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Singularities with exact Poincaré complex but not quasihomogeneous

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ABSTRACT. We give examples of complete intersections in C^3 with exact Poincarè complex but not quasihomogeneous using the classification of C.T.C. Wall and the algorithm of Mora.

0. INTRODUCTION

1971 K. Saito gave a nice numerical characterization for a hypersurface singularity to admit a C^*-action ((X,0) admits a good C^* -action (also called quasihomogeneous) if $\hat{O}_{X,0} \tilde{\to} C[[x_1, \ldots x_m]]/I$ and $I=(p_1, \ldots p_n)$ is an ideal generated by quasihomogeneous polynomials of positive degree d_i with respect to the weights $w_1, \ldots w_m$, ($w_i > 0, w_i \in Z$), i.e. $p_i(\lambda^{w_1}x_1, \ldots \lambda^{w_m}x_m) = \lambda^{d_i}p_i$) (cf. [8]):

Let $(X,0) \subseteq (C^m,0)$ be the germ of an isolated hypersurface singularity and denote by $\mu(X,0)$ the Milnor number of $(X,0)$ - a topological invariant of the singularity - and by $\tau(X,0)$ the Tjurina number of $(X,0)$ - the dimension of the mini-versal deformation of $(X, 0)$, an analytical invariant - then the following conditions are equivalent:

(1) $(X,0)$ admits a good C^*-action

(2) $\mu(X,0) = \tau(X,0)$

(3) the Poincarè complex of $(X,0)$

$$0 \to C \to O_{X,0} \to \Omega^1_{X,0} \to \Omega^2_{X,0} \to \ldots \to \Omega^m_{X,0} \to 0$$

is exact.

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The invarants μ and τ can be computed as the dimension of certain Artinian rings:

If $(X,0)$ is defined by $f=0$, $f \in C\{x_1,\ldots, x_m\}$ a convergent power series then

$$\mu(X,0)=\dim_C C\{x_1, \ldots,x_m\}/ \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_m} \right)$$

$$\tau(X,0)=\dim_C C\{x_1, \ldots,x_m\}/ \left(f, \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_m} \right)$$

A similar numerical characterization of quasihomogeneous singularities (not being hypersurfaces) can be given for curve singularities (cf. [3],[10]). Especially if $(X,0) \subseteq (C^m, 0)$ is a complete intersection curve with isolated singularity, then the following conditions are equivalent:

(1) $(X,0)$ is quasihomogeneous, i.e. admits a good C^*-action

(2) $\mu(X,0)=\tau(X,0)$

If $(X,0)$ is quasihomogeneous then the Poincarè complex

(3) $$0 \to C \to O_{X,0} \to \Omega^1_{X,0} \to \Omega^2_{X,0} \to \ldots \to \Omega^m_{X,0} \to 0$$

is exact.

It is natural to ask whether Saitos result can be generalized i.e. (3) implies (2).

It is the purpose of this note to show that this is not the case. We give examples of complete intersections in C^3 with exact Poincaré complex but not quasihomogeneous.

The idea to construct these examples is as follows:

C.T.C. Wall gave a classification of the unimodal complete intersection singularities, especially he computed the Milnor number

$\mu(X,0)=\dim \Omega^1_{X,0}/d\, O_{X,0}$ (cf. [11]).

If $f, g \in C\{x,y,z\}$ are power series defining an isolated curve singularity $(X,0)$, then

$\tau(X,0)=\dim_C C\{x,y,z,\}/(f,g,M_1, M_2,M_3)$

(cf. [4]), M_i the 2-minors of the Jacobian matrix of f,g.

Using Mora's algorithm to compute a Gröbner base (cf. [6]) one can compute $\tau(X,0)$ and decide whether $(X,0)$ is quasihomogeneous or not.

To prove the exactness of the Poincaré complex

$$0 \to C \to O_{X,0} \to \Omega^1_{X,0} \to \Omega^2_{X,0} \to \Omega^3_{X,0} \to 0$$

we use a result of Reiffen (cf. [7]):

$$\Omega^1_{X,0} \overset{d_2}{\to} \Omega^2_{X,0} \overset{d_3}{\to} \Omega^3_{X,0} \to 0$$

is exact iff

$$(f,g)\Omega^3_{C^3,0} \subseteq d((f,g)\Omega^2_{C^3,0}).$$

This condition is always satisfied if f and g are quasihomogeneous polynomials (not necessarily with the same weights).

