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STABILIZABILITY AND CONTROLLABILITY OF SYSTEMS ASSOCIATED TO LINEAR SKEW-PRODUCT SEMIFLOWS

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Abstract

This paper is concerned with systems with control whose state evolution is described by linear skew-product semiflows. The connection between uniform exponential stability of a linear skew-product semiflow and the stabilizability of the associated system is presented. The relationship between the concepts of exact controllability and complete stabilizability of general control systems is studied. Some results due to Clark, Latushkin, Montgomery-Smith, Randolph, Megan, Zabczyk and Przyluski are generalized.

1 Introduction

A central concern in the study of infinite-dimensional linear control systems with unbounded coefficients is to establish the connections between their asymptotic behaviour and their controllability. It is well known that in Hilbert spaces for a linear control system associated to a C_0 - group its exact controllability is equivalent to its exponential stabilizability backward and forward in time (see [9], [12], [21]). Another important result, in Banach spaces, expresses the relation between uniform exponential stability of an evolution family and the stabilizability and detectability, respectively, of the associated linear control system ([7]).

In recent years, the theory of linear skew-product semiflows has been developed and used to study the asymptotic behaviour of time-varying

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linear systems ([18]). This approach led to the generalization of some classical theorems of dichotomy and stability (see [2]-[6], [10], [11], [13], [14]).

Naturally, the question arises whether the connection between stabilizability and controllability can be extended to systems associated to linear skew-product semiflows.

The purpose of this paper is to answer this question. We shall consider an abstract generalization of systems described by differential equations of the form

$$\begin{cases} \dot{x}(t) &= A(\sigma(\theta,t))x(t) + B(\sigma(\theta,t))u(t) \\ y(t) &= C(\sigma(\theta,t))x(t) \end{cases}$$

where σ is a semiflow on a locally compact metric space Θ. For every $\theta \in \Theta$, the operators $A(\theta)$ are generally unbounded operators on a Banach space X, while the operators $B(\theta) \in \mathcal{B}(U,X)$, $C(\theta) \in \mathcal{B}(X,Y)$, where U, Y are Banach spaces.

We establish the connection between the uniform exponential stability of a linear skew-product semiflow and the stabilizability and detectability, respectively, of the associated control system, using a generalization of a well-known stability theorem of Datko ([8]). Thus we extend a theorem of Clark, Latushkin, Montgomery-Smith and Randolph ([7]).

We also study the relation between the complete stabilizability and exact controllability of a control system associated to a linear skew-product semiflow. The results obtained here generalize some theorems due to Megan, Zabczyk and Przyluski (see [12], [17] and [21]).

2 Preliminaries

Let X be a Banach space, let (Θ, d) be a locally compact metric space and let $\mathcal{E} = X \times \Theta$. We denote by $\mathcal{B}(X)$ the Banach algebra of all bounded linear operators from X into itself and by $\mathbf{R}_+ = [0,\infty)$.

Definition 2.1. *A mapping $\sigma : \Theta \times \mathbf{R}_+ \to \Theta$ is called a semiflow on Θ, if it has the following properties:*

(i) $\sigma(\theta, 0) = \theta$, *for all $\theta \in \Theta$;*

(ii) $\sigma(\theta, s+t) = \sigma(\sigma(\theta, s), t)$, *for all $(\theta, s, t) \in \Theta \times \mathbf{R}_+^2$;*

(iii) *σ is continuous.*

Definition 2.2. *A pair $\pi = (\Phi, \sigma)$ is called a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ if σ is a semiflow on Θ and $\Phi : \Theta \times \mathbf{R}_+ \to \mathcal{B}(X)$ satisfies the following conditions:*

(i) *$\Phi(\theta, 0) = I$, the identity operator on X, for all $\theta \in \Theta$;*

(ii) *$\Phi(\theta, t+s) = \Phi(\sigma(\theta, t), s)\Phi(\theta, t)$, for all $(\theta, t, s) \in \Theta \times \mathbf{R}_+^2$ (the cocycle identity);*

(iii) *$(\theta, t) \mapsto \Phi(\theta, t)x$ is continuous, for every $x \in X$;*

(iv) *there are $M \geq 1$ and $\omega > 0$ such that*

$$||\Phi(\theta, t)|| \leq M e^{\omega t} \tag{2.1}$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$.

The mapping Φ given by Definition 2.2. is called *the cocycle* associated to the linear skew-product semiflow $\pi = (\Phi, \sigma)$.

Remark 2.1. If $\pi = (\Phi, \sigma)$ is a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ then for every $\beta \in \mathbf{R}$ the pair $\pi_\beta = (\Phi_\beta, \sigma)$, where $\Phi_\beta(\theta, t) = e^{-\beta t}\Phi(\theta, t)$ for all $(\theta, t) \in \Theta \times \mathbf{R}_+$, is also a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$.

Example 2.1. Let Θ be a locally compact metric space, let σ be a semiflow on Θ and let $\mathbf{T} = \{T(t)\}_{t \geq 0}$ be a C_0 - semigroup on X. Then the pair $\pi_T = (\Phi_T, \sigma)$, where

$$\Phi_T(\theta, t) = T(t)$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$, is a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$, which is called *the linear skew-product semiflow generated by the C_0 - semigroup $\mathbf{T}$ and the semiflow σ.*

Example 2.2. Let $\Theta = \mathbf{R}_+$, $\sigma(\theta, t) = \theta + t$ and let $\mathcal{U} = \{U(t, s)\}_{t \geq s \geq 0}$ be an evolution family on the Banach space X. We define

$$\Phi(\theta, t) = U(t + \theta, \theta)$$

for all $(\theta, t) \in \mathbf{R}_+^2$. Then $\pi = (\Phi, \sigma)$ is a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ called *the linear skew-product semiflow generated by the evolution family* $\mathcal{U}$.

