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The role of Anschauung in Kant's conception of geometry

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ENG **Abstract:** Far from being outdated, Kant's philosophy of mathematics offers valuable insights to modern scholarship. This paper focuses on Kant's notion of Anschauung (commonly translated as intuition) within geometry. Contrary to a widely held view, it will be shown that appealing to Kantian intuition is, in some cases, still necessary. After reviewing past and recent objections to Kant's arguments, a new interpretation of A 716-717/B 744-745 of the first Critique (*CPR*) will be proposed. The role that Kant assigns to intuition in this passage, which I term 'revelatory,' remains indispensable despite modern advances in mathematics and logic. Notably, almost all English translations of the *CPR*, as well as the official translations in French and Greek, have disregarded this function of intuition. This paper aims to show why these interpretations are inconsistent with Kant's philosophy of mathematics and to argue that the revelatory function of intuition withstands objections to the classical reading.
Keywords: Kant, Intuition, Geometry, Methodology.

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0. Introduction

As with any great invention or discovery, Kant's work has been the subject of intense appraisal but also of sharp criticism from his day up to the present. Much of this criticism has focused on his conception of geometry. Kant's position is straightforward: The propositions of mathematics, including geometry, are of an a priori synthetic nature.

According to Kantian epistemology, this means that, on the one hand, the propositions of geometry can be known pre-empirically, that is, independently of any empirical content (except, of course, the content which we need in order to grasp the concepts), but, on the other, these propositions depend on the *pure* forms of intuition, which are none other than space and time. To know[1] geometric propositions, therefore, we need the assistance of the intellect (in order to grasp and formulate the concepts) and the assistance of that "mysterious" faculty which Kant calls *intuition* (Anschauung)[2].

Up to this point in the history of philosophy, mathematical propositions were characterized either as purely a priori, starting from Plato, or were characterized as products of empirical thought which, after a process of generalization or idealization, led to mathematical concepts. The first to suggest an empiricist philosophy of mathematics was Aristotle, while systematic attempts took place in the Anglo-Saxon world after Kant's death, specifically in the 19th and 20th centuries with John Stewart Mill and Willard Van Orman Quine as the main proponents (Linnebo 2017). In this context, Kant's conception was radical, as he integrated elements from both views to address the limitations that neither could fully resolve.

1 To be precise, Kant held that *almost all* propositions of geometry are a priori synthetic (Shapiro 2000). There are propositions that can be known purely analytically, such as the proposition "Every triangle has three angles". Of course, it has been questioned (Drossos 2006) whether in these cases the involvement of intuition is not required. That is, how does the concept of the three-edge figure follow from the concept of a triangle? One could also think of cases where a square has 4 corners but this does not resemble the shape of the square we have in mind, with its 4 perpendicular sides. It could consist of 3 sides and 4 corners, two curved and two non-curved. Does this lead us to the conclusion, contra Kant, that *all* propositions of mathematics must be a priori synthetic?

2 The interpretive problems of the concept of Anschauung begin already with the rendering of the concept in English. In German, there exist the words "Intuition" and "Anschauung" which both refer to intuition, the latter although having a more philosophical use. For example, the English phrase "female intuition" cannot be translated as "weibliche Anschauung", because Anschauung usually refers to super-sensible things, like "Weltanschauung". Therefore, it is to be expected that, by translating both German words as "Intuition" in English, philosophical confusions are about to occur.

There were many, however, who were not satisfied with Kant's endeavor and, in particular, with his conception of intuition. As Coffa (1991) has observed, "A main item on the agenda of philosophy throughout the nineteenth century was to account for the prima facie necessity and a priori nature of mathematics and logic without invoking Kantian intuition." (quoted in Shapiro 2000, p. 99) This trend continued in the following years and I can say that it reaches to our days. Except for Brouwer and Heyting's theory of intuitionism, which had obvious Kantian origins, the modern currents of neo-logicism and structuralism either try to reduce mathematics to logic (the neo-logicists) or to study mathematics as abstract structures, usually without ontological content (the structuralists). Consequently, modern mathematics is dominated by the analytic a priori[3] and intuition has been set aside as something which, with the tools of modern logic, is no longer useful or is not something which we actually need. That being said, we should not be surprised by statements by post-Kantian scholars, such as the following one by Russel (1919, p. 145-146):

