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On the provenience and meaning of the concept "exponent" in Kant's Critique of pure reason*

Sobre la proveniencia y el significado del concepto "exponente" en la Crítica de la razón pura, de Kant

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Abstract

In this text I shall explore the meaning of the concept "exponent" in the *first Critique* by resorting to its provenience. Beginning with a brief analysis of the two meanings Kant ascribes to it the *Critique*, the exponent of a series and the exponent of a rule, I intend to point out that by means of Kant's concept of analogy, intimately linked with proportion, we can find a route into some of the mathematics textbooks of the 18th century, which shed great light in the matter. Thereafter, as a transition for returning to the *Critique*, we shall see how, in the *Duisburgscher Nachlass*, the exponent plays a central role for Kant as he thinks the emergency and necessity of rules in Philosophy, in comparison to Mathematics. In this way I hope to show how the "exponent" is taken up by Kant and made fruitful, especially for the *Analogies of experience*.

Keywords

analogy, exponent, judgment, proportion, rule

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Resumen

En este texto exploraré el significado del concepto "exponente" en la *primera Crítica* recurriendo a su proveniencia. Partiendo de un breve análisis de los dos significados que Kant le atribuye en la *Crítica*, exponente de un serie y exponente de una regla, pretendo señalar que, mediante el concepto kantiano de analogía, íntimamente ligado a la proporción, podemos encontrar una ruta a algunos de los manuales de matemáticas del siglo XVIII, que lanzan gran luz al este respecto. A partir de esto, como transición para volver a la Crítica, veremos cómo, en el *Duisburgscher Nachlass*, el exponente desempeña un papel central para Kant al pensar la emergencia y la necesidad de las reglas en Filosofía, en comparación con la Matemática. De esta manera espero mostrar cómo Kant incorpora el "exponente" y lo hace fecundo, especialmente para las Analogías de la experiencia.

Palabras clave

analogía, exponente, juicio, proporción, regla

Introduction

The concept of exponent is mentioned very few times in the *Critique of pure reason*. This fact, at least according to a quantitative criterion, seems to attest in favor of its irrelevance. It literally occurs in four passages: two belonging to *Transcendental Analytic* and two, to *Dialectic*. In the former case, the first mention occurs when systematically representing the principles of pure understanding and, the second, in a conclusive excerpt from the *Third Analogy*; in the latter case, we find it in the section *On transcendental ideas* and, for the last time, in the first section of the *Antinomies*, the *System of cosmological ideas*. Located sparsely, in different registers of the work, this attestation indicates nothing about an alleged unit of signification or an alleged explanatory capacity of the concept in question; on the contrary, it could be said that the exponent is a pre-critical residue of a book empirically composed in different periods, and nothing more.

However, it is worth noting that three of the four occurrences explicitly share at least one aspect, namely: the notion of series, in view of the moments of relation; in particular, regarding the pure concept of causality. Thus, if in the *Analogies* we have interconnection of phenomena subordinated to certain exponents[1], in the first book of *Dialectic* the exponent

[1] "Our analogies expound, properly, the unity of nature [in empirical sense, AP] in the interconnection of all phenomena under certain exponents that express nothing more than the relation of time (insofar as it contains in itself all existence) to the unity of apperception, which can only take place in the synthesis according do rules" (KrV, A216/B263). I shall quote the works of Kant according to the AA (with exception to the KrV, in which case, as usual, the excerpts are referred to by A and B). All translations are mine unless otherwise stated.

makes possible the progression of the series of mediated inferences in general[2], and, in the *System of cosmological ideas*, it is attributed to the pure concept of causality the exponent of a series, as a series of causes (as conditions) to a given effect (as conditioned)[3]. In contrast, the first occurrence attributes the exponent of a rule in general to the principles of pure understanding, as a condition under which, given the concrete case through experience, the particular laws of nature, subject to these principles, would be possible[4].

In this sense, it is a question of how the relationship between rule and series in general works (be it causes and effects, reasoning etc.) and, more importantly, what the exponent means for, or what it can clarify about, this relationship. The serial aspect of the concept in question would point to its mathematical provenience and its formulation in judicative terms would be indicative of the appropriation carried out by Kant. Thus, in order to avoid a genetic path that would have as a starting point other texts than that of our author (what, therefore, would make the presentation of the argument somewhat arbitrary) and some notes by Kant himself in loose sheets or in marginalia of other works (in which case the bet on the cohesion of the material would be very uncertain), we shall start our route by considering the judicative element to establish a framework from which I will carry out a regressive analysis of some mathematical sources, in order to approach with greater certainty the serial character. I hope to make clear how, by means of this regressive analysis, the exponent plays a relevant role in Kant's thinking, especially in establishing what would become the *Analogies of experience*.

[2] "Now, every series whose exponent (of categorial or hypothetical judgment) is given, may be continued; therefore, this very action of reason leads to the *ratiocinatio polysyllogistica*, which is a series of inferences that may be continued until undetermined distances, be it by the side of conditions (*per prosyllogismos*), be it by that of the conditioned (*per episyllogismos*)." (KrV, A331/B287-8).

[3] "The same is also valid for substances in community, which are mere aggregates and have no exponent of a series (…). In consequence, only the category of *causality remains*, which offers a series of causes for a certain given effect, whereof we can add that, from the later, as the conditioned, to the formers, as conditions -[…]-" . (KrV, A414/B441-2).

[4] These [superior principles of the understanding, AP] alone, however, provide the concept that contains the conditions and, so to say, the exponent of a rule in general; whilst experience provides the case that is under the rule" (KrV, A159/B198).

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I

As to the four passages referred to above, the one that offers more elements for understanding the judicative meaning of the exponent is the one that mobilizes it to explain the generation of *ratiocinatio polysyllogistica*. In the register of a strictly logical use of our superior cognitive capacity, a syllogism or inference of reason is explained as follows: in the major we have a universal rule; in the minor, the subordination of the condition here expressed to that of the major; finally, the conclusion expresses the assertion of the predicate of the universal rule to the subsumed case. Schematically, according to KrV, A304/ B360-1 and A330/B386:

[All] S is P - universal rule that expresses the assertion of the predicate (P) to the condition (all S)

[All] Q is S - subordination of the condition of the minor (Q) to the condition of the major (all S)

[All] Q is P – assertion of the predicate (P) of the universal rule (all S is P) to the subsumed case (Q)

For the sake of clarity, let us take the judgment "Socrates is mortal". Such a judgment can also be acquired through experience, insofar as I recognize mortality as a ground of cognition, a *ratio cognoscendi*, of the intuitions of Socrates, Plato et al. Unlike the context of the experience, however, two aspects need to be highlighted: i) in the register of logical use, on the one hand, it does not matter the transcendental reference of the concept-subject to intuition. The singularity of intuition, which accounts for the immediacy in which it represents the object, implies that it has no extension (otherwise it would be an infimal concept[5]). However, precisely because it has no extension, what is asserted of Socrates could never refer solely to part of what we represent therein. And ii), since intuition and concept do not differ by the object represented or by what the representation is referred to, but only by the mode or form of the representation[6] (singular, universal; immediate, mediate), it is perfectly licit that, in the syllogism, I represent the singular Socrates *as if* it had common validity, as if it had an extension, so that it can, thus, integrate the purely logical consideration proper to the formal register with which we now deal - something like "all Socrates ", as its total extension, is mortal, where the predicate subordinates, without exception, everything that is under the concept-subject. In this sense, Kant says that "an

[5] Cf. See the specification principle KrV, A655-6/ B683-4; compare with the elucidation of quantity in the table of logical functions, KrV, A71/B98.
[6] See KrV, A320/B376-7.

inference of reason is itself a judgment that is determined, a priori, in the entire extent of its condition" (KrV, A322/B378)

Given the judgment, it is required to find a concept that contains the condition by means of which the assertion (of the predicate mortality) befits Socrates - in this case, it is the concept of man, that is, the condition of mortality; i.e, Socrates, taken in all its extension, expressed in the concept of humanity. Thus, the representation Socrates (condition of the minor) is entirely subordinate to the medium-term humanity (condition of the major, and assertion in the minor), from which we have the minor premise: Socrates is man. Finally, the universal rule that subordinates humanity and provides its predication to man is sought: all men are mortal. In short: the rule "all men are mortal" establishes the predication of something universal (mortality) to Socrates, under the condition of humanity.