Example 2.3. Let X be a Banach space, Θ be a compact metric space and let $\sigma : \Theta \times \mathbf{R}_+ \to \Theta$ be a semiflow on Θ. Let $A : \Theta \to \mathcal{B}(X)$ be a continuous mapping. If $\Phi(\theta, t)$ denotes the solution operator of the linear differential system

$$\dot{u}(t) = A(\sigma(\theta, t))\, u(t), \quad t \geq 0.$$

then the pair $\pi = (\Phi, \sigma)$ is a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. Often, these equations arise from the linearization of nonlinear evolution equations (see e.g. [18] and the references therein).

Example 2.4. On the Banach space X, we consider the time-varying differential equation

$$\dot{x}(t) = a(t)\, x(t), \quad t \geq 0$$

where $a : \mathbf{R}_+ \to \mathbf{R}_+$ is a continuous function such that there exists $\alpha := \lim\limits_{t \to \infty} a(t) < \infty$.

Let $C(\mathbf{R}_+, \mathbf{R})$ be the space of all continuous functions $f : \mathbf{R}_+ \to \mathbf{R}$. This space is metrizable with the metric

$$d(x, y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{d_n(x, y)}{1 + d_n(x, y)}\,,$$

where $d_n(x, y) = \sup\limits_{t \in [0, n]} |x(t) - y(t)|$.

If we denote by $a_s(t) = a(t + s)$ and by $\Theta =$ closure $\{a_s : s \in \mathbf{R}_+\}$ then

$$\sigma : \Theta \times \mathbf{R}_+ \to \Theta, \quad \sigma(\theta, t)(s) := \theta(t + s)$$

is a semiflow on Θ,

$$\Phi : \Theta \times \mathbf{R}_+ \to \mathcal{B}(X), \quad \Phi(\theta, t)x = exp\,(\int_0^t \theta(\tau)\, d\tau)\, x$$

is a cocycle and hence $\pi = (\Phi, \sigma)$ is a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$.

If U, Y are Banach spaces we denote by $\mathcal{B}(U,Y)$ the space of all bounded linear operators from U into Y. If Θ is a locally compact metric space, we denote by $\mathcal{C}_s(\Theta,\mathcal{B}(U,Y))$ the space of all strongly continuous bounded mappings $H:\Theta\to\mathcal{B}(U,Y)$, which is a Banach space with respect to the norm

$$||H|| := \sup_{\theta\in\Theta}||H(\theta)||.$$

If $H\in\mathcal{C}_s(\Theta,\mathcal{B}(U,Y))$ and $G\in\mathcal{C}_s(\Theta,\mathcal{B}(Y,Z))$ we shall denote by GH the mapping $\Theta\ni\theta\mapsto G(\theta)H(\theta)$.

Theorem 2.1. *Let $\pi=(\Phi,\sigma)$ be a linear skew-product semiflow on $\mathcal{E}=X\times\Theta$. If $P\in\mathcal{C}_s(\Theta,\mathcal{B}(X))$ there is an unique linear skew product semiflow $\pi_P=(\Phi_P,\sigma)$ on $X\times\Theta$ such that*

$$\Phi_P(\theta,t)x=\Phi(\theta,t)x+\int_0^t\Phi(\sigma(\theta,s),t-s)\,P(\sigma(\theta,s))\,\Phi_P(\theta,s)x\,ds \quad (2.2)$$

for all $(x,\theta,t)\in X\times\Theta\times\mathbf{R}_+$.

Proof. First, we shall show that for every $\theta\in\Theta$ and every $t\geq 0$ the integral equation (2.2) has a solution which is a bounded linear operator on X. Therefore we define:

$$\Phi_0(\theta,t)x=\Phi(\theta,t)x$$

and

$$\Phi_1(\theta,t)x=\int_0^t\Phi(\sigma(\theta,s),t-s)\,P(\sigma(\theta,s))\,\Phi_0(\theta,s)x\,ds$$

for all $(x,\theta,t)\in X\times\Theta\times\mathbf{R}_+$.

Let M and ω given by (2.1). We have

$$||\Phi_1(\theta,t)||\leq M^2\,||P||\,t\,e^{\omega t},$$

for all $(\theta,t)\in\Theta\times\mathbf{R}_+$. We prove that for every $x\in X$ the function $(\theta,t)\mapsto\Phi_1(\theta,t)x$ is continuous. Let $x\in X$ and $(\theta_0,t_0)\in\Theta\times\mathbf{R}_+$. Because (Θ,d) is a locally compact metric space there exists $r>0$ such that $V=\overline{D}_d(\theta_0,r)$ is a compact neighbourhood of θ_0.

