1\}}$*-space, where* $\{0, 1\}$ *is endowed with the discrete topology.*

Theorem 5: *The following properties are equivalent for a zerodimensional topological space X.*

(a) X is an ultra k-space.

(b) $A\subset X$ *is clopen if and only if* $A\cap K$ *is clopen in K for each compact set K in X.*

(c) X is a k_Y*-space for every separated zerodimensional topological space Y.*

(d) *X is a $k_{\mathbb{K}}$-space for every non-archimedean valued field $\mathbb{K}$.*

(e) *There exists a non-archimedean valued field $\mathbb{K}$ for which X is a $k_{\mathbb{K}}$-space.*

Proof: (a)$\Longrightarrow$(b). Let $A \subset X$ be such that $A \cap K$ is clopen in K for every compact set K in X and let $f = \xi_A : X \to \{0, 1\}$ be the characteristic function of A. Then, f/K is continuous for every compact set K and hence f is continuous; that is, A is clopen.

(b)$\Longrightarrow$(c), (c)$\Longrightarrow$(d) and (d)$\Longrightarrow$(e) are obvious. In order to prove (e)$\Longrightarrow$(a) it is enough to notice that $\{0, 1\}$ has the topology of a subspace of $\mathbb{K}$.

Remarks: (1) The above theorem suggests the following question: Is every zerodimensional ultra k-space a k-space? The answer is no. The space $\mathbb{N}^I$ ($\mathbb{N}$ with the discrete topology and I an uncontable index set) endowed with the product topology is a zerodimensional $k_{\mathbb{R}}$-space (which implies it is an ultra k-space) but is not a k-space (see [1], p. 65).

(2) The preceding remark leads to the following open question: Is every zerodimensional ultra k-space a $k_{\mathbb{R}}$-space?

(3) There are examples of zerodimensional spaces which are not ultra k-spaces; that is the case of the so-called space of Arens (see [12], p. 77).

Also, if $\mathbb{K}$ is not locally compact, $c_o = C_\infty(\mathbb{N})$ with the weak topology $\sigma(c_o, l^\infty)$ is another zerodimensional space which is not an ultra k-space: the unit ball $\{x \in c_o : \|x\| \leq 1\}$ is not clopen for $\sigma(c_o, l^\infty)$ whereas its intersection with every weakly compact set K is clopen in K because on K the norm topology and the weak topology coincide [18, theorem 3.8].

3. EQUICONTINUOUS SETS IN C(X)

Now we are going to consider the space $C(X)$ endowed with the topology of uniform convergence on compact sets.

Our first result, related to completeness of $C(X)$, is an obvious consequence of our theorem 5 and theorem 3.2 in [11].

Proposition 6: *The following properties are equivalent,*

(a) *C(X) is complete.*

(b) *C(X) is quasicomplete (that is, every bounded and closed subset of C(X) is complete).*

(c) *X is an ultra k-space.*

Theorem 7: *If X is an ultra k-space then, every compactoid subset in $C(X)$ is equicontinuous.*

Proof: Let $H: X \to C(X)'$ be defined by $H(x) = H_x$ where as in theorem 1 $H_x(f) = f(x)$. It is obvious that H is continuous if we choose the topology $\sigma(C(X)', (C(X))$ on $C(X)'$.

Also, for a given compact K in X, $H(K)$ is equicontinuous because $H(K) \subset \{f \in C(X): \sup_{x \in K} |f(x)| \leq 1\}^\circ$. Since on equicontinuous sets of the dual of a locally convex space the weak topology coincides with the topology τ_{co} of uniform convergence on compactoids (see [17], lemma 10.6), we deduce that $H: X \to (C(X))', \tau_{co})$ is continuous on compact sets of X. Hence, as X is an ultra k-space, it follows that H is continuous.

Now let $\mathscr{F}$ be a compactoid in $C(X)$. By continuity of H, given $\epsilon > 0$ and $x \in X$ there is a neighborhood U of x in X and $v \in \mathbb{K}$ with $|v| < \epsilon$ such that.

$$y \in U \Rightarrow H_y - H_x \in v\,\mathscr{F}^\circ \Rightarrow |f(y) - f(x)| \leq |v| < \epsilon \text{ for each } f \in \mathscr{F}.$$

Thus, every compactoid subset of $C(X)$ is equicontinuous.

Theorem 8: *Let X be an ultra k-space and let $\mathscr{F} \subset C(X)$. Then $\mathscr{F}$ is compactoid if and only if $\mathscr{F}$ is equicontinuous and $\mathscr{F}(x)$ is bounded in $\mathbb{K}$ for every $x \in X$.*

Proof: By theorem 7, $\mathscr{F}$ compactoid implies $\mathscr{F}$ equicontinuous and it is obvious that $\mathscr{F}(x)$ is bounded in $\mathbb{K}$ for every $x \in X$.