This condition-conditioned relation explains the predication exemplified under two aspects: the subordination of Socrates-man-mortal, in this order, each included under the sphere of the other, as partial representations of the entire extension of the concept, each time, superior; on the other hand, the inclusion of mortal-man-Socrates, in this order, in the intension of the other, insofar as they play the role of ground of cognition[7]. For this reason, the production of polysilogisms can take place in two complementary directions. Reason intends either to find the entirely determined concept (the infimal species) of an individual (therefore, the one subordinated to the extension of all the others that are included in its intension), or the supreme concept or superior ground of cognition (the one included in the intension of all the concepts that are part of its extension).

Putting aside the question of legitimacy, in the first case, of this claim of reason, it is interesting to point out that the continuation of the chain, according to its strictly logical operation, depends on the acquisition of the ground according to which condition and assertion (S and is P, in our example) will be bound in the major premise of each syllogism, in one word, the fundament of the rule. According to this, there will be three types of fundament or ground: the bond in a subject, the bond of the chained members and the bond

[7] Concerning *Merkmal* as partial representation and ground of cognition, see: AA XVI, R2283, 299 (ca.1780-1789? [1776-1778??]); R2285, 299 (ca. 1780-1783? 1776-1779??); R2286, 299-300 (ca.1780-1783). Concerning this subject see the doctoral thesis of Luciano Nervo Codato (*Forma lógica na Crítica da razão pura*, 2003), particularly pp.89-97. Part of the content was published in *Extensão e forma lógica na Crítica da razão pura* (Revista Discurso [USP], n.34, 2004, pp.145-202), particularly pp. 189-194.

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of the parts in a whole. Through them, reason seeks, respectively, the subject who is not predicate, the presupposition that does not presuppose anything else, and the aggregate of divided members for whose completion no more division is required[8]. Precisely these grounds of the universal rule are called exponents: "Now, every series whose exponent (of the categorical or hypothetical judgment) is given can be continued; therefore, the very same activity of reason leads to *ratiocinatio polysyllogistica*" (KrV, A331/B387). We therefore have three modes of syllogism, a typification that elicits its diversity from the universal rule expressed in the major premise.

In these terms, the concept in question is also present on in some *Reflexionen* on logic[9]. For example, we find, in the well-known R3202, the following: "a rule is an assertion under a universal condition. The relation of the condition to the assertion, as the latter is under the former, is the exponent of the rule" (AA XVI, 710 $_{8-10}$, 1790s). Also, in a set that deals strictly with judgments, we read:

"The relation of concepts (exponent):

The subject to predicate
The ground – consequence — form of judgements
Whole – part

Categorical, hypothetical, disjunctive". (AA XVI, R3063, 636 $_{4-11}$ (ca.1776-1779?, [1773-1775?], 1780-1783??).

Indeed, the essential of the operation made possible by the given exponent resides in the logical bond between condition and assertion, whose judicative expression constitutes a universal rule. To the extent that, in this register, one does not go beyond formal consideration, the exponents that provide the chain of the inferences mentioned are not used only in syllogisms. They also apply to judgments that, regardless of the origin of the expressed knowledge or cognition, can play the role of universal rule (or law), i.e., they can be used as a premise in a potential reasoning.

Thus understood, all judgments, as universal propositions, apt to be used as a major premise, are called principles. Note, however, that fundamental or superior propositions of different origin, such as mathematical axioms (e.g.: between two points there can be only

[8] See KrV, A323/B379-381
[9] Mentioned and briefly analyzed by K. Reich (2001, 80-81 [66-67]).

one straight line), principles of pure understanding (e.g.: everything that happens has a cause), or even propositions acquired from experience via induction (e.g.: all bodies are heavy), can be used as principles. If one considers them as to their origin, this denomination would be, strictly speaking, inappropriate, since a principle, in an absolute sense, should provide synthetic knowledge of the particular in the universal through concepts alone[10]. In the case of axioms it is not possible for me to know such a property of the line by concepts alone, but only in pure intuition; as for the principles of pure understanding, although they originate from concepts, they need a confluence from the sensible conditions of a possible experience (one cannot, for example, derive from the mere concept of happening that everything that happens has a cause); in the case of universal propositions acquired by induction and formed by empirical concepts, sensation is required. Regarding, however, the cases that can be subsumed in each of these domains, it is correct to call them, comparatively, principles; in their specific domain of application they configure superior knowledge[11]. For this reason, the meaning of principle carries within it equivocation. According to our author: "the expression of a principle is equivocal, and commonly it means only a knowledge that can be used as a principle, even if, in itself and according to its own origin, it is not a principium at all" (KrV, A300/ B356)[12]. Although the author resorts to this common meaning, there is a preference for characterizing a principle in relation to the scope in which this name can be taken in the strict sense:

> Therefore, I would call knowledge by principles that in which I know the particular in the universal by concepts. Thus, every syllogism is a form of deriving a knowledge from a principle. For the major premise always offers a concept which makes that everything that is subsumed under its condition be known by means of it according to a principle. Now, since all universal knowledge can serve as a major premise in a syllogism, and understanding offers such universal propositions *a priori*, then these can be called principles, in view of their possible use. (KrV, A300 / B357)

In a transcendental register, which includes the question of origin, understanding does not yield principles, since its superior propositions are not based on mere thinking, that is, they do not offer knowledge by concepts alone, but by concepts and intuition. Nevertheless, the consideration of the use abstracts from the transcendental conditions of

[10] See. KrV, A300/B356-7.

[11] In the case of Mathematics this in mentioned in A300/B356. Regarding principles of pure understanding, in KrV, A148/B188 and A302/B358.

[12] The difference between principle and principium as already mentioned in the *System of principles*. See KrV, A160/B199.

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acquisition of these propositions and attends only to what is essential in a rule, in general. Although the *raison d'être* of the principles of pure understanding in the argumentative economy of the text is linked to the reference to intuition (to the form of possible experience), considered *a priori* synthetic judgments, the logical-formal character is inseparable from what they have of fundamental, the logical aspect of the judicative form, rooted in the domain of thinking, of which knowing is only a case. We mean to point out that there is a non-excluding character in what concerns the relation between the registers of general logic and that of transcendental logic, as, for example, the objective validity of concepts, as bound in judgments, which is sought out by the latter must still and necessarily bear the form of thought, with which the first deals.