It remains to check that

$ker\ d_2 = d_1 O_{X,0}$, i.e.

$$\begin{aligned}\mu(X,0) = \dim\Omega^1_{X,0}/d_1O_{X,0} &= \dim \Omega^1_{X,0}/\ker d_2\\ &= \dim d_2\Omega^1_{X,0}\\ &= \dim \ker d_3\\ &= \dim \Omega^2_{X,0} - \dim \Omega^3_{X,0}\end{aligned}$$

Now $\dim\Omega^3_{X,0} = C\{x,y,z\}/\ (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \overline{\frac{\partial g}{\partial z}})$ and $\dim \Omega^2_{X,0} = C\{x,y,z\}^3/M$,

M generated by

$(f,0,0), (0,f,0), (0,0,f), (g,0,0), (0,g,0), (0,0,g),$

$(\frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, 0), (\frac{\partial f}{\partial x}, 0, -\frac{\partial f}{\partial z}), (0, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}), (\frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}, 0),$

$(\frac{\partial g}{\partial x}, 0, -\frac{\partial g}{\partial z}), (0, \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}).$

Using again Mora's algorithm we can compute $\dim\Omega^3_{X,0}$ and $\dim\ \Omega^2_{X,0}$ and decide whether the Poincarè complex is exact or not. We were guided by a computer giving us the Gröbner bases and the corresponding dimensions for many examples. Especially we will give the invariants $\tau(X,0)$, $\dim\Omega^2_{X,0}$, $\dim\ \Omega^3_{X,0}$ for all singularities in C.T.C. Wallss classification (§ 3).

Also as an application we compare in § 4 Walls classification of quasihomogeneous singularities with the corresponding classification of Aleksandrov (cf. [1]).

In § 1 we will give the idea of Mora's algorithm to be able to follow our computations.

In § 2 we give the examples mentioned above.

1. MORA'S ALGORITHM

We will describe the algorithm to compute a Gröbner base of a submodule of a free module. This algorithm is a modification of Buchberger's algorithm to the case of modules over local rings (cf. [2], [6]). The actual implementation by the authors on Atari, IBM-compatible PC and MacIntosh is much more sophisticated (using some extra results and ideas to shorten the computations) as the idea described here.

Let $R=C\{x_1, ..., x_m\}$ and $x^{\underline{a}}=x_1^{a_1}...x_m^{a_m}$, $x^{\underline{b}}=x_1^{b_1}...x_m^{b_m}$ be monomials in R.

Definition 1: *$x^{\underline{a}}<x^{\underline{b}}$ if $\underline{a}\neq\underline{b}$ and either $\sum a_i<\sum b_i$ or $\sum a_i=\sum b_i$ and $a_1=b_1, ..., a_r=b_r, a_{r+1}<b_{r+1}$. We will write $x^{\underline{a}}\leq x^{\underline{b}}$ if $x^{\underline{a}}<x^{\underline{b}}$ or $x^{\underline{a}}=x^{\underline{b}}$.*

Definition 2: *Let $x_i^{\underline{a}}=(0,..., x^{\underline{a}}, 0...)$, $x_j^{\underline{b}}=(0,...,x^{\underline{b}}, 0...)$ be vectors of R^s ($x^{\underline{a}}$ at the i^{th} componente, $x^{\underline{b}}$ at the j^{th} componente). $x_i^{\underline{a}}<x_j^{\underline{b}}$ if either $i<j$ or $i=j$ and $x^{\underline{a}}<x^{\underline{b}}$. Furthermore we will use the notation $x_i^{\underline{a}} \mid x_j^{\underline{b}}$ iff $i=j$ and $x^{\underline{a}} \mid x^{\underline{b}}$.*