Conversely let K be a compact subset of X. By corollary 3, $\mathscr{F}/K = \{f/K: f \in \mathscr{F}\}\}$ is compactoid in $C(K)$. This implies that for a given $\epsilon > 0$ there exist $f_1, \ldots, f_n \in C(K)$ such that

$$\mathscr{F}/K \subset \{g \in C(K): \sup_{x \in K} |g(x)| \leq \epsilon\} + c_o\{f_1, \ldots, f_n\}$$

Now, if we extend each f_i to a continuous map $\hat{f}_i: X \to \mathbb{K}$ [16, theorem 5.24], we have,

$$\mathscr{F} \subset \{g \in C(K): \sup_{x \in K} |g(x)| \leq \epsilon\} + c_o\{\hat{f}_1, \ldots, \hat{f}_n\}$$

which implies that $\mathscr{F}$ is compactoid.

4. THE CASE OF THE STRICT TOPOLOGY

The strict topology in the space $BC(X)$ of all bounded continuous functions $f: X \to \mathbb{K}$ was introduced in the non-archimedean setting by J.B. Prolla [14, chapter 9] in case X is locally compact. For general zerodimensional spaces X the strict topology has been studied by A.C.M. Van Rooij [16] and A.K. Katsaras ([9] and [10]).

This topology is defined by the family of seminorms $\{p_\varphi : \varphi \in B_\infty(X)\}$ where $B_\infty(X)$ is the set of all bounded functions $\varphi: X \to \mathbb{K}$ which vanish at infinity and

$$p_\varphi(f) = \sup_{x \in X} |\varphi(x) f(x)|$$

The strict topology τ_β in $BC(X)$ is between the topology τ_c of uniform convergence on compact sets and the topology τ_u of uniform convergence; this is $\tau_c \leq \tau_\beta \leq \tau_u$ ([10], 2.10).

In particular for $X = \mathbb{N}$ with the discrete topology, the strict topology in l^∞ coincides with the natural topology in the sense of perfect spaces of sequences (see [3]).

Proposition 9: *The following properties are equivalent for the strict topology in $BC(X)$,*

(a) $BC(X)$ is complete.

(b) $BC(X)$ is quasicomplete.

(c) X is an ultra k-space.

Proof: (a)$\Longrightarrow$(b) is obvious. In order to prove (b)$\Longrightarrow$(c) we consider $f: X \to \{0, 1\}$ which is continuous on compact sets. Let for every compact subset K in X, $\hat{f}_K: X \to \mathbb{K}$ be a continuous extension of f/K to X such that

$$\sup_{x \in X} |\hat{f}_k(x)| = \sup_{x \in K} |f(x)| \leq 1$$

[16, theorem 5.24]. Let us see that $A = \{g \in BC(X): \|g\|_\infty \leq 1\}$ is τ_c-closed (and hence τ_β-closed); assume $g \notin A$ and choose $x \in X$ such that $|g(x)| > 1$. Then, $\{h \in BC(X): |h(x) - g(x)| < 1\}$ has empty intersection with A. Also A is τ_β-bounded, which implies A is complete for the strict topology. Furthermore, A is complete for the topology τ_c of uniform convergence on compact sets because τ_c coincides with the strict topology on uniform bounded sets ([10], 2.9).

If we denote by $\mathscr{K}$ the directed set of all compact subsets of X ordered by inclusion, it is easy to check that $(\hat{f}_K)_{K \in \mathscr{K}}$ is a Cauchy net in A for the topology of uniform convergence on compact sets; let $g \in A$ be its limit. On the other hand it is obvious that for each $x \in X$, $f(x) = \lim (\hat{f}_K(x))_{K \in \mathscr{K}}$. Hence, we conclude that $f = g$ is continuous. The proof of (c)$\Longrightarrow$(a) is the same as its archimedean counterpart in which X is assumed to be a k-space (see [7], theorem 9, p. 72).

Theorem 10: *Let X be an ultra k-space. A subset $\mathscr{F} \subset BC(X)$ is compactoid for the strict topology if and only if the following properties are satisfied,*

(a) $\sup\{|f(x)| : f \in \mathscr{F}, x \in X\} < \infty$.

(b) $\mathscr{F}$ *is equicontinuous.*

Proof: First assume that $\mathscr{F}$ is compactoid. Then, $\mathscr{F}$ is also compactoid in the topology of uniform convergence on compact sets, which implies that $\mathscr{F}$ is equicontinuous (theorem 7). Also if $\mathscr{F}$ is compactoid, then $\mathscr{F}$ is τ_β-bounded which implies (a) [10, prop.2.11].

