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A glimpse at the theory of Jordan-Banach triple systems

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ABSTRACT. In this article, a survey of the theory of Jordan-Banach triple systems is presented. Most of the recent relevant results in this area have been included, though no proofs are given.

0. PRELIMINARIES AND GEOMETRIC BACKGROUND

In what follows, E denotes a complex Banach space and $D\subset E$ is a domain. Two domains $D_1\subset E_1$ and $D_2\subset E_2$ are *holomorphically isomorphic* (briefly, isomorphic) if there is a biholomorphic bijection $f:D_1\to D_2$ of D_1 onto D_2. An isomorphism of D onto itself is called an *automorphism* of D. The set Aut$D$ of all automorphisms of D is a group in a natural way.

Only the special class consisting of bounded symmetric domains is going to be considered. An automorphism $s\in$ Aut D is said to be a *symmetry* of D around the point $x_0\in D$ if s is involutive (i.e, $s^2=s$) and x_0 is an isolated fixed point for s. A symmetry of D at x_0, if it exists, is unique and D is said to be *symmetric* if there is a symmetry $s_x\in$ Aut$D$ for each $x\in D$. Thus, for symmetric domains D the group Aut D is plentiful.

A holomorphic vector field $X(x)\dfrac{\partial}{\partial x}$ is said to be *complete* in D if, for every $x_0\in D$, the initial value problem

$$\frac{d}{dt}f(t)=X[f(t)],\quad f(0)=X_0$$

has a solution which is valid on the whole real line $\mathbb{R}$. The set aut$D$ of complete holomorphic vector fields has a natural Lie algebra structure.

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In 1976, Upmeier and Vigué independently proved the following infinite dimensional version of H. Cartan's theorem on groups of holomorphic transformations.

1. Theorem. *Let D be a bounded symmetric domain in a complex Banach space E. Then*

a) AutD is a real Banach-Lie group which acts analytically (in the real sense) on D.

b) autD is a purely real Banach-Lie algebra which is isomorphic to the Lie algebra of AutD.

The way in which the underlying topological and manifold structures in Aut$D$ and aut$D$ were constructed is not relevant for our pourpose.

Let us fix a point $p \in D$ (with no loss of generality, one may assume that p is the origin 0 of E). With respect to a suitable chart, the symmetry s of D at 0 gives a splitting of aut D into a topologically direct sum of linear subspaces:

$$autD = K \oplus P \tag{1}$$

For us, the important fact is that P consists of quadratic vector fields that have the form

$$X(x)\frac{\partial}{\partial x} = (a - q_a(x))\frac{\partial}{\partial x} \tag{2}$$

for some $a \in E$ and some continuous homogeneous polynomial q: $E \to E$ of degree 2, $q_a \in Q(^2E)$, the mapping $a \to q_a$ being continuous and conjugate linear.

Vigué also proved the Banach version of the Harish-Chandra realization for bounded symmetric domains in $\mathbb{C}^n$:

2. Theorem. *Let D be a bounded symmetric domain in a complex Banach space E. Then D is isomorphic to a (bounded symmetric) balanced domain* $\tilde{D}$ *of E.*

A decisive step was given by Kaup and Upmeier who made a close study of the *orbit* Aut$D(0) = \{f(0): f \in \text{Aut}D\}$ of the origin under the group Aut D. They proved

3. Theorem. *Let D be a bounded circular (but not necessarily symmetric) domain in a Banach space E. Then*

a) There is a closed complex subspace F of E such that $AutD(0) = D \cap F$.

b) The Lie algebra autD splits in the form (1) and the mapping $F \mapsto Q(^2E)$ *given by* $a \mapsto q_a$ *is an isomorphism of the underlying real Banach spaces F and* $Q(^2E)$.

c) Any $f \in AutD$ splits in the form $f = L \cdot M$ where L is (the restriction to D of) a surjective linear isometry of E and M, called a Möbius transformation, is the solution of the initial value problem

$$\frac{d}{dt} y(t) = a - q_a[y(t)], \quad y(0) = b$$

for some $a \in F$ and $b \in D \cap F$. The domain D is symmetric if, an only if, $F = E$.