Using this order we can associate to $(g_1, ...,g_s)\in R^s$ $(\neq(0,...,0))$ the leading vector and leading coefficient: Let i be minimal such that $g_i\neq 0$ and $g_i=\sum c_{\underline{a}}x^{\underline{a}}$. Let $\underline{a}$ be defined by $c_{\underline{a}}\neq 0$ and $c_{\underline{b}}\neq 0$ implies $x^{\underline{a}}\leq x^{\underline{b}}$. Then $L(g_1,...,g_s)$: $=x_i^{\underline{a}}$ and $c(g_1,...,g_s)$: $= c_{\underline{a}}$. Similar we associate to any submodule $M\subseteq R^s$ the leading module $L(M)$ generated by $\{L(x), x\in M, x\neq 0\}$. The leading module gives us the possibility to compute the dimension of R^s/M in a combinatorial way: $\dim_C R^s/M = \dim_C R^s/L(M)$,
i.e. $\dim_C R^s/M$ is the cardinality of the set $\{x_i^{\underline{a}}, x_i^{\underline{a}}$ is not dibvisible by $L(g)$ for all g in a Gröbner base of $M\}$.

The algorithm of Mora constructs for a module $M \subseteq R^s$ generators of $L(M)$:

Definition 3: *Let $M \subseteq R^s$ be generated by $f_1, \ldots, f_n$. $\{f_1, \ldots, f_n\}$ is called a Gröbner base of M if $L(M)$ is generated $L(f_1), \ldots, L(f_n)$.*

To describe the construction of the Gröbner base we need the following notations:

(1) Let $(f_1, \ldots, f_s) \in R^s$, $k(f_1, \ldots, f_s) := \min\{i, f_i \neq 0\}$, $k(0, \ldots, 0) := 0$

(2) Let $f, g \in R^s$, $k := k(f) = k(g) > 0$, $L(f) = x^{\underline{a}}_k$, $L(g) = x^{\underline{b}}_k$, and $m = \mathrm{lcm}(x^{\underline{a}}, x^{\underline{b}})$ then $S(f,g) := \dfrac{m}{x^{\underline{a}}} f - \dfrac{c(f)m}{c(g)x^{\underline{b}}} g$. This procedure associating to f and g the S-vector $S(f, g)$ cancels the leading terms of f and g and creates a new leading term if $S(f,g) \neq 0$.

(3) Let $f \in R$ be a polynomial, $f = \sum c_{\underline{a}} x^{\underline{a}}$ and let $\underline{a}^{(1)}$, $\underline{a}^{(2)}$ be defined by
- $c_{\underline{a}}^{(1)} \neq 0$, $c_{\underline{a}}^{(2)} \neq 0$
- $c_{\underline{a}} \neq 0$ implies $x^{\underline{a}(1)} \leq x^{\underline{a}} \leq x^{\underline{a}(2)}$

We define the maxmin-degree of f by $d\mathrm{maxmin}(f) := \sum(a_i^{(1)} - a_i^{(2)})$. Let $f = (f_1, \ldots, f_s) \in R^s$ and $k := k(f) > 0$ then $d\mathrm{maxmin}(f) := d\mathrm{maxmin}(f_k)$, $d\mathrm{maxmin}(0) := 0$.

The algorithm to construct the Gröbner base uses the following algorithm to decide whether a given vector is in the submodule or not:

Let $S \subseteq R^s$ be a finite set of vectors of polynomials, $0 \neq h \in R^s$ a vector of polynomials and M the submodule of R^s generated by S.

1st step
Let $T_1 := S$ and $h_1 := h$

ith step
Suppose T_{i-1} and h_{i-1} are defined and $h_{i-1} \neq 0$. Suppose $L(h_{i-1})$ is divisible by $L(g)$ for some $g \in T_{i-1}$ then we choose $g \in T_{i-1}$ with the following properties:

(1) $L(g) \mid L(h_{i-1})$
(notice that this implies $k(g) = k(h_{i-1})$ by definition!)

(2) max ($d\mathrm{maxmin}(g)$, $d\mathrm{maxmin}(h_{i-1})$) is minimal between all possible choices of g with property (1).

We define $h_i := S(h_{i-1}, g)$ and $T_i := T_{i-1} \cup \{h_{i-1}\}$.
If $L(h_{i-1})$ is not divisible by $L(g)$ for all $g \in T_{i-1}$ then $h \notin M$.