Now, on the one hand, taken as judgments in general, the superior propositions presented in the *System of principles*, as laws, must bear, in their judicative "structure", the relation between condition and assertion, thought in the exponent of the rule (for the form of thinking); on the other hand, taken as *a priori* synthetic judgments, a characterization relevant only in a transcendentalized logic, there must be a determined reference to sensibility (for the real, not just logical, possibility of the conceptual bond). We must then grasp how can we, is this case, attribute objective reality to the logical relation between concepts.

II

When it comes to investigating the concept of exponent, this cleavage between the strictly logical relation of concepts and their possible reference to something = x is especially significant in the *Analogies of experience*. The three superior propositions (which, as such, are laws that have the exponent of a rule) of pure understanding found in this section of the *System of Principles*, establish a domain called nature (in empirical sense), by which it is understood "the interconnection of the phenomena according to their existence, according to necessary rules, that is to say, according to laws" (KrV, A216/ B263). These fundamental laws, the synthetic *a priori* judgments of relation, taken together, exhibit the result of the *Analogies*: "our analogies expound, properly, the unity of nature in the interconnection of all phenomena under certain exponents that express nothing more than the relation of time -

[…]- to the unity of apperception" (KrV, A216/B263). It is not clear, however, what specific contribution is required from exponents so that, subordinating the phenomena, these can be known as interconnected, as to existence. More than that, it would be hasty to assume the meaning of exponent in this excerpt is the same as that found in the *Dialectic*, that is: that the principles of relation, as judgments, can be interpreted according to the relation of condition and assertion found by the exponent does not seem, at this point, too problematic; but that the interconnection of the phenomena occurs under exponents, this seems to require a different meaning of the term than that linked to the judicative form.

At this point, I would like to suggest that attention to the concept of analogy may indicate a promising way to undo the knot. In the context in which this concept comes to the fore, in an introductory section of the *Analogies*, Kant is busy indicating why the principles of dynamical use differ from those of mathematical use. One of the focal points of this difference lies in the constitutive character of the latter, as opposed to the regulatory of the former. This character is linked with a difference in the kind of synthesis (whose superior concept is that of "bond": *conjunctio*, *Verbindung*), specific to each variant. Avoiding entering into all the intricacies of these notions, it should be said that the principles of mathematical use operate through the kind of bond called composition (*compositio*, *Zusammensetzung*), whereas those in dynamic use do it through connection (*nexus*, *Verknüpfung*)[13].

The distinctive feature of the composition (homonymous of the mathematical operation[14]) is that the bound elements are homogeneous with each other and, therefore, it is licit to consider them mathematically; however, they are not necessarily mutually implicated. Both in the case of composition by aggregation and by coalition, homogeneity resides in that all composite members are acquired by the construction act itself, since it is a limitation of the parts of space, or of time. Thus, precisely in virtue of this homogeneity, these constructed elements do not belong to each other because the construction procedure

[13] See KrV, B201-2, note

[14] Stricly speaking, one cannot identify the procedure of composition in the case of principles of mathematical use and in that of Mathematics. In a context in which it is a question of differentiating Philosophy and Mathematics, Kant says that: "In fact, I referred in the *Analytic*, in the table of principles of pure understanding, to certain axioms of intuition; but the principle there introduced was not itself an axiom, but served only to indicate the principium of possibility of axioms in general and, itself, was only a principle from concepts"(KrV, A733/B761). See also KrV, A732/B760 and KrV, A733/B761.

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is, at the same time, arbitrary: in the aggregation of a square by the conjunction of the base of two triangles, or in the coalition of an intensive magnitude by composition from a gradation of infinite intermediate points between 0 and 1, none of the composite members is required to conceive another; that is: unlike an accident in relation to a substance or an effect in relation to a cause (whose bond involves some contingency, that of existence), the constructed elements are, concerning their conception, independent of each other. In this sense, the principles of mathematical use are characterized as constitutive, since homogeneity and reciprocal independence allow me, so to say, to acquire all the members of the composition (leastwise virtually) in the aggregation or coalition procedure itself; in a word, the constructed concept engenders the represented object itself.

With the bond by connection, it happens differently. This is due precisely to the fact that the link of existences takes place (the perceptions of effective objects among themselves, in which case the connection is physical, or those perceptions with the superior cognitive capacity, in which case the connection is metaphysical). Regarding the principles of dynamic use, but above all those of the *Analogies*, the regulatory character concerns the fact that it is not possible to indicate the members connected in a determined way *a priori*. Indeed, the contingency of the way in which perceptions meet with each other is an index that it is not possible to establish a priori the perceptual data itself, which always involves existence. According to the text:

> Here, it is not to think of axioms, nor anticipations; but, if a perception in the temporal relation with others (even if indeterminates) is given to us, it is not possible to say *a priori*: *which* other perception or *how great*, but only how it is bound necessarily with the other in what concerns existence, in this modo of time (KrV, A179/B222)

As can be seen, since the phenomenon cannot be constructed as to existence, the physical connection will not exactly refer to the terms bound, but only to the relation between them; that is why these principles are called regulatory. Thus, even if they represent the real connection in an experience, to the *Analogie*s can only be attributed a priori knowledge of existence in a comparative way[15]. The proper reference, e.g., of the principle of causality, would not reside in denoting the necessity of the *terms* bound in a causal connection; rather, its referent would be the form without which such a relation could not be thought necessarily, insofar as by means of it we can advance the series of existences through the relation of

[15] See KrV, A225/B273 and A226/B279

possible perceptions, since we know the *form* in which this relation must take place. In one word, by means the principle stated in the *Second Analogy* one does not know effectivity itself, but the form according to which possible effectivity must take place as to be thought in a determine matter.

Now, the impossibility of referring, *a priori*, to the terms bound in physical connection is part of the core of the concept of analogy in Philosophy, as opposed to its mathematical meaning. In the introductory section to the *Analogies* we read that:

> In Philosophy the analogies mean something quite different from that which they represent in Mathematics.

In Mathematics

> -[…]- they are formulae that enunciate the equality of two quantitative relations, and [they are, AP] always *constitutive*, in such a manner that, if two members of the proportion are given, with it the third is also given [AA: three members… the fourth[16]], that is, can be constructed.

In Philosophy

> -[…]- analogy is not the equality of two *quantitative* relations, but *qualitative*, in which from three given members I can know and give *a priori* the *relation* to a fourth, but not *this* fourth *member* itself;

And, in Philosophy, the way in which the other members are found:

> however I have indeed a rule to search for it in experience, and a characteristic sign [*Merkmal*] to find it therein (KrV, A179-180/B222).

The approximation, albeit by means of difference, with the mathematical analogy holds rich elements to understand philosophical analogy. Before looking more closely at the similarities and dissimilarities between the two models, I will try to go a little deeper into the notions of mathematical analogy and proportion to better support my argumentative proposal. For now, one should note only that here we have an indication that the way in which the exponent of the rule (relation between condition and assertion in a judgement) and

[16] In the AA it was intended to correct Kant's text. As we will see below, when investigating the mathematical notion of (geometric) proportion, it is true that the acquisition of a fourth unknown term requires three known ones; however, in a progression, from the relationship of at least two members, as Kant says, it is possible to acquire the ratio by which the others are found.