> No appeal to common sense, or "intuition", or anything except strict deductive logic, ought to be needed in mathematics after the premises have been laid down. Kant, having observed that the geometers of his day could not prove their theorems by unaided argument, but required an appeal to the figure, invented a theory of mathematical reasoning according to which the inference is never strictly logical, but always requires the support of what is called "intuition". What can be known, in mathematics and by mathematical methods, is what can be deduced from pure logic. What else is to belong to human knowledge must be ascertained otherwise — empirically, through the senses or through experience in some form, but not *a priori*. (emphasis in the original)

In the present paper, I will argue that Kant's conception of geometry is not just part of the history of philosophy but is of interest today as well. In particular, I will claim that the appeal to Kantian intuition is, in some cases, still necessary. In the first section of this paper, I will analyze the classical reading of the role of Kantian intuition in geometry, based on the distinction that Brittan (2006) proposes. I will mention the objections that have been presented against Kant's arguments, both in today's context and in the context of his time. In the second section I will propose a reading of the A 716–717 / B 744–745 of the *CPR*, where I believe that the use of Kantian intuition remains timely and philosophically important. This role of intuition, which I call "revelatory", remains indispensable despite the advances that have been made in mathematics and logic since Kant's time. Furthermore, I will argue that almost all of Kant's interpreters that have translated the *CPR* in the English language have disregarded this function of intuition, which also holds for the official translations in French and also in Greek. The aim of this paper is to show why these interpretations are inconsistent with the context in which Kant develops his philosophy of mathematics, as well as to argue why the revelatory function of intuition is not offended by the objections that have been set against the classical reading of Kant.

1. Intuition as the guarantor of the geometer or, in other words, how to overcome the problems of Aristotelian logic

In this section I will examine the classical reading of Kant's function of intuition, which is none other than the role of intuition as the helper or guarantor of the geometer. In short, this reading argues that the intellect (Verstand) is not sufficient for the production of mathematical knowledge and that an appeal to intuition is necessary. Since, for Kant, the realm of the knowable is divided into concepts (Begriff) and intuitions (Anschauung)[4], what is not a concept or does not suffice to be a concept, as in the case of mathematics, can only be an intuition. As Brittan (2006) summarizes it, the advocates of this reading of Kant can be divided into two groups. The first group stresses the role of the premises in the deductive process, which are basic proposition whose truth is assumed.

As can already be seen in the *Critique of Pure Reason* (*CPR*), Kant's mathematical paradigm is Euclid's *Elements* (Hintikka 1992), in which all propositions are either proved from their antecedents or are assumed to be known from the outset. In the second category belong the propositions which are called postulates, axioms or common notions. As previously mentioned, these propositions cannot be established through a purely logical process, leaving intuition responsible for this task. On this reading, propositions such as the first Euclidean postulate, which states "To draw a straight line from any point to any point", need the aid of intuition in order to be considered true and, at the same time, necessarily true. No human mind in Kant's time would have questioned this reasoning. Kant's innovation is that he recognized that these propositions were not grounded on logic but on a *transcendental* necessity, arguing that our human minds could not conceive of any way so that this proposition is rendered false.

It is well known, of course, that Kant's argument was considered problematic by many, leading to attempts to establish a logical foundation for the first principles of mathematics, thereby eliminating the need to rely on intuition. The most important proponents of this position were Frege (1884) in arithmetic and Hilbert (1899) in geometry.

As the father of the logicism project, Frege believed there was a "remarkable difference" in how the fundamental principles of geometry and arithmetic are grounded (Frege 1874, p. 50). While the principles of arithmetic, according to Frege, can be derived solely from logic, he explicitly endorsed a Kantian view of geometry,

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3 Analyticity is defined by Kant in propositions of the form "subject-predicate". Later philosophers (starting with Frege) used broader notions of analyticity, while retaining the original term.