Let $\varepsilon>0$. For $h>0$ and $\theta\in V$ we have:

$$||\Phi_1(\theta,t_0+h)x-\Phi_1(\theta_0,t_0)x||\leq\int_0^{t_0}||\varphi(\theta,h,s)-\varphi(\theta_0,0,s)||\,ds+$$

$$+ \int_{t_0}^{t_0+h} ||\Phi(\sigma(\theta,s), t_0+h-s)\, P(\sigma(\theta,s))\, \Phi(\theta,s)x||\, ds$$

where:

$$\varphi : V \times [0,1] \times [0,t_0] \to X,$$

$$\varphi(\theta,h,s) = \Phi(\sigma(\theta,s), t_0+h-s)P(\sigma(\theta,s))\Phi(\theta,s)x.$$

The function φ is continuous on $V \times [0,1] \times [0,t_0]$ and hence it is uniformly continuous. Then there exist $\delta_1 \in (0,1)$ and $r_1 \in (0,r)$ such that

$$||\varphi(\theta,h,s) - \varphi(\theta',h',s')|| < \frac{\varepsilon}{2(t_0+1)} \tag{2.3}$$

for all $(\theta,h,s), (\theta',h',s') \in V \times [0,1] \times [0,t_0]$ with $|h-h'| < \delta_1$, $|s-s'| < \delta_1$ and $d(\theta,\theta') < r_1$.

Let $\tilde{\delta} \in (0,\delta_1)$ such that

$$\int_{t_0}^{t_0+h} ||\Phi(\sigma(\theta,s), t_0+h-s)\, P(\sigma(\theta,s))\, \Phi(\theta,s)x||\, ds \leq$$

$$\leq h\, M^2\, e^{\omega(t_0+h)}\, ||P||\, ||x|| < \frac{\varepsilon}{2}, \tag{2.4}$$

for all $h \in [0,\tilde{\delta})$. Using (2.3) and (2.4) we obtain that

$$||\Phi_1(\theta, t_0+h)x - \Phi_1(\theta_0,t_0)x|| < \varepsilon \tag{2.5}$$

for all $h \in [0,\tilde{\delta})$ and $\theta \in D_d(\theta_0, r_1)$.

Similary, one can show that there is $\delta \in (0,\tilde{\delta})$ and $r_0 \in (0,r_1)$ such that

$$||\Phi_1(\theta, t_0-h)x - \Phi_1(\theta_0,t_0)x|| < \varepsilon$$

for all $h \in (0,\delta)$ and $\theta \in D_d(\theta_0,r_0)$, so the function $(\theta,t) \mapsto \Phi_1(\theta,t)x$ is continuous on $\Theta \times \mathbf{R}_+$ for every $x \in X$.

Inductively we define

$$\Phi_{n+1}(\theta,t)x = \int_0^t \Phi(\sigma(\theta,s), t-s)\, P(\sigma(\theta,s))\, \Phi_n(\theta,s)x\, ds,$$

for all $(n,\theta,t,x) \in \mathbf{N} \times \Theta \times \mathbf{R}_+ \times X$. Then for every $n \in \mathbf{N}$ and $x \in X$ the function $(\theta,t) \mapsto \Phi_n(\theta,t)x$ is continuous on $\Theta \times \mathbf{R}_+$ and

$$||\Phi_n(\theta,t)|| \leq M\, e^{\omega t}\, \frac{(M||P||t)^n}{n!}, \tag{2.6}$$

for all $(n, \theta, t) \in \mathbf{N} \times \Theta \times \mathbf{R}_+$. It has sense to define:

$$\Phi_P(\theta, t) = \sum_{n=0}^{\infty} \Phi_n(\theta, t), \tag{2.7}$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$. So $\Phi_P(\theta, 0) = I$ for every $\theta \in \Theta$. Using (2.6) we obtain that for every $t \geq 0$ and $\theta \in \Theta$ $\Phi_P(\theta, t) \in \mathcal{B}(X)$ and

$$||\Phi_P(\theta, t)|| \leq M\, e^{(\omega + M\,||P||)t},$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$.

Let $x \in X, (\theta_0, t_0) \in \Theta \times \mathbf{R}_+$ and let V be a compact neighbourhood of θ_0. Since the series (2.7) converges uniformly on $V \times [0, t_0 + 1]$ by the continuity of Φ_n, we obtain that the function $(\theta, t) \mapsto \Phi_P(\theta, t)x$ is continuous in (θ_0, t_0). Moreover:

$$\Phi_P(\theta, t)x = \Phi(\theta, t)x + \sum_{n=1}^{\infty} \Phi_n(\theta, t)x =$$

$$= \Phi(\theta, t)x + \sum_{n=1}^{\infty} \int_0^t \Phi(\sigma(\theta, s), t - s)\, P(\sigma(\theta, s))\, \Phi_{n-1}(\theta, s)x\, ds =$$

$$= \Phi(\theta, t)x + \int_0^t \Phi(\sigma(\theta, s), t - s)\, P(\sigma(\theta, s))\, \Phi_P(\theta, s)x\, ds,$$

so Φ_P verifies the equation (2.2).

Using (2.2) and Gronwall's lemma it is easy to show that Φ_P verifies the cocycle identity.

Finally, suppose that Φ'_P is a cocycle which verifies (2.2). Then we have:

$$||\Phi_P(\theta, t)x - \Phi'_P(\theta, t)x|| \leq \int_0^t M\, ||P||\, e^{\omega(t-s)}\, ||\Phi_P(\theta, s)x - \Phi'_P(\theta, s)x||\, ds.$$

From Gronwall's lemma it follows that $\Phi_P(\theta, t) = \Phi'_P(\theta, t)$, for all $t \geq 0$ and $\theta \in \Theta$.

Remark 2.2. The linear skew-product semiflow $\pi_P = (\Phi_P, \sigma)$ given by Theorem 2.1. is called *the linear skew-product semiflow generated by the pair* (π, P).