Mora proved (cf. [6]) that there is an l such either $h_l = 0$ or $L(h_l)$ is not divisible by $L(g)$ for all $g \in T_{l-1}$.
We define $\text{reduction}_S(h) := h_l$.

The algorithm:

Let $M \subseteq R^s$ be a submodule generated by $f_1, \ldots, f_n$; f_i vectors of polynomials.

We construct the Gröbner base inductively:

1st step
We defined $S_1 := \{f_1, \ldots, f_n\}$, $P_1 := \{(f_i, f_j),\ i<j\}$.

ith step
Suppose S_{i-1} and P_{i-1} are defined.
If $P_{i-1} = \varnothing$ then S_{i-1} is a Gröbner base of M.
If $P_{i-1} \neq \varnothing$ we choose $(f,g) \in P_{i-1}$.
Let $s = S(f,g)$ and $h = \text{reduction}_{S_{i-1}}(s)$.
If $h=0$ then $\underline{S_i := S_{i-1}}$ and $P_i := P_{i-1} \setminus \{(f,g)\}$.
If $h \neq 0$ then $S_i := S_{i-1} \cup \{h\}$ and $P_i := (P_{i-1} \setminus \{(f,g)\}) \cup \{(v,h),\ v \in S_{i-1}\}$.

Now $L(M)$ is finitly generated. This implies that for some N always reduction$_{S_N}(s) = 0$, i.e. the algorithm will stop and S_N is a Gröbner base.

2. SINGULARITIES WITH EXACT POINCARE COMPLEX BUT NOT QUASIHOMOGENEOUS

Let $(X,0)$ be the germ of the curve singularity in $(C^3,0)$ defined by the zero set of the polynomials $f = xy + z^{l-1}$ and $g = xz + y^{k-1} + yz^2$, $4 \leq l \leq k$ and $5 \leq k$. This singularity is denoted by $FT_{k,l}$ in C.T.C. Walls classification and has Milnor number $\mu(X,0) = k + l + 2$ (cf. [11]). This singularity is not quasihomogeneous. To prove this we have to compute the Tjurina number $\tau(X,0) = \dim_C C\{x,y,z,\} / (f,g,M_1,M_2,M_3)$, M_i being the 2-minors of the Jacobian matrix

$$J\ (f,g) = \begin{pmatrix} y & x & (l-1)z^{l-2} \\ z & z^2 + (k-1)y^{k-2} & x + 2yz \end{pmatrix}$$

$M_1 = xz - yz^2 - (k-1)y^{k-1}$
$M_2 = xy + 2y^2z - (l-1)z^{l-1}$
$M_3 = x^2 + 2xyz - (l-1)z^l - (l-1)\ (k-1)y^{k-2}z^{l-2}$

We use Mora's algorithm to compute a Gröbner base of (f,g,M_1,M_2,M_3):
$h_1:=S(g,M_1)=2yz^2+ky^{k-1}$
$h_2:=S(M_2,f)=2y^2z-lz^{l-1}$
$h_3:=S(h_1,h_2)=lz^{l-1}+ky^{k-1}$
$h_4:=\text{reduction}_{\{f,g,M_1,M_2,M_3\}}(S(f,g))=((4\text{-}(2\text{-}l)(2\text{-}k))/2l)y^k$

Let $S:=\{f,g,M_3,h_1,h_2,h_3,h_4\}$ then one can check that $\text{reduction}_S(S(h,k))=0$ for all $h,k\in S$, i.e. S is a Gröbner base of the ideal (f,g,M_1,M_2,M_3). This implies

$$\begin{aligned}\tau(X,0)&=dim_C C\{x,y,z\}\ /\ (f,g,M_1,M_2,M_3)\\ &=dim_C C\{x,y,z\}\ /\ (xy,\ xz,\ x^2,yz^2,y^2z,z^{l-1},y^k)\\ &=l+k+1\\ &<l+k+2=\mu(X,0),\end{aligned}$$

i.e. $(X,0)$ is not quasihomogeneous.