4 "An objective perception is knowledge. The latter is either an intuition or a concept (intuitus vel conceptus)", *CPR*, A 320/ B 377, Guyer and Wood translation.

where construction in pure intuition plays an essential role. He characterized the propositions of geometry as synthetic a priori, observing that "[t]he elements of all geometrical constructions are intuitions, and geometry refers to intuition as the source of its axioms" (Frege 1874, p. 50).

Although Schirn (2018, 2019) has argued that, despite Frege's explicit acknowledgment of adopting Kant's views on the subject, their positions on geometry might not fully align, there are nonetheless significant similarities between the two thinkers. One such similarity relates to Kant's notion of construction (Konstruction). As Kant states in the Doctrine of Method, "mathematical cognition [acquired through a priori construction] considers the universal in the particular, indeed even in the individual" (A 714/ B 742). Similarly, Frege notes that in geometry, "points, lines, or planes that are intuited are not really particular ones and hence can serve as representatives for the whole of their kind" (cited in Schirn 2017, p. 32).

That said, Frege views intuition as crucial in geometry not only because it validates its first principles but also because it plays a pivotal role in the process of mathematical proof. The derivational role of intuition will be addressed in the next part of this essay.

In contrast to Frege, Hilbert argued in favor of a Kantian conception of arithmetic. For Hilbert, mathematics could never be fully reduced to logic, and he believed that Frege's and Dedekind's attempts to do so were destined to fail (Hilbert 1926, p. 170-171). Although Hilbert discusses mathematics broadly, he specifically engaged with Kant's view of arithmetic as having an a priori synthetic character. According to Hilbert, these fundamental principles could not be reduced to logic alone. However, he also believed that intuition in geometry could be eliminated—this was one of the central aims of his *Grundlagen der Geometrie*.

Advocating for a form of formalism known as *deductivism*, Hilbert emphasized the importance of the consistency of an axiomatic system, particularly in the case of geometry. For Hilbert, the central concern was not the ontological status of mathematical objects or whether mathematical propositions corresponded to an external reality, but rather whether the system itself was internally coherent and free from contradictions. By focusing on consistency, he effectively relieved mathematical propositions of their ontological commitments, meaning they were no longer tied to questions of existence or truth in an absolute sense. Instead, mathematical statements were seen as valid or invalid based solely on their adherence to the rules and axioms of the system.

This shift allowed Hilbert to sidestep philosophical debates about the nature of mathematical objects, reframing mathematics as a purely formal, symbolic system whose significance lies in its internal structure rather than its connection to the external world. In discussing his view on the existence and truth of mathematical objects, Hilbert noted in a letter to Frege:

> If the arbitrarily given axioms do not contradict each other with all their consequences, then they are *true* and the things defined by them *exist*. This is for me the criterion of truth and existence. [Letter to Frege 29/12/1899. Quoted in Shapiro (2000, p. 156), my emphasis.]

As anyone who has seriously engaged with Kant's philosophy, even partially, can appreciate, Kant's assertion that geometric propositions are necessarily true aligns with his broader claim of the empirical objectivity and reality of the natural world. For Kant, the synthetic a priori nature of geometric truths supports the structured experience of space and time, which underpins our understanding of the physical world. Even if Kant had access to Frege's advanced logical framework or Hilbert's deductivist approach, it is likely he would still have rejected formalism due to its "idealistic"[5] implications. Formalism, by abstracting mathematics from its grounding in human intuition and experience, would conflict with Kant's insistence on the essential role of intuition in the construction of mathematical knowledge. Thus, Kant's critique of formalism would remain rooted in his commitment to the transcendental idealism that preserves the connection between human cognition and the empirical reality it seeks to explain.

The second group that Brittan distinguishes from the proponents of the classical reading of Kant consists of those who emphasize the role of *inferences* in geometric proofs. This group focuses on the role of intuition as a crucial guarantor of the validity of mathematical arguments. In their view, intuition becomes necessary when logical inference alone is insufficient to derive a seemingly true conclusion. To better illustrate this perspective, let us consider the very first proposition (1.1) of Euclid's *Elements*.

The first proposition of the *Elements* is, perhaps, the one that has received the most criticism of all the propositions in the book. Its apparent simplicity and elegance leave a strong impression when one begins to notice the logical gaps it contains. Let us now present its content.