This result reveals the existence of a close connection between the orbit $\mathrm{Aut}D(0)$, the quadratic vector fields $(a - q_a(x))\frac{\partial}{\partial x}$ and the mapping $a \mapsto q_a$ and it suggests the notion of a *Jordan-Banach triple system* (or JB*-triple) which shall be dealt with in the next section. However, triple systems had already been introduced by Koecher and Loos as a vehicle for classifying bounded symmetric domains in the $\mathbb{C}^n$ setting.

1. JB*-TRIPLE SYSTEMS. ELEMENTARY PROPERTIES

A JB*-triple is a complex Banach space E with a ternary law of composition $E \times E \times E \to E$, denoted by $\{x, y^*, z\}$ and called the *triple product*, with the following properties:

J_1: The triple product $\{x, y^*, x\}$ is continuous in (x, y, z), symmetric and linear in the external variables x, z, and conjugate linear in the middle variable y.

Let $a \Box b^*$ stand for the bounded linear operator $x \mapsto \{a, b^*, x\}$. Then, for $x, y, u, v \in E$

J_2: $[x \Box y^*, u \Box v^*] = \{x, y^*, u\} \Box v^* - u \Box \{v, x^*, y\}^*$
where $[A, B] =: AB\text{-}BA$ is the commutator product in $\mathcal{L}(E)$.

J_3: For $a \in E$, $a \Box a^*$ is a hermitian positive element of the algebra $\mathcal{L}(E)$.

J_4: For $a \in E$, one has $\|a \Box a^*\| = \|a\|^2$

Axiom J_1 appears as an abstract formulation of the properties prosessed by the function $q_a(x, y) = \{x, a^*, y\}$, i.e. the symmetric bilinear mapping corresponding to the homogeneous component of the quadratic vector field $(a - q_a(x))\frac{\partial}{\partial x}$. Axiom J_2 is also known as the *Jordan Identity*. Motivations for axioms J_2, J_3 and J_4 can be found, in the $\mathbb{C}^n$ setting, in Loos [L1].

Homomorphisms, isomorphisms and automorphisms between JB*-triples can be introduced in the usual manner. Thus, JB*-triples form a category.

In §0(Th.3.b), a JB*-triple was associated to each bounded symmetric domain D in E. Notice that a point (supposed to be the origin and referred to as the *base point* of D) has been distinguished when constructing the chart at 0 in which the component in P of any complete vector field has the form

$$f(x)\frac{\partial}{\partial x}=(a-q_a(x))\frac{\partial}{\partial x}$$

If two bounded symmetric domains are isomorphic, then their associated JB*-triples are also isomorphic. If a JB*-triple E is given, one can find a bounded symmetric domain D having E as associated system, whence the following result holds

4. Theorem. *There is an equivalence between the category of JB*-triples and that of bounded symmetric domains with base point.*

Thus, JB*-triples appear as a natural algebraic-metric setting for the study of bounded symmetric domains. There are some other reasons for the study of this structure.

Firstly, this category is large enough to contain several others that are well known in Functional Analysis:

1) Any complex Hilbert space H with scalar product $(.|.)$ becomes a JB*-triple in the product

$$2\{x,y^*,z\}=:(x|y)z+(z|y)x$$

2) Any complex C*-algebra becomes a JB*-triple in the product

$$\{x,y^*,z\}=:\frac{1}{2}(xy^*z+zy^*x)$$

3) Any complex Jordan-Banach algebra (briefly JB*-algebra) with product $\circ$ and involution $*$ is a JB*-triple by setting