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the exponent of the series (interconnection of the phenomena under the exponents) correlate must be linked, in some way, to proportion.

III

As K. Reich has pointed out[17], the concept of proportion appears in eighteenth-century mathematics manuals linked to the concept of relation, in the wake of Euclid's book V of the Elements. In an exemplary way, G. S. Klügel (1739-1812) explains, in his Mathematics Dictionary (5 vols.) The notion of relation as follows:

> Relation (ratio, λόγος) of two homogeneous quantities to each other is, according to Euclid (V. explan. 3), the mutual reference in which both these quantities are to each other, regarding their quantities, so that, thus, the concept of a relation in general arises from the comparison of two homogeneous quantities with each other. (KLÜGEL, 1831, T.5, B.2, 728).

A mutual correlation that takes place between two homogeneous quantities (a condition without which they could not be compared) does not, strictly speaking, constitute a relation if the quantities are equal. More precisely, a relation takes place by the inequality of the compared quantities; comparison by which the relation is established. Thus, it can be said that the question that poses the terms in which the relation will take place is a question concerning the *how* of mutual reference between quantities; question related to *how much* or *how many times*. As the comparison is linked to one or the other, the answer to the question determines an arithmetical or a geometrical relation[18] (although this denomination has already been considered inappropriate[19]).

[17] REICH, 2001, 80-82 [66-68]. Reich also indicates, in these pages, the textbooks we shall investigate in the sequence.

[18] As L. Euler puts it: "(...) when one asks about inequality, it can occur in two ways; then, either it is asked how much one [quantity] is greater than the other, or how many times one is greater than the other. Both types of determination are called relation; the first one is usually named arithmetical and the second, geometrical relation" (EULER, 1911 [[1]1770], §390, p.148). In the same way Kästner: "when we investigate, through subtraction, how great one number is in comparison to another, we consider both these numbers in their arithmetical relation (ratio arithmetica). When, however, we investigate, through division, how many times one number in contained in the other, or what sort of part it is from the other, we consider them in their geometrical relation (ratio geometrica)" (KÄSTNER, 1786 [[1]1758], V, p.129).

[19] For Euler: "this denomination has nothing in common with the subject matter itself, but was adopted arbitrarily" (EULER, 1911 [1770], §390, 148); In the same way Klügel: "Both names [arithmetical and geometrical] are inappropriate" (KLÜGEL, 1831, T.5, B.2, 729). Also, Kästner notes that: "according to the use of the ancients, the name relation pertains only to the geometrical. The moderns also attributed it to the

The relation is understood as inequality between numbers, and when considering its members (as correlated quantities by comparison), one investigates how one quantity can emerge from the other; in which case, of a known quantity, it is possible to discover the other, at first unknown. In an arithmetic relation, this inequality is called difference (See EULER, 1911 [[1]1770], §§381-3, 146; also KLÜGEL, 1831, T.5, B.2, 729): it is the quantity that must be added to the antecedent to arrive at the consequent, since it is obtained by subtracting the smallest from the largest, such as in the relation between *a* and *b*, in which *b* is greater and *a*, smaller, the difference, *d*, is obtained by $b - a$. On the other hand, when it comes to the geometrical relation, the inequality between the members is obtained when, in relation $a : b$, the consequent is divided by the antecedent (or on the contrary, since the order of the members is indifferent), so that if *a* is known, *b* is reached by multiplying the antecedent by the *inequality* (for antecedent 4 and consequent 2, we have inequality 1/2). Euler calls this inequality, which is expressed in a fraction whose numerator is the consequent and the denominator the antecedent, the name or denomination (*Benennung*) (EULER, 1911 [[1]1770], §§441-445, 164-166). Klügel indicates, in another way, that the inequality can also be called exponent: "The number *e*, by which to multiply A, to obtain B is called exponent or name of A: B" (KLÜGEL, 1831, T. 5, B.2, 729). So does A. G. Kästner, "in whose hands", says Kant, "everything becomes precise, understandable and pleasant" (AA II, NG, 170)[20]. According to the famous professor: "The exponent or name (exponens sive nomen) of a relation is the number that indicates how many times the antecedent member is contained in the consequent" (KÄSTNER, 1786 [1758], V, 134)[21].

arithmetical comparison between two numbers. Therefore, when this word is put without one of its epithets, one must understand, every time, geometrical relation" (KÄSTNER, 1786 [[1]1758], V, 129).

[20] In the context of the compliment Kant indicates the way, the clearest and most determined, with which Kästner works with the concept of negative quantities, referring to the *Anfangsgründe der Arithemtik*, cited above, on pp. 59-62 of Kant's edition.

[21] For Euler, differently from Kästner and Klügel, the exponent is the number that indicates the grad of a power, like its current meaning: a^{100}, pronounced *a* elevated [*elivirt oder erhaben*] to a hundred, expresses the hundredth power of *a*. The number written on top, in our case 100, is usually named exponent" (EULER, 1911 [1770], §172, 64). Klügel, in his Dictionary, points out a cleavage in the term in question: "Exponent is, firstly, the number that indicates the grad in a power, and can be a whole, fractioned, rational or irrational, positive or negative number. Secondly, it is the number by means of which one must multiply the consequent member of a relation as to arise the antecedent member [as the order in here indifferent, AP] (KLÜGEL, 1805, T.2, B.1, 170-171). We should note that the difference in both meanings is more one of context than a conceptual one. In the one case, a relation (a : b); in the other a series (a, b, c, d…), such that if the initial relation were a : a.a, the series of powers could be a, a^2, a^3, in which case the exponent is *a*. As a grad of a power, expressed as $a_n = R^{n-1}$, the exponent (n-1) indicates the position of a term in the series, the reason why Klügel considers more

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Our interest, therefore, is the geometrical relation. It, as we see, has three elements: antecedent, consequent and exponent or name, so that it is possible to express the relation between *a* and *b* as *a* : *ae*, where *e* is the exponent. Note, moreover, that it is also possible to establish a relation of equality between two relations (therefore, four members are required) whose internal members are unequal (which is true for both arithmetical and geometrical). This occurs to the extent, therefore, that there is an equality of inequalities, that is, that the relations have, if geometrical, the same exponent. In this case, the mutual reference between the relations constitutes a proportion.

> Geometrical relations are *equal* to each other when their exponents are equal to each other, so thus its evident that in both relations the second member emerges in a single way from the first, and two equal geometrical relations form a *geometrical proportion*. (proportio, ἀναλογία). (KLÜGEL, 1831, T.5, B.2, 732)[22].

Here we have a fundamental property of geometric proportion, synonymous with analogy[23]. In two equal relations, A: B and C: D, not only is the division of the first and second member equal to that of the third and fourth, but also the product of the external terms (first and fourth) must be equal to that of the middle terms (second and third) - multiplying both fractions of the ratio A / B = C / D by B, we have A = BC / D; multiply once more by D, then we have AD = BC. From this criterion or characteristic property, the relation of analogy A: B :: C: D can be modulated (A: C = B: D; D: B = C: A; D: C = B: A), and the proportion remains the same as long as the product of middle terms remains the same as that of external terms (AD = BC). Thus, if three terms are given (A: B :: C: ...), we find the proportional fourth = BC / A, by Regula de tri or Regel detri (EULER, 1911 [1770], §§471, 478, pp.174 and 176; also KLÜGEL, 1831, T.5, B.2, 747). The continued application of the three-term rule allows the subsequent acquisition of *n* proportional relations, by the successive search for the proportional fourth in each case, all under the same exponent (if the rule used is composed, it is called, according to the number of given relations, e.g.: regula quinque, septem etc.).