In this proposition, Euclid constructs an equilateral triangle given a line segment, as shown in the figure above. The proof is carried out by constructing two circles using the third postulate, one centered at A with radius AB and the second centered at B with radius AB. Then, Euclid draws the line segments CA and CB from the point C where the two circles intersect, using the first postulate. The proof concludes with some simple arguments, using definitions and common notions, proving that the sides CA, CB and AB are all equal to each other and the triangle is equilateral, what was to be proved. At first glance, the outline of the proof presents no particular difficulties in terms of understanding. However, a closer examination of each step reveals a significant issue: The existence of point C is not proved by postulates 1 and 3[6], but only becomes manifest from the construction of the figure!

5 I use quotation marks here to indicate that the term "idealistic" is used in its broad sense, i.e. to deny the objective existence of the natural world. Hilbert either denies or has no problem denying that Euclidean geometry describes the physical world. In a discussion with Blumenthal in 1891, Hilbert had said "one must always be able to say, instead of "points, straight lines, and planes", "tables, chairs and beer mugs" [Hilbert 1935, p. 403. Quoted in Shapiro (2000, p. 151)].

6 For the sake of completeness, we set forth the postulates 1 to 3 (as translated by Heath): Postulate 1: To draw a straight line from any point to any point. Postulate 2: To produce (extend) a finite straight line continuously in a straight line. Postulate 3: To describe

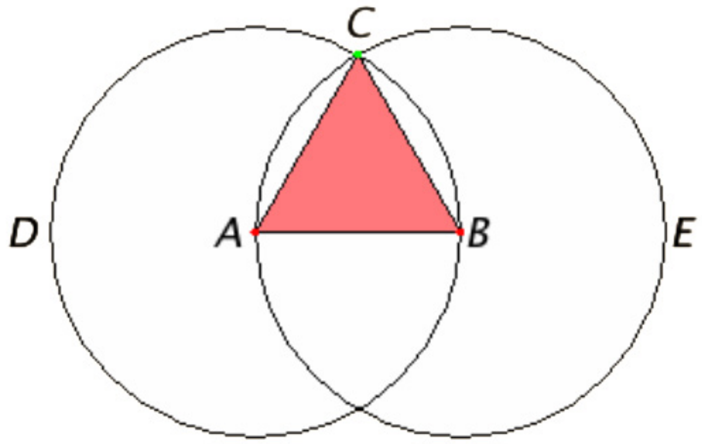

Figure 1. The figure of proposition 1.1 (Joyce 2010).

This reading of Kant is already evident in Russel (1919), as can be seen from the text I have quoted in the introduction (p. 252). Perhaps the predominant contemporary Kant scholar who has argued for this position is Micheal Friedman. Friedman (1992) emphasizes the role of monadic (Aristotelian) logic in Kant's work and in particular the question of existence that arises from it. As Friedman observes, the existence of more than two points in a plane cannot be logically derived in the Euclidean system. The key, then, to secure existence in geometric proofs is given by the concept/process of *construction*. The concept of construction is possibly the most central to Kant's view of geometry and the term "Konstruktion" is one of those most frequently found in Kant's references to the subject.

Kant uses the term "construction" in two related but distinct ways. The first refers to *empirical* construction which, simply put, is the process by which a geometer, using a straightedge and compass, constructs the points needed for geometric proofs by applying Postulates 1 to 3 of the *Elements*. Next, using the mental faculty of intuition, the geometer confirms, each time, the existence of the respective points. This is why I called intuition the geometer's guarantor in the title of this section. Without intuition, the existence of these geometric points is far from guaranteed.

Although empirical, this notion of construction is not *a posteriori.* This is because construction, while relying on a schema, does not, as Kant puts it, borrow any elements from actual experience. It abstracts from the particularities of individual shapes, focusing only on the general structure. When discussing triangles, Kant emphasizes the crucial role of intuition in both of his notions of construction.