We will now prove that the Poincaré complex of $(X,0)$ is exact. We have to prove (cf. introduction) that $\mu(X,0)=dim_C\Omega^2_{X,0}-dim_C\Omega^3_{X,0}$

$$\Omega^3_{X,0}=C\ \{x,y,z,\}\ /(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z},\frac{\partial g}{\partial x},\frac{\partial g}{\partial y},\frac{\partial g}{\partial z})$$

$=C\ \{x,y,z\}/(x,y,z)$
i.e. $\dim\ \Omega^3_{X,0}=1$.

We have to prove that $\dim\Omega^2_{X,0}=l+k+3$.
$\Omega^2_{X,0}=\underline{C\ \{x,y,z,\}^3/M\text{ and }M}$ is generated by
$f_1:=(f,0,0)=(xy+z^{l-1},0,0)$
$f_2:=(0,f,0)=(0,xy+z^{l-1},0)$
$f_3:=(0,0,f)=(0,0,xy+z^{l-1})$
$f_4:=(g,0,0)=(xz+yz^2+y^{k-1},0,0)$
$f_5:=(0,g,0)=(0,xz+yz^2+y^{k-1},0)$
$f_6:=(0,0,g)=(0,0,xz+yz^2+y^{k-1})$
$f_7:=(\frac{\partial f}{\partial y},\frac{\partial f}{\partial z},0)=(x,(l-1)z^{l-2},0)$

$f_8:=(\frac{\partial f}{\partial x},0,-\frac{\partial f}{\partial z})=(y,0,-(l-1)z^{l-2})$

$f_9:=(0,\frac{\partial f}{\partial x},\frac{\partial f}{\partial y})=(0,y,x)$

$f_{10}:=(\frac{\partial g}{\partial y},\frac{\partial g}{\partial z},0)=(z^2+(k-1)y^{k-2},x+2yz,0)$

$f_{11}:=(\frac{\partial g}{\partial x},0,-\frac{\partial g}{\partial z})=(z,0,-x-2yz)$

$f_{12}:=(0,\frac{\partial g}{\partial x},\frac{\partial g}{\partial y})=(0,z,z^2+(k-1)y^{k-2})$

Let $S_1:=(f_1,\dots,f_{12})$.
$h_1:=\text{reduction}_{S_1}(S(f_{10},f_{11}))=(0,x+2yz,xz+2yz^2+(l-1)(k-1)y^{k-2}z^{l-2})$
$S_2:=S_1\cup\{h_1\}$
$h_2:=\text{reduction}_{S_2}(S(f_4,f_{11}))=(0,0,x^2+2xyz+(l-1)z^{l+1}+(l-1)y^{k-2}z^{l-1})$
$S_3:=S_2\cup\{h_2\}$
$h_3:=\text{reduction}_{S_3}(S(f_6,f_3))=(0,0,(l/2-1)z^l+y^k)$
$S_4:=S_3\cup\{h_3\}$
$h_4:=\text{reduction}_{S_4}(S(f_8,f_{11}))=(0,0,2y^2z-lz^{l-1})$
$S_5:=S_4\cup\{h_4\}$
$h_5:=\text{reduction}_{S_5}(S(f_9,f_{12}))=(0,0,-2yz^2-ky^{k-1})$
$S_6:=S_5\cup\{h_5\}$
$h_6:=\text{reduction}_{S_6}(S(h_4,h_5))=(0,0,((2l)/(l-2)-k)y^k)$
$S_7:=S_6\cup\{h_6\}$

Now one can check that $\text{reduction}_{S_7}(S(g,h))=0$ for all $g,h\in S_7$, i.e. S_7 is a Gröbner base of M. Especially $L(M)$ is generated by $(x,0,0)$, $(y,0,0)$, $(z,0,0)$,$(0,x,0)$,$(0,y,0)$, $(0,z,0)$, $(0,0,xy)$, $(0,0,xz)$, $(0,0,yz^2)$, $(0,0,x^2)$, $(0,0,y^2z)$, $(0,0,z^l)$, $(0,0,y^k)$. This implies that $\dim_C\Omega^2_{X,0}=l+k+3$.