$$\{x,y^*,z\}=x\circ(y^*\circ z)-y^*\circ(z\circ x)+z\circ(x\circ y^*)$$

4) Let H and K be complex Hilbert spaces, and let $L(H,K)$ be the space of bounded linear operators with the operator norm. A norm-closed complex subspace U of $L(H,K)$ is a J*-algebra if $AA^*A\in U$ whenever $A\in U$. Here, A^* stands for the usual adjoint operator of A. Any J*-algebra becomes a JB*-triple in the product

$$\{A,B^*,C\}=:\frac{1}{2}(AB^*C+CB^*A)$$

Notice that in examples 2 and 3, the binary product is respectively associative (but non-commutative) and commutative (but non-associative) whereas in example 4 there is no binary product. (See [HA.1] for an account of J*-algebras)

On the other hand, the category of JB*-triples behaves reasonably well so as to have nice properties, and it is closed under many usual operations in Functional Analysis:

1) Any JB*-triple E is locally isomorphic to a commutative C*-algebra. More precisely, the subtriple generated by a single element $a \in E$ is isomorphic to (the triple corresponding by example 3 to) the C*-algebra $C_0(\Omega)$ of continuous functions that vanish at infinity in $\Omega(=$the spectrum of the operator $a\Box a^* \in \mathcal{L}(E))$. This is important because many problems can be solved locally.

2) Suppose $f: E \mapsto F$ is an *algebraic homomorphism* between JB*-triples, that is, f is a linear mapping such that

$$f(\{x,y^*,z\}) = \{f(x), f(y)^*, f(z)\} \qquad (x,y,z \in E)$$

Then f is a contraction, i.e. $\| f \| \leqslant 1$. In particular, any algebraic homomorphism of JB*-triples is continuous, any algebraic isomorphism is an isometry, the norm and the triple product are uniquely determined by each other, and

$$\|\{x,y^*,z\}\| \leqslant \|x\| \; \|y\| \; \|z\| \qquad (x,y,z \in E)$$

3) Let $(E_i)_{i\in I}$ be an indexed family of JB*-triples, and set

$$E =: \bigoplus_{\infty} E_i =: \{(x_i)_{i\in I} \in \Pi E_i : \sup_{i\in I} \|x_i\| < \infty\}$$

Then E with the supremun norm and the coordinatewise triple product $\{(x_i),(y_i)^*,(z_i)\} =: (\{x_i y_i^* z_i\})_{i\in I}$ becomes a JB*-triple.

4) Since in the triple product, the middle variable does not behave like the external ones, one is led to define the *ideals* of a JB*-triple E as those linear subspaces F of E for which $\{F,E^*,E\} \subset F$ and $\{E,F^*,E\} \subset F$ (3)

The kernel $f^{-1}(0)$ of any homomorphism between JB*-triples is an ideal. For closed ideals F, the quotient space E/F with the quotient norm and the triple product: $\{x+F,\ y^*+F,\ z+F\} =: \{x,y^*,z\}+F$ $(x,y,z \in E)$ is a JB*-triple.

5) Let E be a JB*-triple, and assume that $P \in L(E)$ is a contractive projection, (i.e. $P^2 = P$ and $\|P\| \leqslant 1$). Then $F =: P(E)$ is a JB*-triple in the product

$$\{x,y^*,z\}_F =: P(\{x,y^*,z\}_E), \qquad (x,y,z \in F)$$

6) Let E be a JB*-triple, I a set of indices and $U \subset P(I)$ an ultrafilter in I. Denote by $l^\infty(E,I)$ *the* l^∞-direct sum of the spaces $E_i = E$ as defined in example 3. The set of the U-null sequences $N =: \{(x_i)_{i \in I}: \lim_U \|x_i\| = 0\}$ is a closed subspace of $l^\infty E,I)$. Denote by $E^u =: l^\infty(E,I)/N$ the quotient space and put $\tilde{x} =: = (x_i) + N$ for the equivalence class of $(x_i)_{i \in I} \in l^\infty(E,I)$. Then E is a JB*-triple in the norm $\|\tilde{x}\| = \lim_U \|x_i\|$ and triple product

$$\{(x_i)^\sim, (y_i)^{\sim *}, (z_i)^\sim\} =: (\{x_i y_i^* z_i\})^\sim_{i \in I}$$