Corollary 2.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. If $P \in \mathcal{C}_s(\Theta, \mathcal{B}(X))$ then the linear skew-product semiflow $\pi_P = (\Phi_P, \sigma)$ generated by the pair (π, P) verifies the equation*

$$\Phi_P(\theta, t)x =$$

$$= \Phi(\theta, t)x + \int_0^t \Phi_P(\sigma(\theta, s), t - s))\, P(\sigma(\theta, s))\, \Phi(\theta, s)x\, ds \tag{2.8}$$

for all $(\theta, t, x) \in \Theta \times \mathbf{R}_+ \times X$.

Proof. It is easy to verify that

$$\Phi_n(\theta, t) = \tilde{\Phi}_n(\theta, t), \tag{2.9}$$

for all $(n, \theta, t) \in \mathbf{N} \times \Theta \times \mathbf{R}_+$ where $\tilde{\Phi}_n(\theta, t)$ is defined by:

$$\tilde{\Phi}_0(\theta, t) = \Phi(\theta, t)$$

$$\tilde{\Phi}_n(\theta, t)x = \int_0^t \tilde{\Phi}_{n-1}(\sigma(\theta, s), t - s)\, P(\sigma(\theta, s))\, \Phi(\theta, s)x\, ds$$

for all $(n, t, \theta, x) \in \mathbf{N}^* \times \mathbf{R}_+ \times \Theta \times X$. Then using (2.7) and (2.9) we obtain

$$\Phi_P(\theta, t)x = \Phi(\theta, t)x + \sum_{n=1}^{\infty} \tilde{\Phi}_n(\theta, t)x =$$

$$= \Phi(\theta, t)x + \sum_{n=1}^{\infty} \int_0^t \Phi_{n-1}(\sigma(\theta, s), t - s)P(\sigma(\theta, s))\Phi(\theta, s)x\, ds =$$

$$= \Phi(\theta, t)x + \int_0^t \Phi_P(\sigma(\theta, s), t - s)P(\sigma(\theta, s))\Phi(\theta, s)x\, ds,$$

for all $(x, \theta, t) \in \mathcal{E} \times \mathbf{R}_+$, which ends the proof.

Definition 2.3. *A linear skew-product semiflow $\pi = (\Phi, \sigma)$ on $\mathcal{E} = X \times \Theta$ is called uniformly exponentially stable if there are $N \geq 1$ and $\nu > 0$ such that*

$$||\Phi(\theta, t)|| \leq N e^{-\nu t}$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$.

Example 2.5. Consider the linear skew-product semiflow $\pi_\beta = (\Phi_\beta, \sigma)$ where

$$\Phi_\beta(\theta, t) = e^{-\beta t}\, \Phi(\theta, t), \quad \beta \in \mathbf{R}_+$$

and $\pi = (\Phi, \sigma)$ is the linear skew-product semiflow given in Example 2.4. It is easy to see that π_β is uniformly exponentially stable if and only if $\beta > \alpha$.

A sufficient condition for uniform exponential stability of linear skew-product semiflows is given by:

Proposition 2.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. If there are $t_0 > 0$ and $c \in (0, 1)$ such that*

$$||\Phi(\theta, t_0)|| \leq c,$$

for all $\theta \in \Theta$, then $\pi = (\Phi, \sigma)$ is uniformly exponentially stable.

Proof. Let $M \geq 1$ and $\omega > 0$ given by (2.1) and let $\nu > 0$ such that $c = e^{-\nu t_0}$.

Let $\theta \in \Theta$. For every $t \in \mathbf{R}_+$ there are $n \in \mathbf{N}$ and $r \in [0, t_0)$ such that $t = nt_0 + r$. Then we obtain:

$$||\Phi(\theta, t)|| \leq ||\Phi(\sigma(\theta, nt_0), r)|| \, ||\Phi(\theta, nt_0)|| \leq M e^{\omega t_0} \, e^{-n\nu t_0} \leq N e^{-\nu t},$$

where $N = M e^{(\omega+\nu)t_0}$. So π is uniformly exponentially stable.

Now, we give a characterization of uniform exponential stability of linear skew-product semiflows, which generalizes the well-known theorem of Datko ([8]).

Theorem 2.2. *The linear skew-product semiflow $\pi = (\Phi, \sigma)$ is uniformly exponentially stable if and only if there are $K > 0$ and $p \geq 1$ such that*

$$\int_0^\infty ||\Phi(\theta, t)x||^p dt \leq K||x||^p, \tag{2.10}$$

for all $(x, \theta) \in \mathcal{E}$.

Proof. *Necessity.* If $\pi = (\Phi, \sigma)$ is uniformly exponentially stable and $N \geq 1$, $\nu > 0$ are given by Definition 2.3. it follows that

$$\int_0^\infty ||\Phi(\theta, t)x||^p \, dt \leq \frac{N^p}{\nu p} \, ||x||^p,$$

for all $(x, \theta) \in \mathcal{E}$ and $p \geq 1$.