Let $(X,0)$ be the germ of the curve singularity in $(C^3,0)$ defined by the zero set of the polynomials $f=xy$ and $g=xz+yz^2+z^3+z^{3+i}$, $i>1$. This singularity is denoted by $FZ_{0,i}$ in C.T.C. Walls classification and has Milnor number $\mu(X,0)=10+i$ (cf. [11]). As before one compute $\tau(X,0)=9+i$, $\dim_C\Omega^3_{X,0}=1$ and $\dim_C\Omega^3_{X,0}=11+i$. This implies that $(X,0)$ is not quasihomogeneous but the Poincarè complex is exact, because f and g satisfy Reiffens condition $(f,g)\,\Omega^3_{C^3,0}\subseteq d((f,g)\Omega^2_{C^3,0})$ (cf. introduction).

3. THE INVARIANTS OF THE UNIMODAL COMPLETE INTERSECTION SINGULARITIES

type	*equations*		μ	τ	$\dim\Omega^2$	$\dim\Omega^3$
$P_{k,l}$	$xy, x^k + y^l + z^2$		$k+l+1$	$k+l+1$	$k+l+2$	1
$FT_{4,4}$	$xy + z^3,\ xz + y^3 + ayz^2$	$a \in C - \{1/4\}$	10	10	11	1
$FT_{k,l}$	$xy + z^{l-1},\ xz + y^{k-1} + yz^2$	$k \geq l \geq 4, k > 4$	$k+l+2$	$k+l+1$	$k+l+3$	1
FW_{13}	$xy + z^3,\ xz + y^4$		13	13	14	1
FW_{14}	$xy + z^3, xz + zy^3$		14	14	15	1
$FW_{1,0}$	$xy + z^3, xz + z^2y^2 + ay^5$	$a \in C - \{0, -1/4\}$	16	16	17	1
$FW_{1,p}$	$xy + z^3, xz + z^2y^2 + ay^{5+p}$		$16+p$	$14+p$	$16+p$	1
$FW^*_{1,p}$	$xy + z^3, xz + 2z^2y^2 - y^5 + 2y^2 l_p(z,y)$		$16+p$	$14+p$	$16+p$	1
FW_{18}	$xy + z^3, xz + zy^4$		18	18	19	1
FW_{19}	$xy + z^3, xz + y^6$		19	19	20	1
FZ_{6m+6}	$xy, xz + z^3 + y^{3m+1}$		$6m+6$	$6m+6$	$6m+7$	1
FZ_{6m+7}	$xy, xz + z^3 + zy^{2m+1}$		$6m+7$	$6m+7$	$6m+8$	1
FZ_{6m+8}	$xy, xz + z^3 + y^{3m+2}$		$6m+8$	$6m+8$	$6m+9$	1
$FZ_{m-1,0}$	$xy, xz + z^3 + z^2y^m + ay^{3m}$	$a \in C - \{0, -4/27\}$	$6m+4$	$6m+4$	$6m+5$	1
$FZ_{m-1,p}$	$xy, xz + z^3 + z^2y^m + y^{3m+p}$		$6m+4+p$	$5m+4+p$	$5m+6+p$	1
G_{n+8}	$x^2 + z^3, y^2 + z l_n(x,z)$		$n+8$	$n+8$	$n+10$	2
HA_{p+11}	$xy + z^3, x^2 + z^3 + yz^2 + y^{3+p}$	$p \geq 0$	$p+11$	$11 (p=0)$	$13 (p=0)$	2
				$p+10$	$p+12$	
HB_{p+13}	$xy + z^3, x^2 + yz^2 + y^{4+p}$	$p \geq 0$	$p+13$	$p+12$	$p+14$	2
HC_{13}	$xy + z^3, x^2 + z^3 + y^4$		13	13	15	2
HC_{14}	$xy + z^3, x^2 + z^3 + xy^3$		14	14	16	2
HC_{15}	$xy + z^3, x^2 + z^3 + y^5$		15	15	17	2
HD_{13}	$xy + z^3, x^2 + zy^2$		13	13	15	2
HD_{14}	$xy + z^3, x^2 + y^3$		14	14	16	2
J_{6m+7}	$xy + z^2, xz + y^{3m+3}$		$6m+7$	$6m+7$	$6m+8$	1
J_{6m+8}	$xy + z^2, xz + zy^{2m+2}$		$6m+8$	$6m+8$	$6m+9$	1
J_{6m+9}	$xy + z^2, xz + y^{3m+4}$		$6m+9$	$6m+9$	$6m+10$	1
$J_{m+1,0}$	$xy + z^2, xz + z^2y^m + ay^{3m+2}$	$a \in C - \{0, -4/27\}$	$6m+5$	$6m+5$	$6m+6$	1
$J_{m+1,p}$	$xy + z^2, xz + z^2y^m + y^{3m+2+p}$		$6m+5+p$	$5m+5+p$	$5m+6+p$	1
K_8	$xy + z^2, x^2 + y^3$		8	8	9	1
K_9	$xy + z^2, x^2 + zy^2$		9	9	10	1
$K_{1,0}$	$xy + z^2, x^2 + z^2y + ay^4$	$a \in C - \{0, 1/4\}$	11	11	12	1
$K_{1,p}$	$xy + z^2, x^2 + z^2y + y^{4+p}$		$11+p$	$10+p$	$11+p$	1
$K^*_{1,p}$	$xy + z^2, x^2 + 2z^2y + y^4 + yz l_p(z,y)$		$11+p$	$10+p$	$11+p$	1
K_{13}	$xy + z^2, x^2 + zy^3$		13	13	14	1
K_{14}	$xy + z^2, x^2 + y^5$		14	14	15	1