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adequate to name (n-1) as position-index or index-number [*Stellenzeiger* oder *Stellenzahl*] (KLÜGEL, 1805, T.2., B.1, 171).

[22] The same applies for both Euler (1911 [1770], §461. 171) and Kästner (1786 [[1]1758], V, 132 and 134).

[23] "Analogie (analogia) is synonymous with proportion. It is the greek word ἀναλογία, by which Euclid expresses the equality of two relations, see proportion" (KLÜGEL, 1803, T.1, B.1, 77)

However, it is interesting to note that Kästner, on the other hand, also uses the Regeldetri in another context[24], that of progression, in which there is properly a series, given the non-indifference of the *order* of proportions. This is evidenced above all by placing the use of this operation not for the search of the proportional fourth, for which three known terms are required, but based on two given terms, for which the third is sought, in a serial relation. According to the author: "a geometric series is given when either its first and second members are given or, instead, its exponent is given; for all the following members are found through the three-term rule" (KÄSTNER, 1786 [1758], VI, 148). In this case, in order to configure a series based on the relations of the same exponent, they must have equal middle terms, such as a: b :: b: c :: c: d. Otherwise, it could not be said that in the series, as a progression of proportions, the terms that follow or precede have the same exponent. Thus, see the series:

$$a^4/b^3 :: a^3/b^2 :: a^2/b :: a :: b :: b^2/a :: b^3/a^2 :: b^4/a^3 \ldots b^n/a^{n-1}$$

The first thing to note, for the sake of clarity, is that the term "a" can be read, in its complete "expression" as a^1/b^0 and, the term "b" as b^1/a^0. Secondly, less important, is that in the form A:B :: B:C, the progression would be written $a^4/b^3 : a^3/b^2 :: a^3/b^2 : a^2/b$ etc., which makes it unnecessary to repeat, regarding the expression, the middle term. In any case, it is understood that the exponent between any two proportions of this progression is always the relation b/a, since the consequent term, divided by the antecedent (t_{n+1}/t_n) is always equal to b/a.

Now, when it comes to the acquisition of the chain or series, in the quote above Kästner had indicated that there are two ways in which this is possible: either at least two members are given, or the exponent is given. In the first case, from a specific and localized relation, I can, by means of a comparison procedure, find the exponent, the universal relation between the terms (the *terminus generalis*); in the second, in possession of the latter, I can specify it, and determine the singular relations between the members, provided that at least two members are given. However, there is an important difference between the two procedures, in which, it is true, I manage to construct the chain ad infinitum, but in a different way. By comparing two given relations, I find the exponent b/a and, in possession of this exponent, by the three-term rule, form the series. When comparing the terms 1/8 :: 1/4, I find

[24] As to the fist context, that of *proportion*, see KÄSTNER, 1786 [1758], V, 137.

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ratio 2, and I get 1/8 :: 1/4 :: 1/2 :: 1 :: 2 :: 4 :: 8 etc. I therefore discover how to find a term *c*, since $c_n = (c_{n-1})2$. On the other hand, when the exponent is given, it is not necessary that the constructed series is equal to the series in the first case. Here it is much more about building a family of series or possible series[25]: both 1 :: 2 :: 4 :: 8 and 3/2 :: 3 :: 6 :: 12, in such a way that it is possible to obtain the law of progressions in general (not just those of exponent = 2, as in the example), namely, that $c_n = e^{n-1}$, where *e* is the exponent, since a term of position *n* is the exponent multiplied by itself n-1 times, whatever the exponent. To put in another way, in the first case I have the exponent for the construction of particular relations; in the second, the universalized exponent, for which the construction of serial relations between proportions is, firstly, possible.

IV

We have seen that the use by Kant of the exponent is aimed, on the one hand, to explain the strict logical relation between condition and assertion in a judgment in general (the exponent of the rule), and, on the other, to explain relation by means of which we think a series of causes-effects (the exponent of the series). Now, in the *Lose Blätter aus dem Duisburg'schen Nachlass* (henceforth DN), we find a conception of the exponent that takes place a sensibility-oriented register, that is to say, not solely in a logical-formal sense, but *also* in the logical-transcendental sense. As we shall see, it will be possible to gain some terrain as to clarify both the meaning of condition and assertion, and to deepen our understanding concerning the sense in which the philosophical use of the exponent differs from the mathematical.

Thus, in these reflections of the mid-1770s, we find the concept of rule worked out in relation to the exponent. In DN Kant names what is required for the emergence of a rule. According to the author:

> For the emergence of a rule three parts are required: 1. *x* as the datum for a rule (object of sensibility or, better, real sensible representation). 2. *a*, the *aptitudo* for a rule or the condition by means of which it [the real sensible representation, AP] is, in general, referred to a rule. 3. *b*, the exponent of the rule (AA XVII, 4676, 656 $_{8\text{-}12}$ [1773-1775]).

[25] I should thank my colleague, Paulo Borges de Santana Junior, for bringing to my attention the case with the family of possible series.

The insertion of a datum for a rule in the form of *something* (*Etwas*) *x*, is an additional element in relation to the strictly formal conception of "assertion under a universal condition", as we saw above. Understanding *x* as the datum for the rule, as an indeterminate *sensible something* that is virtually determinable when represented as subordinate to the rule, implies conceiving or anticipating the object of a possible judgment, whose consummation is set in terms of a determined reference of the relation between condition and assertion. Here, in the DN, however, the pressing issue is more that of acquiring the rule, than that of the convenience of the rule to the case. Thus, there is a demand for the satisfaction of the condition, designated by the symbol *a*, by which *x* (at first refractory to conceptual universality) may befit the rule (at first, refractory to the individuality of the sensible data). Properly, *a* is what offers determinability to *x*, that is to say, it is that by virtue of which *x*, as something intuited, can be converted into something thought; thus, *a* is at the same time a condition *sine qua non* of the thinkability of *x* and of the concrete reference of the rule: *a* designates *x* as a *thought* object. In this condition, *a* is defined as a concept (among others: "-[…]- the phenomenon *x*, of which *a* is a concept" [AA XVII, 4680, 665 $_{5\text{-}6}$, 1773-1775]; "*x* always means the object of the concept *a* [AA XVII, 4674, 644 $_{27\text{-}28}$, 1773-1775]).

It would now be tempting to assign to *b*, the exponent of the rule, the role of concept-predicate. In some *Reflexionen* of the κ phase (1769), the symbols *x*, *a* and *b* were mobilized to explain the insertion of an object, while something indeterminate, in the judicative structure[26]. From the usual model of the categorical form (or predicative, in this case) a judgment whose referent is potentially something = x takes place "-[…]- when something *x* that I know through the representation *a* is compared with another concept (^{g} b), in order to include or exclude it "(AA XVII, 3920, 344 $_{24\text{-}26}$ [1769]). Aside from the fact that the judicative form is limited to affirmation or denial, it is necessary to note that the relevant change at the time of DN resides in that *b* is no longer a mere predicate and, as in the R4676 mentioned above, responds to a function of the relation between concepts in a judgment. Although in the same DN we find some passages that seem to take *b* only as the concept-predicate in a categorical judgment[27], here the concept of rule, by means of the exponent (*b* as a relation between concepts, and not just as a concept-predicate), must apply (or be

[26] See SCHULTHESS, P., 1981, pp.78-86.
[27] E.g..: "The proposition of identity contains the comparison of two predicates, *a* and *b*, with *x* –[…]-" (AA XVII, R4676, 653 $_{12\text{-}13}$, [1773-1775]).