As a consequence of this [D.1]

7) the bidual E^{**} of a JB*-triple E is again a JB*-triple, the canonical inclusion $J: E \to E^{**}$ is a homomorphism of triples, and the triple product in E^{**} extends that of E. Moreover, if B_E is the open unit ball of E, any holomorphic automorphism $f \in \mathrm{Aut} B_E$ (see section §0 extends to a holomorphic automorphism $f^{**} \in \mathrm{Aut} B_{E^{**}}$ of the unit ball of E^{**}.

2. TRIPOTENTS, PEIRCE DECOMPOSITION AND EXTREME POINTS

An element e of a JB*-triple E is a *tripotent* if $\{e,e^*,e\} = e$. In the study of triple systems, tripotents play the same role of projections in C*-algebras. Due to the Jordan identity, if e is a tripotent in E, the operator $e \square e^* \in \mathcal{L}(E)$ has the eigenvalues 0,1/2, 1, and E splits into a direct topological sum of the corresponding eigenspaces $E = E_0 \oplus E_{1/2} \oplus E_1$ which are JB*-subtriples of E. This is the *Peirce decomposition* of E relative to e. Besides, E_1 is a JB*-algebra in the product $x.y =: \{x,e^*,y\}$ for which e is a unit.

A tripotent e is regular if $e \square e^*$ is a regular element of the algebra $\mathcal{L}(E)$. One has

5. Theorem. *For any tripotent $e \in E$, the following assertions are equivalent:*

1) e is a regular tripotent.
2) the 0-Peirce projector of e, $P_0(e)$, is null.
3) e is a real extreme point of the unit ball B_E.
4) e is a complex extreme point of B_E.

Notice that statement 2 above characterizes the extreme points (either real or complex) of B_E in purely algebraic terms. The set of regular tripotents is preserved by JB^*-isomorphisms and by all transformations in AutB_E. If extr$\bar{B}_E$ is not empty, then any JB*-isomorphism of E and any $f \in$ AutB_E is uniquely determined by its values at that set. However, extr$\bar{B}_E$ is sometimes a plentiful set (as occurs when E is a Hilbert space), but it may be empty (as occurs when E is the JB*-triple $C_0(H)$ of compact operators on H). By the Krein-Milman theorem, in order to ensure the existence of extreme points, one is led to consider triples that are dual Banach spaces.

3. JBW*-TRIPLES, IDEALS AND STRUCTURE THEORY

A JB*-triple E is called a *JBW*-triple* if E is the dual of a Banach space E_* and the triple product is $\sigma(E,E_*)$-continuous in each variable separately. In that case, E_* is referred to as a *predual* of E.

Peirce projectors and JB*-automorphisms in a JBW*-triple E with predual E_* are $\sigma(E,E_*)$-continuous. However, holomorphic automorphisms of B_E, i.e. the elements of AutB_E, may fail to be so. See [SI.1] for a discussion of this problem.

In [BT.1] Barton and Timoney improved Dineen's ultrafilter argument to prove that, in the second dual E^{**} of a JB*-triple E, the triple product is separately $\sigma(E^{**},E^*)$-continuous, thus providing an important family of JBW*–triples.