with $l_{2i}(x,y) = xy^i$ and $l_{2i+1}(x,y) = y^{i+2}$

4. THE QUASIHOMOGENEOUS UNIMODAL SINGULARITIES

Here we compare Wall's classification with the classification of Aleksandrov (cf.[1]).

type (Wall)	*type (Aleksandrov)*	*weights; degress*
$P_{k,l}$	$l=3{:}T_{k+4}$ $l=4{:}R_{k+5}$ $l\geq 5{:}L_{2,l,k}$	$2l,2k,kl;2(k+1),2k+l$
$FT_{4,4}$	V_{10}	2,1,1;3,3
FW_{13}	Q_{13}	11,4,5;15,16
FW_{14}	Q_{14}	9,3,4;12,13
$FW_{1,0}$	Q_{16}	7,2,3;9,10
FW_{18}		12,3,5;15,17
FW_{19}		17,4,7;21,24
FZ_{6m+6}	$V_{12}(FZ_{12})$	$6m+2,3,3m+1;6m+5,9m+3$
FZ_{6m+7}	$V_{13}(FZ_{13})$	$4m+2,2,2m+1;4m+4,6m+3$
FZ_{6m+8}	$V_{14}(FZ_{14})$	$6m+4,3,3m+2;6m+7,9m+6$
$FZ_{m-1,0}$	$V_{10}(FZ_{0,0})$ $V_{16}(FZ_{1,0})$	$2m,1,m;2m+1,3m$
G_{n+8}	M_{n+8}	$6,n+5,4;12,2n+10$ (n even) $3,[n/2]+3,2;6,2[n/2]+6$ (n odd)
HA_{11}	Y_{11}	3,2,2;6,5
HC_{13}	$L_{3,2,4}$	3,6,4;12,9
HC_{14}	G_{14}	9,4 ,6;18,13
HC_{15}	$L_{3,2,5}$	6,15,10;30,21
HD_{13}	H_{13}	7,5,4;12,14
HD_{14}	H_{14}	9,6,5;15,18
J_{6m+7}	$U_{13}(J_{13})$	$6m+5,3,3m+4;6m+8,9m+9$
J_{6m+8}	$U_{14}(J_{14})$	$4m+4,2,2m+3;4m+6,6m+7$
J_{6m+9}	$U_{15}(J_{15})$	$6m+7,3,3m+5;6m+10,9m+12$
$J_{m+1,0}$	$U_{11}(J_{2,0})$	$2m+1,1,m+1;2m+2,3m+2$
K_8	Y_8	6,4,5;10,15
K_9	M_9	5,3,4;8,10
$K_{1,0}$	M_{11}	4,2,3;6,8
K_{13}	N_{13}	7,3,5;10,14
K_{14}	N_{14}	10,4,7;14,20

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