> For the construction of a concept, therefore, a *non-empirica*l intuition is required, which consequently, as intuition, is an *individual* object, but that must nevertheless, as the construction of a concept (of a general representation), express in the representation universal validity for all possible intuitions that belong under the same concept. Thus I construct a triangle by exhibiting an object corresponding to this concept, either through mere imagination, in pure intuition, or in paper, in empirical intuition, but in both cases completely *a priori,* without having to borrow the pattern for it from any experience. (A 713/B 741, emphasis in the original. Guyer and Wood translation)

The second notion of construction is intimately related to what Kant terms "construction in pure intuition." Unlike physical construction, which involves tangible tools like a ruler and compass, this form of construction occurs within the realm of pure intuition—specifically, the pure intuition of space and the workings of imagination. In this intellectual process, concepts are not just abstract ideas; they are vividly represented in the mind's eye. For instance, a geometer performs this type of construction by mentally visualizing geometric objects and their interrelationships. Through this mental exercise, the geometer can manipulate these objects according to the principles and concepts pertinent to geometry, all while remaining within the framework of pure intuition. Guided by intuition, the geometer is able to solve problems concerning existence in geometric proofs, with intuition confirming the existence of each point when it is constructed.

In modern mathematics, where the use of universal and existential quantifiers is considered fundamental, the existence of mathematical objects is established purely through logical means. This stands in contrast to Kant's reliance on intuition, which is grounded in the spatio-temporal nature of geometric objects. While Kant's argument hinges on the idea that geometric figures possess this spatio-temporal character, this notion seems inconceivable to modern mathematicians, who approach existence from a more formal and abstract standpoint.

The significance of Kant's position, however, lies in the fact that without this spatio-temporal framework[7], the entire foundation of Euclidean geometry—the dominant model for understanding the natural world in his time—would be philosophically unestablished. Since, as Kant points out, mathematicians are not directly

a circle with any center and distance (radius).

7 "I cannot represent to myself any line, no matter how small it may be, without drawing it in thought, i.e., successively generating all its parts from one point, and thereby sketching this intuition. [...] On this successive synthesis of the productive imagination, in the generation of shapes, is grounded the mathematics of extension (geometry) with its axioms, which express the conditions of

concerned with questions of existence ("But in mathematical problems the question is not about this nor about existence as such at all" A 719/B 747, Guyer and Wood translation), it falls to philosophers to properly ground these concepts. Working within the limitations of the logic available in his day, Kant did his best to provide this necessary philosophical support[8].

2. The revelatory role of intuition

In this section, I propose an interpretation of passage A 716-717/ B 744-745 from the first *Critique*, arguably one of the most illuminating passages on Kant's conception of mathematical methodology. In it, Kant makes an important clarification: purely analytic thinking, such as that used by philosophers when dealing with concepts, is ineffective in the field of geometry. Kant references (without explicitly stating) proposition 1.32 from Euclid's *Elements*, where the task is to prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Kant asserts that no amount of philosophical meditation on the abstract concepts of "triangle" and "angle" can lead to the desired conclusion. Simply analyzing these concepts is insufficient to produce the property the proposition aims to establish. In contrast, the geometer begins by constructing a triangle, considers previously proven properties of triangles, extends one side of the triangle, and—through a series of further constructions and logical steps—reaches the desired result.

The difference between the philosopher and the geometer is not a difference of ability, since both are used to dealing with abstract structures and concepts. The difference for Kant is that the former does not use the *whole* of his mental faculties. These consist not only of understanding (Verstand), but also of *intuition* (Anschauung). A sterile rationalism cannot justify the latter, while a sterile empiricism cannot justify the universality of geometric results.

Kant's solution offers crucial insight into the relationship between the philosophy of mathematics and the methodology of the working geometer. The geometer does not merely contemplate the abstract concept of a triangle; rather, they engage with the problem through action (either in pure intuition through imagination or in empirical intuition by tracing the figure on paper), constructing the figure and intuitively "seeing" the proof unfold. Kant describes the solution of the problem guided by intuition as "einleuchtenden", which could be literally translated as that which "sheds" light on[9] something[10]. This is, I believe, the main role of intuition that most Kant scholars overlook: The *revelatory* one.