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universalized) to all relations between concepts in a judgment, therefore, also to hypothetical and disjunctive forms[28].

Now, in the DN Kant recognizes, according to the relations between concepts in a judgment, three exponents: "there are, in this, three exponents: 1. of the relation to the subject, 2. of the relation of consequence between them, 3. of gathering-together [*Zusammennehmung*] "(AA XVII, 4674, 647 $_{17\text{-}19}$ [1773-1775]); and according to the three exponents, three possible relations of *a* and *b* in a judgment whose referent is something sensible = x: "In judgments, however, there is a relation of *a* : *b*, which both refer to *x*. *a* and *b* in *x*, *x* by means of *a* : *b*, finally *a* + *b* = *x*." (AA XVII, 4676, 657 $_{8\text{-}10}$ [1773-1775]). Although the categorical exponent, "of the relation to the subject", when expressed as "a and b in x" may suggest the predication of the subject-concept and the predicate-concept to some sensible object, the attribution of the relation between condition and assertion thought of in the exponent cannot be identified with a judicative "formula" (such as subject, verb and predicate), whose expression is a proposition; rather, the aforementioned relations should be understood acts of thinking, as it is highlighted in the sequence of R4674:

> There are, in this, three exponents: -[…]-. The determination of *a* in these *momentis* of apperception is the subsumption under one of these *actibus* of thinking; one knows the concept *a* ([g] as in itself determinable and, therefore, objectively) when one subsumes it under one of these universal acts of thinking, through which it ends up under a rule. (AA XVII, 4674, 647 $_{16\text{-}24}$ [1773-1775])

We see here that *b*, the exponent, is a determination of *a*. This determination is understood in terms of subsuming the condition of the rule under one of the three universal actions or acts of the mind. This determination, in turn, allows the formation of the rule or, to put it another way: in what appears to be an "ascendant" movement, will permit the intellectualization of *x* as the exponent, *b*, determines the way in which *a* is to be thought in regard with *x*; that is to say, *b* determines *a* in *x*. In this way, is not just the case that the exponent is determinant in relation to the concept *a* by means of which we think *x* (as thought of *as* independent of its singularity[29]), but also, in some way, *x* determines *a* (insofar as *x* specifies the concept, or concepts, under which it is thought). Thus, in this context, to say

[28] For a slightly different interpretation see SCHULTHESS, 1981, 252-253.

[29] As Kant says in R4674: "-[…]- such proposition [of subsumption of *a* under one of the universal acts of thinking, AP] is a principle of the rule, therefore, of the knowledge of the appearance by the understanding, [rule, AP] by means of which it [the appearance, AP] is considered as something objective that is thought in itself independently of the singularity of what was given" (XVII, 4674, 647 $_{24\text{-}27}$, 1773-1775).

that the determination of *a* resides in its subsumption under one of the three universal acts of thinking, is to say that the exponent provides us with the way in which the representation of *x* as something thought (*a*) must, for its turn, be thought or conceptualized. By itself, unregarded its expression in a concrete judgment, the exponent, strictly speaking, is not itself a determination *of* something, as a predicate of a *rule* in a real relation would be; but it is the *manner in which* this determination ought to take place by means of the virtually acquired rule[30]. In these terms, we can say that the exponent *b* can be characterized as the *function of the rule*. According to R4680: "In the appearance *x*, whereof *a* is a concept, there must be contained, in addition to what is thought by *a*, conditions of its specification, which make a rule necessary[31], whose function is expressed by *b*" (AA XVII, 4680, 665 $_{5\text{-}8}$, 1773-1775).

In light of what was said, it is possible to better understand the meaning of the relation *a* : *b*, in which both symbols refer to *x*. It should first be noted that there are three cases wherein this can occur: mathematical construction, exposition, and observation. As Kant says, respectively: "In the first case the relation *a* : *b* follows from the construction of *a* = *x*. In the second, it is drawn from the sensible condition of the intellection of *a*; in the third, from the observation. The first two synthesis are *a priori* (all three objective)" (AA XVII, 4676, 655 $_{6\text{-}10}$, 1773-1775). The difference of these modes of reference to *x* is grounded in the kind of sensible condition that is taken up to form a rule. As far as we are concerned only with the first two modes[32], it should be said that in the construction, or synthesis through composition, what matters is the condition by means of which the object is *given*, that is to say, the conditions of *sensible intuition*. E.g.: in the pure form of space, I can, solely by the possession of the concept "triangle", compose a figure whose concept necessarily contains the procedure for its construction, that is, three angles whose sum equals 180 degrees; or, to further exemplify, in the pure form of time, I can, by means of the concept of a geometrical[33]

[30] I must say that given the character of text we now have to deal with, it is by no means undisputed if some characterizations of key concepts that occur in these *loose sheets* should be interpreted in a univocal, restricted manner; it seems to be the case that conflicting conceptions stay side by side here. E.g., the addition of the same period to the R4674 "x is therefore the determinable (object) that I think through the concept *a*, and *b* is its determination ([g] or the manner to determine it)" (AAXVII, 4674, 645 $_{28\text{-}29}$, 1773-1775). It seems to me that the determination itself and the way it must occur, therefore, something as its prescriptive character, should not be identified.

[31] That is not to say that the rule is required or that we necessitate it (in the sense of nötig); but that it applies unrestrictedly (notwendig).

[32] In any case, see R4678 (661 $_{8\text{-}12}$, 1773-1775) for the exemplification of "observation"

[33] Although "geometrical" brings to mind spatiality, we must remember that, as Euler, Kästner and also Klügel have noted, this nomenclature is entirely arbitrary, and has no bearing on the subject proper.

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series in which the procedure of construction, $c_n = e^{n-1}$ is contained, compose all possible series in which the progressions consists in multiplying the *ratio* for itself *n* times. The strict universality of this kind of concept resides in the fact that the concept and, at the same time, the procedure for the construction of the *x* it implies, are valid for any and all triangles (be isosceles, scalenus etc.) or for any and all geometrical progressions (be the *ratio* 2, 1/3 etc.). To say then that in this case $a : b$ follows from the construction of $a = x$ means that the sensible condition for *a*, and by consequence, the condition for a rule, is given by the arbitrary construction of *x*, in which case, so to say, there is no gap between object and concept..

Now, although in the case of exposition the model of necessity seems to be closely inspired by mathematical construction, these two ways of determining *a* in *x* can, by no means, be identified. In R4684 Kant says that "Therefore, we represent us the object by means of an analogon of construction" (AA XVII, 4684, 670 $_{20\text{-}21}$, 1773-1775)[34]. Here, he has in mind the concept of triangle, which gives us a rule of composition that is valid for any and all triangles; the same kind universal validity of the rule should somehow apply to those cases in which, e.g., something that occurs always follows from something that precedes. This representation, as a universal act for determination of appearances, must yield a rule *as if* the connection therein represented was constructed in the inner sense. The problem here is that this kind of connection cannot, on the one hand, be constructed, nor is given by (which is absent in mathematics) sensation alone. The reason thereof is that the kind of object that in this case must be thought under *a* is always linked to *existence*[35], which, on the one hand, is only possible *a posteriori* and, on the other, does not present itself and by itself to us as *connected* with other existences (therefore necessarily), albeit conjoined (just generally). The main question for Kant is then what, if anything, can we know *a priori* and necessarily about this existences or, to rephrase it in terms of the DN: how, from this contingent existences, can emerge an universal and necessary rule or law[36]?