Sufficiency. Let $t \geq 1$ and $m = \int_0^1 M^{-p} e^{-p\omega\tau} d\tau$, where $M \geq 1$ and $\omega > 0$ are given by (2.1). Then using the cocycle identity we obtain from (2.10) that

$$m||\Phi(\theta, t)x||^p \leq ||\Phi(\theta, t)x||^p \int_0^t M^{-p} e^{-p\omega(t-s)}\, ds \leq$$

$$\leq \int_0^t ||\Phi(\theta, s)x||^p ds \leq K||x||^p,$$

and hence

$$||\Phi(\theta, t)|| \leq M_1 = \max\left\{ Me^{\omega}, (\frac{K}{m})^{\frac{1}{p}} \right\}$$

for all $(t, \theta) \in \mathbf{R}_+ \times \Theta$. Setting $t_0 = 2^p M_1^p K$ we deduce:

$$t_0||\Phi(\theta, t_0)x||^p \leq M_1^p \int_0^{t_0} ||\Phi(\theta, s)x||^p\, ds \leq K M_1^p\, ||x||^p$$

and hence $||\Phi(\theta, t_0)|| \leq \frac{1}{2}$, for all $\theta \in \Theta$. From Proposition 2.1. it results that π is uniformly exponentially stable.

We denote by $\mathcal{M}(X)$ the linear space of all strongly Bochner measurable functions $u : \mathbf{R}_+ \to X$ identifying the functions which are equal almost everywhere. For every $p \in [1, \infty)$ the linear space

$$L^p(\mathbf{R}_+, X) = \{u \in \mathcal{M}(X) : \int_0^\infty ||u(t)||^p dt < \infty\}$$

is a Banach space with respect to the norm:

$$||u||_p := \left(\int_0^\infty ||u(t)||^p dt \right)^{\frac{1}{p}}.$$

We shall denote by $L^1_{loc}(\mathbf{R}_+, X)$ the set of all locally integrable functions $u : \mathbf{R}_+ \to X$.

Let U, Y be two Banach spaces and

$$\{A_\theta : L^1_{loc}(\mathbf{R}_+, U) \to L^1_{loc}(\mathbf{R}_+, Y), \theta \in \Theta\}$$

a family of linear operators.

Definition 2.4. *The family* $\{A_\theta\}_{\theta\in\Theta}$ *is said to be uniformly* $(L^p(\mathbf{R}_+, U)$, $L^p(\mathbf{R}_+, Y))$ *- stable if the following conditions are satisfied:*

(i) $A_\theta u \in L^p(\mathbf{R}_+, Y)$, *for all* $(u, \theta) \in L^p(\mathbf{R}_+, U) \times \Theta$;

(ii) *there is* $L > 0$ *such that* $||A_\theta u||_p \leq L||u||_p$, *for all* $(u, \theta) \in L^p(\mathbf{R}_+, U) \times \Theta$.

Remark 2.3. If the families $\{A_\theta\}_{\theta\in\Theta}$, $\{B_\theta\}_{\theta\in\Theta}$ are uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, Y))$ - stables and the family $\{C_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, Y), L^p(\mathbf{R}_+, Z))$ - stable then

(i) the family $\{A_\theta + B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, Y))$ - stable;

(ii) the family $\{C_\theta A_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, Z))$ - stable.

Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. For every $\theta \in \Theta$ we define the operator

$$P_\theta : L^1_{loc}(\mathbf{R}_+, X) \to L^1_{loc}(\mathbf{R}_+, X), (P_\theta u)(t) = \int_0^t \Phi(\sigma(\theta, s), t - s)u(s)\, ds \tag{2.11}$$

Another characterization of uniform exponential stability of linear skew-product semiflows has been treated in [13] and it is given by:

Theorem 2.3. *Let* $\pi = (\Phi, \sigma)$ *be a linear skew-product semiflow on* $\mathcal{E} = X \times \Theta$. *Then* π *is uniformly exponentially stable if and only if the family* $\{P_\theta\}_{\theta\in\Theta}$ *is uniformly* $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ *- stable.*

Proof. see [13], Theorem 3.2.

Remark 2.4. The above result is an extension of a well-known theorem of Perron type, proved by Datko in [8]. Other approaches of this theorem have been presented by Neerven in [16] for the particular case of C_0-semigroups, employing a complex analysis technique and by Clark, Latushkin, Montgomery-Smith and Randolph in [7], for the case of evolution families, applying Neerven's result for the associated evolution semigroup.

3 Stabilizability and detectability of linear control sytems

In this section we shall establish the connection between the uniform exponential stability of a linear skew-product semiflow and the stabilizability and detectability of the system associated to the linear skew-product semiflow. Thus, we shall extend a result due to Clark, Latushkin, Montgomery-Smith and Randolph ([7]).

Let X, Y, U be Banach spaces and let Θ be a locally compact metric space. Let $B \in \mathcal{C}_s(\Theta, \mathcal{B}(U, X))$ and $C \in \mathcal{C}_s(\Theta, \mathcal{B}(X, Y))$. Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$.

Consider the system $S = (\pi, B, C)$ described by the following integral model

$$\begin{cases} x(\theta, t, x_0, u) &= \Phi(\theta, t)x_0 + \int_0^t \Phi(\sigma(\theta, s), t-s)B(\sigma(\theta, s))u(s)\, ds \\ y(\theta, t, x_0, u) &= C(\sigma(\theta, t))x(\theta, t, x_0, u) \end{cases}$$

where $t \geq 0, (x_0, \theta) \in \mathcal{E}, p \in [1, \infty)$ and $u \in L^p_{loc}(\mathbf{R}_+, U)$.