In [HO.1] Horn showed that any JBW*-triple E has a unique predual E_*, and that in a JBW*-triple which has a unique predual E_*, the triple product is separately $\sigma(E,E_*)$-continuous. These results were again improved by Barton and Timoney who showed the following:

6. Theorem. *Let E be a JB*-triple which is a dual Banach space. Then E has a unique predual E_* and the triple product is separately $\sigma(E,E_*)$-continuous.*

Notice that this theorem is not implied by any of the partial results mentioned before. As a consequence, the requirement concerning the separate $\sigma(E,E_*)$-continuity of the triple product in a JBW*-triple is automatically satisfied, and may be dropped from the definition. Also, because of the uniqueness of E_*, a JBW*-triple has a well defined *weak-** topology. By the Krein-Milman theorem, the set extr$\bar{B}_E$ of extreme points is weak-* dense in $\bar{B}_E$ (actually, it is norm-total in E by [HO.1]).

As in any Banach space, in a JB*-triple E one can consider *M-summands* and *M-ideals* as introduced by Alfsen and Effros [AE.1]. One also has the *J*-ideals* (briefly, ideals) that arise from the triple product in E, as defined in (3), and the relationships between these concepts are:

7. Theorem. *The closed ideals of a JB*-triple E are precisely its M-ideals.*

Closed ideals may fail to be M-summands; however for dual triples one has:

8. Theorem. *The weak-* closed ideals of a JBW*-triple are precisely its M-summands.*

For dual C*-algebras and dual JB*-algebras, the set of *projections* is a complete lattice, and so order structures play an important role in their study. But the category of JB*-triples is based only on geometry, and there is no order structure in it. Despite this fact, Friedman and Russo [FR.1] got a characterization of the state space of a JB*-triple similar to that of Alfsen and Shultz [AS.1] for JB*-algebras.

A tripotent e is said to be *minimal* if its 1-Peirce projector $P_1(e)$ satisfies $P_1(e)E = \mathbb{C}e$. If E is JBW*-triple, then any extreme point $a \in extr\mathring{B}_{E_*}$ of the unit ball of E_* is called an *atom*. Friedman and Russo established a bijection between the set of atoms in E_* and the set of minimal tripotents in E, and using this, they proved the following structure theorems.

9. Theorem. *Let E be a JBW*-triple with predual E_*. Then $E_* = A \oplus_{l_1} N$ where A is the norm - closure of the linear span of the atoms of E, and the unit ball of N has no extreme points.*

As a consequence,

10. Theorem. *Let E be a JBW*-triple. Then E splits into an l_∞-direct sum of two ideals $E = A \oplus_\infty N$, where A is the weak-* closure of the linear span of its minimal tripotents, and N has no minimal tripotent.*

A JBW*-triple E for which $N=0$ is said to be *atomic*, and E is called a *factor* if it cannot be written as an l_∞-direct sum of two proper weak-* closed ideals. A factor that contains a minimal tripotent is called a *JBW*-triple factor of type I.* Here are some examples known as *Cartan Factors*, as they were introduced by E. Cartan in 1935 to solve the problem of the analytic classification of bounded symmetric domains in $\mathbb{C}^n$. The infinite-dimensional version of the (non exceptional) Cartan factors is due to Harris, who also noticed that they are J*-algebras (§**1**, example 4).:

Let $U = L(H,K)$ be a J*-algebra of operators, and suppose that Q is a conjugation on H (i.e. a conjugate linear mapping with $Q^2 = I$, $\|Q\| \leqslant I$). Let $A \mapsto A^t$ be its associated transposition on $L(H)$, where $A^t =: QA^*Q$. Then U is said to be a Cartan factor of

Type C_1 if $U=L(H,K)$
Type C_2 if $U=\{A \in L(H): A^t=A\}$
Type C_3 if $U=\{A \in L(H): A^t=-A\}$
Type C_4 if U is a closed complex subspace of $L(H)$ such that $U^* \subset U$ (i.e. U is selfadjoint) and $U^2=:\{A^2: A \in U\} \subset \mathbb{C}I$.