[34] In the same way, the principles of exposition are "-[…]- analoga of axioms, which take place *a priori*, but only as anticipations of all laws of nature in general" (XVII, 4675, 658 $_{1\text{-}2}$, 1773-1775).

[35] Kant refers to it here in the DN as the *real object*. As for the vocabulary of the Critique, it should be understood as the *effective* object, in so far as the *real* of sensation is there connect with quality and is subjective, although virtually objective. In the *System of Principles*, the dynamical categories (relation and modality) deal with effectivity, *Wirklichkeit*, and not just *Realität*.

[36] To be sure, Kant also asks about de conditions of acquisition of determinate rules or empirical laws of nature. However, the most pressing question, as in the Critique, concerns the principles of pure understanding. See,

Whilst construction required nothing more than pure forms of intuition, exposition, in dealing with existence, requires *also* perception, understood here as the way in which existence is posited in the inner sense, e.g.: "perceptions are not solely appearances, i.e., representations of appearances, but of their existence. –[…]– The perception is the position in the inner sense in general" (XVII, 4677, 659 $_{14\text{-}18}$, 1773-1775). The requirement of perception can be understood *grosso modo* by the fact that *our* mind cannot generate objects in regard to their existence (as it would be the case with intellectual intuition), although it *can think* them in a determining manner. In our present context it should suffice to say that "existence" indicates, in opposition to arbitrarily generated mathematical objects in pure intuition, that something effective (and therefore in itself radically independent from our intellectual capacities) is *given* to us via reality in sensation (as matter of perception), that is, is posited to our receptivity. In this sense, *a* must represent the condition whereupon our perceptions have the unity required to form a rule, that is, so that the rule may refer to perceptions in general. In a sense, *a* ought to be both sensible, as to be able to represent the datum *in concreto*, and intellectual, for it must also represent something universal in *x*. As Kant puts it in R4684:

> If a concept is also sensible, but universal, then it must be considered in its *concreto*, for example, triangle in its construction. If the concept does not signify pure, but empirical intuition, i.e., experience, then the *x* contains the condition of relative position (*a*) in space and time; i.e., the condition of determining universally something therein.
>
> Moreover, appearances are determined through time, but in the *synthesi* the time [is determined, AP] through an appearance, e.g., of that which exists or occurs or coexists. These [relations, AP] are the most universal in appearance, whereof reality is the matter. (XVII, 4684, 671 $_{10\text{-}19}$, 1773-1775).

In the case of Mathematics, it could be said that the conditions for conceiving *a in concreto* would be met by the concept of triangle since it both represents triangle *in general* and the very procedure for generating any particular triangle[37]. With empirical intuition, however, we have already seen that it is not possible for human intellect to absolutely posit effectivity itself, that is, to generate the perceptual data. What we can relatively posit is the

for example: "Hence determinate rules of synthesis can be given to us only through experience, but their universal norm [can be given] *a priori*" (XVII, 4679, 663 $_{19\text{-}20}$, 1773-1775). See also the edition of 1781, where Kant says the following: "Now, the representation of a universal condition according to which a certain manifold (…) can be posited is called a rule and, when it must be so posited, it is called a law" (KrV, A104).

[37] By now it should be clear that *a* in the DN (although surely not in a univocal sense) represents what in the *Critique* will take place in the Schematism. Unfortunately, we will not be able, in this paper, to follow this very interesting route.

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data, not made but given elsewhere, in temporal relation. That is also to say that appearances are given to us, due to the forms of our sensibility, in space and time (appearances determined through time), but the act of positing in inner sense *temporalizes* the given, i.e., institutes *order* in time (time determined through appearances). Therefore, the notions, according to the above-mentioned excerpt, of appearances of that which exists in time, that which occurs in time and that which coexists in time will provide us with the sensible, but universal concepts; or, more precisely, concepts *under* sensible conditions [38] (ad 1781 called Schemata), that account for the ways in which time is to be determined. Just as *a* represents the universal in perception, or that which is universal in positing in time, *b* can now be further specified as "-[…]- the exponent of the relations of perceptions, hence of determining their place according to a rule" (XVII, 4676, 655 $_{19\text{-}21}$, 1773-1775).

As we have seen, *a* was the "*aptitudo* for a rule or the condition by means of which it [the real sensible representation] is, in general, referred to a rule"; this reference to the rule ought to arise through the exponent that gives us the manner in which *a* is to be determined. Let us take the concept *a* in the case in which the position in time takes place as something that occurs, that is, in the case in which existence is posited in the mode of succession. Through the exponent of relation of consequence, the particular succession is apt to be thought as a member of a series of possible perceptions, which are chained by means of we relating them to each other in terms of ground and consequence. So, when something *d* occurs, we determine its place in time by positing for it something as its ground (*c*), that is, something from which it came to be. In the series of possible perceptions, the same relation must apply to, say, something *f* that occurs. We would say that just as I must posit something *c* as to determine *d* in its time-relation, I must posit something *e*, so the same may occur with *f*, that is, c : d :: e : f. This series, therefore, would be ordered by means of the exponent of ground and consequence, as the manner to determine the positing in time, *a*.

If, say, the procedure of mathematical construction applied here, not only the effective given would be unnecessary but we would also be able to acquire the complete series, past, present and future, of the relation of consequence for the totality of what occurs, since we would be able to construct all the terms in the chain and arrive at a uncondicioned

[38] "In a synthetical judgment it can never be in relation 2 pure concepts of reason with each other, but one pure concept of understanding with a concept under sensible conditions" (XVII, 4584, 671 $_{24\text{-}26}$, 1773-1775).

(*Unbedignt*), to use the terminology of the *Antinomies*. Thus, it would be possible, e.g., from a given effect, to arrive at the *first cause* in time, by means of a chain of regressive conditions. To mention the same example of the last section, be it the progression:

$$a^4/b^3 :: a^3/b^2 :: a^2/b :: a :: b :: b^2/a :: b^3/a^2 :: b^4/a^3 ...b^n/a^{n-1}$$

Here, any t_{n+1}/t_n means that t_{n+1} is the effect of the cause t_n. By means of the exponent b/a each *causal relation* could be linked by its middle term, so that each consequent *relation* could be taken as an effect of a preceding one, and each antecedent *relation*, as a cause of a consequent one: $a^4/b^3 : a^3/b^2 :: a^3/b^2 : a^2/b$ etc., so that the occurrence a^3/b^2 both causes a^2/b, and is the effect of a^4/b^3 (remembering that, as mentioned above, the identity of the middle term makes it unnecessary to explicitly refer to it every time).