Definition 3.1. *The system $S = (\pi, B, C)$ is called:*

(i) *stabilizable if there exists $F \in \mathcal{C}_s(\Theta, \mathcal{B}(X, U))$ such that the linear skew-product semiflow $\pi_{BF} = (\Phi_{BF}, \sigma)$ generated by the pair (π, BF) is uniformly exponentially stable;*

(ii) *detectable if there exists $K \in \mathcal{C}_s(\Theta, \mathcal{B}(Y, X))$ such that the linear skew-product semiflow $\pi_{KC} = (\Phi_{KC}, \sigma)$ generated by the pair (π, KC) is uniformly exponentially stable.*

For every $\theta \in \Theta$ we define the operators

$$B_\theta : L^1_{loc}(\mathbf{R}_+, U) \to L^1_{loc}(\mathbf{R}_+, X), \quad (B_\theta u)(t) = B(\sigma(\theta, t))u(t)$$

$$C_\theta : L^1_{loc}(\mathbf{R}_+, X) \to L^1_{loc}(\mathbf{R}_+, Y), \quad (C_\theta u)(t) = C(\sigma(\theta, t))u(t)$$

Theorem 3.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$, let $\{P(\theta)\}_{\theta \in \Theta}$ the family associated to π by relation (2.11) and let $p \in [1, \infty)$. The following assertions are equivalent:*

(i) *π is uniformly exponentially stable;*

(ii) *the system $S = (\pi, B, C)$ is stabilizable and the family $\{P_\theta B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, X))$ - stable;*

(iii) *the system $S = (\pi, B, C)$ is detectable and the family $\{C_\theta P_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, Y))$ - stable;*

(iv) *the system $S = (\pi, B, C)$ is stabilizable, detectable and the family $\{C_\theta P_\theta B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, Y))$ - stable.*

Proof. (i) $\Longrightarrow$ (ii) Since π is uniformly exponentially stable, according to Theorem 2.3., the family $\{P_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ - stable. Because

$$||B_\theta u||_p \leq ||B||\,||u||_p,$$

for all $(u, \theta) \in L^p(\mathbf{R}_+, U) \times \Theta$, it follows that the family $\{B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, X))$ - stable, so from Remark 2.3. the family $\{P_\theta B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, X))$ - stable.

The implications (i) $\Longrightarrow$ (iii) and (i) $\Longrightarrow$ (iv) can be obtained in an analogous manner.

(ii) $\Longrightarrow$ (i) Let $F \in \mathcal{C}_s(\Theta, \mathcal{B}(X, U))$ such that $\pi_{BF} = (\Phi_{BF}, \sigma)$ is uniformly exponentially stable. For every $\theta \in \Theta$ we consider the operators

$$G_\theta : L^1_{loc}(\mathbf{R}_+, X) \to L^1_{loc}(\mathbf{R}_+, X), \quad (G_\theta u)(t) = \int_0^t \Phi_{BF}(\sigma(\theta, s), t - s)u(s)\, ds$$

$$F_\theta : L^1_{loc}(\mathbf{R}_+, X) \to L^1_{loc}(\mathbf{R}_+, U), \quad (F_\theta u)(t) = F(\sigma(\theta, t))u(t).$$

Because π_{BF} is uniformly exponentially stable the family $\{G_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ - stable.

Let $\theta \in \Theta$, $t \geq 0$ and $u \in L^1_{loc}(\mathbf{R}_+, X)$. Using Fubini's theorem we obtain

$$(P_\theta B_\theta F_\theta G_\theta u)(t) =$$

$$= \int_0^t \int_0^s \Phi(\sigma(\theta, s), t - s)B(\sigma(\theta, s))F(\sigma(\theta, s))\Phi_{BF}(\sigma(\theta, \tau), s - \tau)u(\tau)\, d\tau\, ds =$$

$$= \int_0^t \int_\tau^t \Phi(\sigma(\theta, s), t - s)B(\sigma(\theta, s))F(\sigma(\theta, s))\Phi_{BF}(\sigma(\theta, \tau), s - \tau)u(\tau)\, ds\, d\tau =$$

$$= \int_0^t [\Phi_{BF}(\sigma(\theta,\tau), t-\tau)u(\tau) - \Phi(\sigma(\theta,\tau), t-\tau)u(\tau)]\, d\tau.$$

So

$$P_\theta = G_\theta - P_\theta B_\theta F_\theta G_\theta, \tag{3.1}$$

for every $\theta \in \Theta$.

Using the hypothesis and the fact that the families $\{F_\theta\}_{\theta\in\Theta}$ and $\{P_\theta B_\theta\}_{\theta\in\Theta}$ are uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, U))$ -stable and $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, X))$ -stable, respectively, we deduce from Remark 2.3. that the family $\{P_\theta B_\theta F_\theta G_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ -stable.

Hence from (3.1) and Remark 2.3. we obtain that the the family $\{P_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ - stable. From Theorem 2.3. it follows that π is uniformly exponentially stable.

(iii) $\Longrightarrow$ (i) Let $K \in \mathcal{C}_s(\Theta, \mathcal{B}(Y, X))$ such that $\pi_{KC} = (\Phi_{KC}, \sigma)$ is uniformly exponentially stable. For every $\theta \in \Theta$ we consider the operators

$$H_\theta : L^1_{loc}(\mathbf{R}_+, X) \to L^1_{loc}(\mathbf{R}_+, X), \quad (H_\theta u)(t) = \int_0^t \Phi_{KC}(\sigma(\theta, s), t-s)u(s)\, ds$$

$$K_\theta : L^1_{loc}(\mathbf{R}_+, Y) \to L^1_{loc}(\mathbf{R}_+, X), \quad (K_\theta u)(t) = K(\sigma(\theta, t))u(t).$$

Because π_{KC} is uniformly exponentially stable the family $\{H_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ - stable. Using an analogous argument as in the proof of (ii) $\Longrightarrow$ (i) one obtain that

$$P_\theta = H_\theta - H_\theta K_\theta C_\theta P_\theta,$$

for all $\theta \in \Theta$. Then we immediately deduce that the family $\{P_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, X), L^p(\mathbf{R}_+, X))$ - stable, so from Theorem 2.3. π is uniformly exponentially stable.