Conversely let $\epsilon > 0$ and $\varphi \in B_\infty(X)$. Let K be a compact set in X such that $|\varphi(x)| < \epsilon$ if $x \in X\text{-}K$ and let $M = \sup_{x \in X} |\varphi(x)|$. Since $\mathscr{F}/K = \{f/K : f \in \mathscr{F}\}$ is compactoid in $C(K)$ (theorem 1), there are $f_1, \ldots, f_n \in C(K)$ such that

$$\mathscr{F}/K \subset \{g \in C(K) : \sup_{x \in K} |g(x)| \leq \epsilon\} + c_o\{f_1, \ldots, f_n\}$$

Let $\hat{f}_i : X \to \mathbb{K}$ $(i = 1, \ldots, n)$ be a continuous extension of f_i such that $\sup_{x \in X} |\hat{f}_i(x)| \leq S$ where $S = \max_i \sup_{x \in K} |f_i(x)|$ [16, theorem 5.24]. Then, $\hat{f}_i \in BC(X)$ for $i = 1, \ldots, n$ and

$$\mathscr{F} \subset \{g \in BC(X) : \sup_{x \in K} |g(x)| \leq \epsilon\} + c_o\{\hat{f}_1, \ldots, \hat{f}_n\}$$

which implies

$$\mathscr{F} \subset \{g \in BC(X) : \sup_{x \in X} |\varphi(x)\, g(x)| \leq C\epsilon\} + c_o\{\hat{f}_1, \ldots, \hat{f}_n\}$$

where $C = \max\{\sup\{|f(x)| : f \in \mathscr{F}, x \in X\}, M, S\}$.

In particular, $\mathscr{F} \subset l^\infty$ is compactoid for the strict topology if and only if $a = \sup\{|f(n)| : n \in \mathbb{N}, f \in \mathscr{F}\} < \infty$. In this case $\mathscr{F}$ is contained in the normal hull of the constant map $g \equiv \lambda$ where $|\lambda| \geq a$. This is a particular case of [3, theorem 3.6] and [13, proposition 2.1] where more results on compactoids in perfect spaces of sequences are found.

References

1. E. BECKENSTEIN, L. NARICI and C. SUFFEL, *Topological algebras*, North-Holland, Amsterdam, 1977.
2. N. DE GRANDE-DE KIMPE, *The non-archimedean space $C^{\infty}(X)$*, Comp. Math. 48 (1983), 297-309.
3. N. DE GRANDE-DE KIMPE, *Nuclear topologies on non-archimedean locally K-convex spaces*, Proc. Kon. Ned. Akad. van Wetensch A90 (1987), 279-292.
4. P. G. O. FREUND and M. OLSON, *Non-archimedean strings*, Phys. Lett. B 199 (1987), 186-190.
5. B. GROSSMAN, *p-adic strings, the Weil conjectures and anomalies*, Phys. Lett. B 199 (1987), 101-104.
6. L. GRUSON and M. VAN DER PUT, *Banach spaces*, Bull. Soc. Math. France, Mem, 39-40 (1974), 55-100.
7. H. JARCHOW, *Locally convex spaces*, Teubner, Stuttgart, 1981.
8. A. K. KATSARAS, *On compact operators between non-archimedean spaces*, Ann. Soc. Sci. Bruxelles. Sér. I 96 (1982), 129-137.
9. A. K. KATSARAS, *Strict topologies in non-archimedean function spaces*, Internat. J. Math. & Math. Sci. 7 (1984), 23-33.
10. A. K. KATSARAS, *The strict topology in non-archimedean vector-valued function spaces*, Proc. Kon. Ned. Akad. van Wetensch A87 (1984), 189-201.
11. A. K. KATSARAS, *Spaces of non-archimedean valued functions*, Boll. Unione Mat. Ital B5 (1986), 603-621.
12. J. L. KELLEY, *General Topology*, Springer, New York, 1955.
13. C. PÉREZ-GARCÍA, *On compactoidity in non-archimedean locally convex spaces with a Schauder basis*, Indag. Math. 50 (1988), 85-88.
14. J. B. PROLLA, *Approximation of vector valued functions*, North-Holland, Amsterdam, 1977.
15. R. RAMMAL, G. TOULOUSE and M. A. VIRASORO, *Ultrametricity for physicists*, Rev. Modern Phys. 58 (1986). 765-788.
16. A. VAN ROOIJ, *Non-archimedean functional analysis*, Marcel Dekker, New York, 1978.
17. W. H. SCHIKHOF, *Locally convex spaces over nonspherically complete valued fields*, Groupe d'etude d'Analyse ultrametrique, 12^{e} annee 24 (1984/85), 1-33.
18. W. H. SCHIKHOF, *On weakly precompact sets in non-archimedean Banach spaces*, Report 8645, Mathematisch Instituut, Katholieke Universiteit, Nijmegen (1986), 1-14.
19. W. H. SCHIKHOF, *Compact-like sets in non-archimedean functional analysis*, Proc. Conference on p-adic analysis, Hengelhoef (1986). Editors: N. De Grande-De Kimpe and L. Van Hamme, 19-30.

Departamento de Matemáticas
Universidad de Cantabria
Facultad de Ciencias
Avda. Los Castros
39071 Santander (España)

Recibido: 19 de agosto de 1988

Departamento de Matemáticas
Universidad de Santiago de Chile
Casilla 5659 C-2
Santiago (Chile)