The revealing function of intuition that Kant presents in A 716–717 / B 744–745 remains highly relevant, capturing the mental state of the practicing geometer with remarkable precision. After all, one of the primary aims of the *Critique of Pure Reason*—if not the main aim—is to establish the existence and necessity of *a priori* synthetic knowledge, an epistemological project that seeks to integrate both rational and empirical elements. If Kant can show that the propositions of mathematics are *a priori* synthetic, he effectively refutes one of the strongest arguments of the rationalists, who regard mathematics as the paradigm of pure reason.

This interpretation is consistent with Kant's broader epistemological endeavor. From the very introduction of first *Critique*, Kant seeks to convince us of the existence of *a priori* synthetic propositions and remains faithful to this objective throughout his work. Another instance in the first Critique where Kant supports this revelatory function of intuition is found in the following passage of the Doctrine of Method.

> From *a priori* concepts (in discursive [philosophical] cognition), however, intuitive certainty, i.e., self-evidence, can never arise, however apodictically certain the judgments may otherwise be. Thus only mathematics contains demonstrations, since it does not derive its cognition from concepts, but from their construction, i.e., from the intuition that can be given *a priori* corresponding to the concepts. [...] [M]athematics can assess the universal *in concreto* (in the individual intuition) and yet through pure *a priori* intuition, where every false step becomes visible. (A 734-735/ B 762-763, Guyer and Wood translation. Emphasis in original.)

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sensible intuition *a priori,* under which alone the schema of outer appearance can come about." (A 162-163/ B 203-204, Guyer and Wood translation)

8 There are, of course, philosophical (and specifically naturalistic) objections to this function of intuition. The main one is given by Kitcher (1992), although a hint of the argument must already be present in Hume. Hume tells as in *Treatise* that "I have already observ'd, that geometry [...] yet never attains a perfect precision and exactness. Its first principles are still drawn from the general appearance of the objects; and that appearance can never afford us any security, when we examine the prodigious minuteness of which nature is susceptible." (Hume 2007, p. 51). Kitcher (1992, p. 126-127), commenting on Kant's argument for the synthetic character of space, argues that Kant's view presupposes that the human eye can indeed perceive infinitely small parts of space, which is impossible (with the data of modern science, of course). Hence, Kant is not justified in believing that the infinite bisection of rectilinear parts of physical space is guaranteed by intuition, as these may not even be (visually) intuitable/detectable.

9 There is a connection with Plato here. This element of revelation that is evident in Kant's text is, in large part, Platonic. I am not suggesting, of course, that Kant is an ontological (and mathematical) realist, but that the intuition that enlightens the geometer and leads him closer to the cognition of mathematics is reminiscent of the case of the cave, where the philosopher comes to light (and to the knowledge of the Forms) through the process of education ("If we're thinking about the effect of *education* —or the lack of it— on our nature, there's another comparison we can make. Picture human beings living in some sort of underground cave dwelling, with an entrance which is long, as wide as the cave, and open to the *light*." *The Republic* 514 a, my emphasis). This education, needless to say, for Plato included decades of intensive study on mathematics. Finally, in the *Symposium*, Diotima speaks of the ladder of Eros (209e-210a), where it is the means to knowledge of the Form of Beauty. What is interesting here is the language that Plato uses, which comes from the Eleusinian mysteries (Nightingale 2021, p. 31. Quoted in Kindi 2023). Especially, the words Plato uses for the supreme initiation into the Form of Beauty are "τέλεα καί ἐποπτικά", which Benardete (2001) translates as "perfect revelation".

10 I quote here the original German text: Er gelangt auf solche Weise durch eine Kette von Schlüssen, immer von der Anschauung geleitet, zur völlig einleuchtenden und zugleich allgemeinen Auflösung der Frage.

In this context, intuition acquires both a positive and negative heuristic role. In its positive sense, intuition guides the geometer through the problem-solving process, directing each step in the construction of the proof until the solution becomes fully illuminating[11](völlig einleuchtenden). In the passage just mentioned, Kant suggests that this intuitive certainty is akin to self-evidence—a kind of immediate justification for the correctness of mathematical judgments[12], a certainty that philosophy cannot attain in the same way.