(iv) $\Longrightarrow$ (i) If $\{H_\theta\}_{\theta\in\Theta}$ and $\{K_\theta\}_{\theta\in\Theta}$ are defined in the same manner as above we obtain that

$$P_\theta B_\theta = H_\theta B_\theta - H_\theta K_\theta C_\theta P_\theta B_\theta,$$

for all $\theta \in \Theta$. Then, using the hypothesis we deduce that the family $\{P_\theta B_\theta\}_{\theta\in\Theta}$ is uniformly $(L^p(\mathbf{R}_+, U), L^p(\mathbf{R}_+, X))$ - stable. Because S is stabilizable we finally conclude that π is uniformly exponentially stable.

Remark 3.1. The above theorem has been obtained by Clark, Latushkin, Montgomery-Smith and Randolph ([7]), for the case of time-varying systems associated to evolution families.

Remark 3.2. Another characterization for exponential stability of linear systems in Hilbert spaces, in terms of dual concepts, have been presented by Weiss and Rebarber in [19]. There, it is proved that a system associated to a C_0-semigroup is exponentially stable if and only if it is optimizable, estimatable and input-output stable.

4 Complete stabilizability and exact controllability

In this section we shall present the connection between complete stabilizability and the exact controllability of a system associated to a linear skew-product semiflow.

Let X, U, Y be reflexive Banach spaces and let Θ be a locally compact metric space. Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$, $B \in \mathcal{C}_s(\Theta, \mathcal{B}(U, X))$ and $C \in \mathcal{C}_s(\Theta, \mathcal{B}(X, Y))$. Let $p \in (1, \infty)$.

Let $S = (\pi, B, C)$ be the system considered in the previous section.

Definition 4.1. *The system $S = (\pi, B, C)$ is said to be exactly controllable if for every $\theta \in \Theta$ there is $t > 0$ such that for all $x_0, x_1 \in X$ there exists $u \in L^p(\mathbf{R}_+, U)$ with $x(\theta, t, x_0, u) = x_1$.*

Remark 4.1. Because the concept of exact controllability does not depend on the mapping C we suppose $C = 0$ and in all what follows we shall denote the system $S = (\pi, B, 0)$ by $S = (\pi, B)$.

For every $(\theta, t) \in \Theta \times \mathbf{R}_+$ consider the bounded linear operator

$$C_S^{\theta,t} : L^p(\mathbf{R}_+, U) \to X,\ C_S^{\theta,t} u = \int_0^t \Phi(\sigma(\theta, s), t-s)\, B(\sigma(\theta, s))\, u(s)\, ds.$$

Lemma 4.1. *Let X be a Banach space and let X' be a reflexive Banach space. If $A \in \mathcal{B}(X', X)$ then A is surjective if and only if there is $c > 0$ such that*

$$||A^* x^*|| \geq c||x^*||,$$

for all $x^* \in X^*$.

Proof. See [21], pp. 207-209.

Proposition 4.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ and $S = (\pi, B)$. The following assertions are equivalent:*

(i) *S is exactly controllable;*

(ii) *for every $\theta \in \Theta$ there is $t > 0$ such that $C_S^{\theta,t}$ is surjective;*

(iii) *for every $\theta \in \Theta$ there are $t > 0$ and $c > 0$ such that $||(C_S^{\theta,t})^* x^*|| \geq c\,||x^*||$, for all $x^* \in X^*$.*

Proof. It is immediate from Definition 4.1. and Lemma 4.1.

As a consequence of Theorem 2.1. and Definition 4.1. we obtain

Proposition 4.2. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ and let $F \in \mathcal{C}_s(\Theta, \mathcal{B}(X, U))$. The system $S = (\pi, B)$ is exactly controllable if and only if the system $S_{BF} = (\pi_{BF}, B)$ is exactly controllable.*

Proof. Let $(\theta, t) \in \Theta \times \mathbf{R}_+$ and $u \in L^p(\mathbf{R}_+, U)$. Using Theorem 2.1. and Fubini's theorem we obtain that

$$C_{S_{BF}}^{\theta,t} u = C_S^{\theta,t}(u + u_1),$$

where

$$u_1(\tau) = \begin{cases} F(\sigma(\theta,\tau)) \int_0^\tau \Phi_{BF}(\sigma(\theta,s), \tau - s) B(\sigma(\theta,s)) u(s) ds & , \tau \in [0,t] \\ 0 & , \tau > t \end{cases}.$$

So Range $C_{S_{BF}}^{\theta,t} \subset$ Range $C_S^{\theta,t}$. In the same way by using Corollary 2.1. we obtain that Range $C_S^{\theta,t} \subset$ Range $C_{S_{BF}}^{\theta,t}$, which ends the proof.