Besides these spaces, there are two *exceptional* Cartan factors C_5 and C_6 which cannot be described as J*-algebras, though they can be isometrically embedded as subtriples of (the JB*-triple asssociated by example 3 in § 1 to) the JB*-albegra $H(\mathbb{O})$ of all 3×3 matrices with entries in the complex Cayley algebra $\mathbb{O}$, which are hermitian with respect to the canonical involution in $\mathbb{O}$. (See [L.1] for details).

A relevant property of these spaces is the following characterization of JBW*-triple factors of type I due to Horn:

11. Theorem. *Every JBW*-triple factor of type I is isomorphic to a Cartan factor (of type C_1 to C_6).*

Another important aspect of them is the following Gelfand-Neimark representation result due to Friedman and Russo [FR.2].

12. Theorem. *Every JB*-triple is isometrically isomorphic to a subtriple of an l_∞-direct sum of Cartan factors, and so isomorphic to a subtriple of $L(H) \oplus_\infty C(S,C_6)$.*

Here H is a suitable Hilbert space, and $C(S,C_6)$ is the JB*-triple of all C_6-valued continuous functions on the Stone-Čech compactification S of a discrete set.

Norm closed subtriples E of $L(H)$ are called *JC*-triples* and weak-* closed (i.e. ultraweakly or σ-weakly closed) subtriples of $L(H)$ are called *JW*-triples.* E is *special* if it is isometrically isomorphic to a JC*-triple, and E is *exceptional* if every representation (or homomorphism) of E into a JC*-triple is zero. One has [BD.1]:

13. Theorem. *Every JBW*-triple E has a unique decomposition $E=E_{sp} \oplus_\infty E_{exc}$ where E_{sp} is a special ideal of E and E_{exc} is a purely exceptional one. Moreover $E_{exc}=C(S',C_6) \oplus C(S'',C_6)$ for some hyperstonean spaces S' and S''.*

4. SOME GEOMETRICAL RESULTS

To conclude this survey, some relevant results of a geometric nature, due to Kaup ([K.2] and [K.3]), are mentioned.

Two elements, u,v of a JB*-triple E are *orthogonal* if $u\Box v^*=v\Box u^*=0$, and in this case one writes $u\perp v$. A set of tripotents $(e_\alpha)_{\alpha\in A}\subset E$ is *complete* if $x\in E$ and $x\perp e_\alpha$ for $\alpha\in A$ implies $x=0$. A JB*-triple which admits a complete system of minimal pairwise orthogonal tripotents is called an *atomic triple.* Two such complete systems have the same cardinal which is called the *rank* of E. A JB*-triple which is a reflexive Banach space is always atomic and has finite rank. Using this fact and the theory of finite rank triple systems, Kaup in 1981-1983 extended Cartan's classification theorem (for bounded symmetric domains in $\mathbb{C}^n$) to the class of reflexive Banach spaces. He proved

14. Theorem. *Every bounded symmetric domain D in a reflexive complex Banach space E can be represented in an (up to order) unique way as an* l_∞*-direct sum* $D=\bigoplus_{j=1}^{k} D_j$ *where each* D_j *is the unit ball of a Cartan factor.*

Except for a few cases in low dimensions [L.2], the unit balls of two Cartan factor $E'=E''$ are not holomorphically equivalent unless $E'=E''$. So this theorem gives a complete classification of bounded symmetric domains within the family of reflexive Banach spaces, and it has been conjectured that, beyond this family, no complete classification of symmetric domains will ever be found.

By ϕ**0**, th.2, every bounded symmetric domain D in an arbitrary complex Banach space E is holomorphically equivalent to a (bounded symmetric) balanced domain $\tilde{D}$, For $E=\mathbb{C}^n$, it was known that this $\tilde{D}$ is also convex, whereby renorming E with the corresponding Minkowski functional, $\tilde{D}$ becomes the unit ball of E. In 1983, Kaup got the infinite dimensional version of this result, thus obtaining a Riemann mapping theorem for Banach spaces [K.4].