In its negative sense, intuition serves to reveal errors or inconsistencies to the geometer. By making "every false step visible," intuition acts as a safeguard, preventing the geometer from falling into logical fallacies or incorrect constructions. This error-detecting function of intuition ensures that each step in the proof is not only guided but also rigorously checked. Thus, intuition operates as both a constructive and corrective tool in mathematical reasoning, providing the geometer with immediate insight into both the accuracy of their reasoning and the identification of any missteps. This dual heuristic role—both positive and negative—demonstrates the indispensable role of intuition in the practice of geometry, a role that is absent in purely conceptual or philosophical analysis.

Now that I have provided some support for the "revelatory" role of intuition (as indicated by Kant's use of "einleuchtenden" when referring to the solution of the geometric problem in A 716–717 / B 744–745), it is important to examine how other scholars have translated this passage of the first Critique. Unfortunately, our interpretation does not align with that of the majority of translators of the *CPR*.

To explore this divergence, I will start by examining the most recent and widely accepted translation of the *CPR* into English which is by Paul Guyer and Allen Wood (abbreviated G & W, 1998). In their translation, G & W render the German term "einleuchtenden" as "illuminating." This choice aligns with our proposed interpretation of the term, which I believe faithfully captures both the textual content and the overall spirit of Kant's work. Notably, this is the only English translation that employs the term "illuminating" for "einleuchtenden."

The first official English translation of the *CPR* was made by Francis Haywood (1838), who translated "einleuchtenden" as "clear". I find this rendering doubtful and see no need for further explanation. The next English translation was made by John M.D. Meiklejohn (1855), who retained Haywood's term. A shift, however, occurred in the next English translation, by Friedrich M. Müller (1896). Müller rendered "einleuchtenden" as "convincing," which seems to me more appropriate than the others that have been used over the years, with the exception of course of G & W's translation. After Müller and up to G & W, the term preferred by all translators (Smith 1929, Pluhar 1996, Weigelt 2007 [based on Müller's (!)]) was "evident". This is akin to the Greek translation composed by Micheal Demetracopoulos (2006), who used the term "σαφής ή προφανής" ("clear or obvious"), while André Tremesaygues and Bernard Pacaud (1975), in the currently predominant French translation of the *CPR*, opted for "claire", a term similar to "clear" as used by Haywood and Meiklejohn.

My position on the above translations (except G & W) can be summarized in the following sentence: The role they assign to intuition is not unrelated to the concept of Anschauung in Kantian epistemology, but it is unrelated to the role Kant assigns to it in A 716–717 / B 744–745. As is well known, Kant attributes two main characteristics to intuition (compared to concepts): Intuitions are immediate and singular, as opposed to concepts that need a means to be understood (they are thus mediate) and generic. The relationship between these two attributes of intuition has been widely discussed in the literature, with Charles Parsons (1992) and Jaakko Hintikka (1992)[13] as the leading commentators. In short, Hintikka argues that the two criteria are ultimately identical, with the immediacy criterion being just another formulation of the criterion of singularity. In contrast, Parsons argues that the two criteria are different, with the criterion of singularity being more general and not reducible to that of immediacy. Regardless of how this debate is resolved, what commentators certainly agree on is that these two criteria are crucial to understanding Kantian epistemology. Kant in Logic (paragraph 1) may only mention the criterion of singularity for intuition, but he is clear in the CPR when he uses both singularity and immediacy to refer to intuition (A 19/ B34 and A 320 / B377. Cited in Wilson 1975).

In this context, the terms "evident" and "convincing" are representative of "einleuchtenden", as both clarity/transparency and persuasiveness satisfy the two above-mentioned criteria of intuition. The reason, in my opinion, why I do not consider these renderings to be appropriate is that they do not relate to the thinking of the geometer during the deductive process. The proposition "Every effect has a cause" is an a priori synthetic proposition for Kant, a law of nature, and can be said to be clear to us. But it does not *lead* us to the production of new knowledge

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11 I am using here Guyer and Wood's translation of "einleuchtenden", which I find most fitting for this interpretation. I will elaborate on this choice, as well as other possible translations, later in the essay.