Definition 4.2. *The system $S = (\pi, B)$ is said to be completely stabilizable if for every $\nu > 0$ there are $N \geq 1$ and $F \in \mathcal{C}_s(\Theta, \mathcal{B}(X, U))$ such that the linear skew-product semiflow $\pi_{BF} = (\Phi_{BF}, \sigma)$ satisfies the inequality*

$$||\Phi_{BF}(\theta, t)|| \leq N e^{-\nu t},$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$.

Now we can give:

Theorem 4.1. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$ with the property that for every $\theta \in \Theta$ there is $t_0 > 0$ such that $\Phi(\theta, t_0)$ is surjective. If the system $S = (\pi, B)$ is completely stabilizable then S is exactly controllable.*

Proof. Suppose the contrary, i.e. there exists $\theta_0 \in \Theta$ such that for all $t > 0$ Range $C_S^{\theta_0,t} \neq X$. It follows from Proposition 4.1. that for every $\varepsilon > 0$ and every $t > 0$ there is $x^*_{t,\varepsilon} \in X^*$ with $||x^*_{t,\varepsilon}|| = 1$ such that

$$||(C_S^{\theta_0,t})^* \, x^*_{t,\varepsilon}|| < \varepsilon. \tag{4.1}$$

Let $t_0 > 0$ such that $\Phi(\theta_0, t_0)$ is surjective. Since X is reflexive from Lemma 4.1. it follows that there exists $k > 0$ with

$$k||x^*|| \leq ||\Phi(\theta_0, t_0)^* x^*||, \tag{4.2}$$

for all $x^* \in X^*$.

Let $\nu > 0$. Since S is completely stabilizable there is $F : \Theta \to \mathcal{B}(X, U)$ a strongly continuous mapping with $||F|| = \sup_{\theta \in \Theta} ||F(\theta)|| < \infty$ and $N \geq 1$ such that

$$||\Phi_{BF}(\theta, t)|| \leq N e^{-\nu t},$$

for all $(\theta, t) \in \Theta \times \mathbf{R}_+$. Using Theorem 2.1. we obtain that

$$\Phi(\theta, t)x = \Phi_{BF}(\theta, t)x - C_S^{\theta,t} \, \Gamma_\theta \, x,$$

for all $(\theta, t, x) \in \Theta \times \mathbf{R}_+ \times X$, where for every $\theta \in \Theta$

$$\Gamma_\theta : X \to L^p(\mathbf{R}_+, U), \quad (\Gamma_\theta \, x)(s) = F(\sigma(\theta, s)) \, \Phi_{BF}(\theta, s)x.$$

Then we have

$$\Phi(\theta, t)^* \, x^* = \Phi_{BF}(\theta, t)^* \, x^* - (\Gamma_\theta)^* \, (C_S^{\theta,t})^* \, x^*,$$

for all $x^* \in X^*$. It follows that

$$||\Phi(\theta, t)^* \, x^*|| \leq N \, e^{-\nu t} \, ||x^*|| + ||(\Gamma_\theta)^*|| \, ||(C_S^{\theta,t})^* \, x^*||, \tag{4.3}$$

for all $(\theta, t, x^*) \in \Theta \times \mathbf{R}_+ \times X^*$. Since

$$||(\Gamma_\theta)^*|| = ||\Gamma_\theta|| \leq N_1 := \frac{||F|| \, N}{\nu p},$$

from (4.3) applied for $x^*_{t_0,\varepsilon}, \theta_0$ and t_0 using (4.1) and (4.2) we obtain that

$$k < Ne^{-\nu t_0} + N_1\varepsilon.$$

Since $\varepsilon > 0$ was arbitrary we obtain from above that

$$e^{\nu t_0} < \frac{N}{k},$$

for all $\nu > 0$, which is absurd.

So the system $S = (\pi, B)$ is exactly controllable.

Remark 4.2. The above theorem generalizes a result obtained by Megan ([12]), Zabczyk ([20], [21] Theorem 3.4., pp. 229-231) and Przyluski ([17]), for systems with control described by C_0 - groups and C_0 - semigroups, respectively.

Remark 4.3. The hypothesis imposed in Theorem 4.1. on the surjectivity of Φ is essential, even for linear skew-product semiflows generated by C_0 - semigroups, as shows:

Example 4.1. Let $\{e_n\}_{n\geq 0}$ be an ortonormal basis in the separable real Hilbert space X and $\mathbf{T} = \{T(t)\}_{t\geq 0}$ be the C_0 - semigroup defined by

$$T(t)x = \sum_{n=0}^{\infty} e^{-nt} x_n e_n, \quad \text{for } x = \sum_{n=0}^{\infty} x_n e_n.$$

Let σ be a semiflow on the locally compact metric space Θ and Φ_T be the cocycle generated by $\mathbf{T}$ and σ. Let $U = X$, $B : \Theta \to \mathcal{B}(X), B(\theta) = I$, the identity operator on X and $p = 2$. Since for every every $t > 0$

$$\text{Range } T(t) = \{x = \sum_{n=0}^{\infty} x_n e_n : \sum_{n=0}^{\infty} e^{2nt} x_n^2 < \infty\}$$

it follows that for every $\theta \in \Theta$, $\Phi_T(\theta, t) = T(t)$ is not surjective.

It is easy to see that $S = (\pi, B)$ is completely stabilizable but

$$\text{Range } C_S^{\theta,t} = \{x = \sum_{n=0}^{\infty} x_n e_n : \sum_{n=1}^{\infty} n^2 x_n^2 < \infty\},$$

for every $(\theta, t) \in \Theta \times \mathbf{R}_+$, so the system $S = (\pi, B)$ is not exactly controllable.

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