15. Theorem. *Every bounded symmetric domain D in a complex Banach space E is biholomorphically equivalent to the open unit ball* $\tilde{D}$ *of a certain Banach space* $\tilde{E}$ *uniquely determined by D up to a linear isometry. The norms in E and* $\tilde{E}$ *are topologically equivalent.*

References

[AE.1] ALFSEN, E. and EFFROS, E. *Structure in real Banach spaces, I, II.* Ann. of Math, 96 (1972), 98-173.

[AS.1] ALFSEN, E. and SHULTZ, F. *State space of Jordan algebras.* Acta Math. 140 (1978), 155-190.

[BDH.1] BARTON, T. DANG, T. and HORN, G. *Normal representations of Banach-Jordan triple systems.* Preprint.

[BT.1] BARTON, T. and TIMONY, R. *Weak-* continuity of Jordan triple products and applications.* Math. Scand.

[BKU.1] BRAUN, R., KAUP, W. and UPMEIER, H. *On the automorphisms of circular and Reinhardt domains in complex Banach spaces.* Math. Math. 25, (1978), 97-133.

[CE.1] CARTAN, E. *Sur les domaines bornés homogenes d'espace de n variables complexes.* Abh. Math. Sem. Univ. Hamburg 11, (1935) 116-162.

[CH.1] CARTAN, H. *Les foncions de deux variables complexes et le probleme de la representation analytique.* J. Math. Pures Appl. 10 (1931) 1-114.

[D.1] DINEEN, S. *Complete holomorphic vector fields on the second dual of a JB*-triple system.* Math. Scand. 59 (1986) 131-142.

[FR.1] FRIEDMAN, Y and RUSSO, B. *Structure of the predual of a JBW*-triple.* J. Reine Ang. Math. 60 (1985), 56-79.

[FR.2] ______________________ *The Gelfand-Neimark theorem for JB*-triples.* Duke math. J. 53 (1986), 139-148.

[HA.1] HARRIS, L.A. *Bounded symmetric domains in infinite-dimensional spaces.* Proceedings on Infinite Dimensional Holomorphy, Springer Lecture Notes 364 (1974), 13-40.

[HA.2] __________ *A generalization of C*-algebras.* Proc. Lond. Math. Soc. 42 (1981), 331-361.

[HO.1] HORN, G., *Klassifikation der JBW*-tripel vom Typ I.* Ph.D. Dissertation Tubingen, 1984.

[KA.1] KAUP, W. *Algebraic characterization of symmetric complex Banach Manifolds.* Math. Ann, 228 (1977), 39-64.

[KA.2] ________ *Uber die Klassifikation der symmetrischen hermiteschen Mannigfaltigkeiten unendlicher Dimension, I.* Math Ann. 257(1981), 363-486; II Math. Ann. 262 (1983), 57-75.

[KA.3] ________ *A Riemann mapping theorem for bounded symmetric do mains in complex Banach spaces.* Math. Z. 183 (1983), 503-529.

[KU.1] KAUP, W. and UPMEIER, H. *Jordan algebras and symmetric Siegel domains in Banach spaces.* Math. Z. 157 (1977), 179-200.

[KO.1] KOECHER, M. *An elementary approach to bounded symmetric domains.* Lecture Notes, Rice University, 1969.

[L.1] LOOS, O. *Bounded symmetric domains and Jordan pairs.* Lecture Notes, Irvine, 1977.

[SI.1] STACHÓ, L. and ISIDRO, J. *Algebraically compact elements of JBW*-triples.* Preprint 1987.

[U.1] UPMEIER, H. *Uber die Automorphismengruppen von Banachmanigfeltigkeiten mit invarianter Metrik.* Math. Ann. 223 (1976) 279-288.

[V.1] VIGUÉ, J.P. *Le groupe des automorphismes analytiques d'un domaine borné d'un espace de Banach complexe.* Ann. Sci. Ecole Norm. Sup. IV, 9 (1976) 203-282.

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