12 An anonymous reviewer (#1) suggested that "There are some particular issues concerning the proposals presented in the text that need clarification or even justification. For instance, the use of 'propositions' and not 'judgments' to talk about intuition'". This is a reasonable claim. When discussing mathematics, referring to "mathematical judgments" instead of "mathematical propositions" may seem unusual. The most accurate Kantian description would be to call them "mathematical constructions," given the constructive nature Kant attributes to mathematical knowledge. Although Kant emphasizes the role of intuition in proving the propositions of Euclid's *Elements* (a few of which are discussed in the *CPR*), I believe it is more appropriate to speak of constructions rather than judgments when assessing Kant's views on geometry. However, in each step of the proof, the act of confirming the existence of geometric elements does involve—or constitutes—a judgment in Kant's terms. That said, I prefer the term "confirm" here, rather than phrasing it as "judging that it exists."

13 There have, of course, been other attempts to explain the relationship between immediacy and singularity. Kirk Wilson (1975) attempted to respond to Parsons and Hintikka by contending that these two criteria are *intentionally* different but *extensionally* identical, seeking a synthesis of the two approaches by using, among other things, elements from mereology. Manley Thompson (1992) agrees with Hintikka that the two criteria are ultimately identical, but disagrees that Kant uses singular terms in the modern sense. Kant's singular terms, Thompson tells us, simply denote *possibilities* and make no claim for the existence of mathematical objects (drawing on Parsons and passage A 719/ B 747 of the *CPR*). See Posy's introduction (1992) for a concise analysis of this discussion.

in the way that intuition does for Kant in the passage[14] A 716–717 / B 744–745. Kant strives to explain the difference between the philosopher and the mathematician by saying that intuition leads the latter to the solution of the mathematical problem that the former, without the use of intuition, cannot achieve ("Now he may reflect on this concept [of the triangle] as long as he wants, yet will never produce anything new" A 716 / B 744). The notion of clarity, then, does not provide intuition that *guiding* element that Kant tries to attribute to it in this passage.

In this sense, the word "convincing" seems to me a more accurate translation, since persuasion is always related to an argument one is trying to make. For example, when we say that "This argument is evident to me", it signifies that it is understandable to me, that I can grasp its meaning. When, on the other hand, we say that "This argument is convincing", something is always implied as to which this argument has the property of persuasion. This, in the case of geometry, is the next step in solving the problem or realizing that what was to be proved was proved. Once again, however, this term is not as appropriate and indicative of Kantian reasoning as the term "illuminating".

3. Conclusion

In this essay, we explored various aspects of Kant's notion of intuition in geometry, focusing primarily on his views in the Doctrine of Method in the first Critique. I specifically argued for a guiding or heuristic role of intuition, which I termed "revelatory." This function, I contend, is not undermined by the objections raised against the classical interpretation of Kantian intuition, as discussed in the first section of this essay. In this sense, the "revelatory" role of intuition remains relevant today. However, I acknowledge that this claim is not fully substantiated without considering more recent developments in the philosophy of mathematics, beyond the frameworks of Kant's immediate successors, such as Frege and Hilbert. Addressing this would go beyond the scope of this essay. Instead, this work should be viewed as an introductory step toward examining this perhaps overlooked function of intuition.

This need becomes even more pressing when we consider the translations of the A 716–717 / B 744–745 passage of the first Critique. As I have attempted to show, only one English translation has accurately rendered the term "einleuchtenden" in a manner that is faithful both to the guiding role of intuition in Kant's philosophy of mathematics and to the overall spirit of his text. The primary goal of this essay was to emphasize the importance of further studying the revelatory role of Anschauung, both in the philosophy of mathematics and in the history of philosophy. I hope this essay marks the first step in both directions.

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[14] In her recent article, "Two Models of Kantian Construction" (2023), Aljoša Kravanja argues that Kant employs two distinct models of construction in the Doctrine of Method: one she calls the "equivalence model" and the other the "overstepping model." In the latter, intuition plays a key role in the generation of new knowledge. According to Kravanja, this model asserts that "intuition conveys more than the concept," and to construct in this sense "means to go beyond the concept." In the Doctrine of Method, particularly in A 716-717 / B 744-745, Kant's use of intuition in constructing mathematical knowledge—and in distinguishing the mathematical from the philosophical methodology—aligns with the overstepping model, as outlined by Kravanja.

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