But, strictly speaking, mathematical construction is not justified in *a priori* knowledge of experience *as such*. Here, by an equality of proportions, an analogy, we cannot provide *a priori* the *terms* related themselves, as if our minds posited existences. To refer to the introductory section of the *Analogies*, in Philosophy: “analogy is not the equality of two *quantitative* relations, but *qualitative*, in which from three given members I can know and give *a priori* the *relation* to a fourth, but not *this* fourth *member* itself” (KrV, A179-180/B222). According to its mathematical provenience in geometrical proportions, also in Philosophy the exponent has a role to play in determining a series. Here, however, given the impossibility of using our representations in a constitutive manner, the possession of the exponent does *not* allow us, *a priori*, to provide any *determine* causal relation and, *a posteriori*, to provide the terms of the series for that which there is no corresponding datum. What we nevertheless know is that *if* existence as succession is posited to our receptivity, then it *must* be brought under a rule by means of a given exponent (in this case, the relation of consequence), as we can only know *a priori* the relation of possible perceptions, never the perceptions themselves.

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As we have just seen, there is something like an anticipatory character linked to the operation the exponent represents. However, this sort of *a priori* anticipation of an aspect of the form of experience does not, for Kant, *determines*, but as he puts it:

> -[…]- it only says that something is, according to a rule still to be found, determinable according to a certain given exponent. It serves, therefore, to try this determination and to expose the appearance, being the *principium* of judging [*Beurtheilung*] the appearance, e.g., what occurs has in some precedent its ground. (XVII, 4677, 659 $_{22\text{-}24}$, 660 $_{1\text{-}3,}$ 1773-1775).

Albeit the exponent being the determination of *a*, e.g., the existence as succession (*a*) thought as relation of consequence (*b*), it is not a determining judgment as the universal rule presented, e.g., in *Second Analogy*, which states that "all changes occur in accordance to the law of the connection of cause and effect" (KrV, B232), or a particular rule, say, that all changes in matter have an exterior cause[39]. Instead, the exponent provides determinability to the given, by means of which the determination can be "tried", that is, by means of which the given may be recognize as a case of a (potential) rule. The procedure, so to say, as to make the datum determinable, so it can be determinately thought under concepts, is to expose its appearance. Accordingly, Kant names this procedure exposition, which is an alternative to construction[40].

By exposition of appearances it is understood "-[…]- the determination of the ground whereupon the interconnection of sensations reside", as the "ground of all relation and of concatenation [*Verkettung*] of representations" (XVII, 4674, 643 $_{20\text{-}22,\ 10\text{-}11}$, 1773-1775). By its turn, concatenation is conceived as "-[…]- the representation of the inner action of the mind to connect representations" (XVII, 4674, 643 $_{14\text{-}15}$, 1773-1775); connection that can occur, as we have seen, in terms of a relation to a subject, of a relation of consequence or of a relation of gathering-together. Therefore, although the exponent is not itself a determining judgment, it is the ground by means of which the determination is made possible. These inner actions account for the very condition of unity of our conscious representations, that is, that in connected or related representations by means of this inner action "there is unity not in virtue of: in which; but: through which the manifold is brought in one" (XVII, 4674,

[39] See AA IV, MAdN, 543

[40] "We must expose [exponieren, AP] concepts when we cannot construct them" (XVII, 4678, 660 26-27, 1773-1775).

643 17-18, 1773-1775). This "through which" is understood a little later in the same R4674 as "-[…]- the subjective representation (of the subject) itself, but made universal, for I am the original of all objects. Therefore, it is conjugation as function, which makes the exponent of a rule" (XVII, 4674, 646 12-14, 1773-1775).

Cum grano salis, this bears an interesting resemblance to what Kant calls, in the §15 of B-Deduction, *qualitative unity.* Here, he is discussing the concept of bond, *Verbindung*, as representation of the synthetic unity of the manifold:

> The representation of this unity cannot emerge from the bond; rather, as it is added to the representation of the manifold, it makes possible, in the first place, the concept of bond. This unity, which precedes *a priori* all concepts of bond, is not that category of unity (§10); for all categories are based upon logical functions in judgments, but in these bond is already thought and, thus, unity of given concepts. (…) Therefore, we must search even higher for this unity (as qualitative, §12), namely in that which contains in itself the ground of the unity of different concepts in judgments and, thus, of the possibility of the understanding, even in its logical use. (KrV, B130-1).

This qualitative unity is that of the logical functions, as for this excerpt, a pre-categorial condition of unity. Here I would like to suggest that though this unity is the ground of conceptual unity in judgments it does not have to be, itself, a conceptual unity; like the "I think", as the continuation of the text (§16) might suggest. Anyhow, the "I think" is not original, say, in the sense that it precedes that inner action of the mind as its condition. Rather: it is only by means of a *recognition* of a common subjective ground in all binding activities that it is possible to think something as a numerical identity therein, something *thought of* as that which is the "transcendental actor", as the condition of possibility, of these acts. However, given my scope here, I do not intend to deal with the difficulties involving the conditions for self-consciousness or self-ascribed representations, nor with a sound characterization of the synthetic unity of consciousness. For me, it suffices to frame this unity in terms of activity[41].

[41] Here it is posed, so to say, the limit of my argument. These last parts, namely, what concerns §15 of the B-Deduction and §12 are not yet for me clearly connect to the DN. This possible relation was, as I recollect, indicated by Prof. Placencia in last years "Primeras Jornadas", for which I am very grateful; as also was pointed out, as a hypothesis, what seems to me now the logical continuation of the analysis, to step in the *third Critique*, as the exponent appears there as means in which we bring representations of imagination to concepts, as opposed to demonstration, in which in expose our concepts in sensibility, or at least a common *Merkmal* in them (see AA V, KdU, §57, Anm. I, 341-344).

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Looking a little further into it, as for the reference to §12, Kant notes the following concerning unity:

> For in all knowledge of a object there is *unity* of the concept, which one may call *qualitative unity*, insofar as by it only the unity the comprehension [*Zusammenfassung*] of the manifold of knowledge is thought, as, say, the unity of the theme in a drama, in a speech, in a fable. (KrV, B114)

The qualitative unity as mentioned here seems a much, so to say, looser one than that strictly represented by concepts. It is analogous of that of a drama, a fable etc. It seems to be the unity of conceiving in one, but not necessarily in a conceptual, reflected manner. As will be said later in the B-Deduction, the *Zusammenfassung* of the manifold according to the forms of sensibility yields formal intuition (KrV, B161n). Although we read in the continuation that this representation is the determination of sensibility by the understanding, the unity of this intuition pertains to space and time and not to concepts of the understanding. The seems very much alike the way in which, as seen in the DN, in the existence as succession (which by itself has some form of unity, though not that of a case of a given universal rule) we would arrive at an order in succession by means of the relation of consequence; but not at the judgment that states that everything that occurs presupposes a cause. In any case, the exponent operates as that by means of which a given particular must be posited in time as to be brought (or thought of as contained) under the universal even if, as the above passage indicates, this universal is not currently given. As mere inner action of mind to connect representations, it should not (though made universal, as the subjective form of our concepts) be hastily equated with a determining function in judgement. It seems to be much more the case to conceive it as mere spontaneity of thinking, as we read, e.g. in a passage from the Paralogisms: "Thinking, taken by itself, is merely the logical function, hence the sheer spontaneity of the bond of a manifold of a merely possible intuition -[…]-" (KrV, A428).

To be sure, if my argument so far is, albeit not in every bit, leastwise sound, it indicates how Kant could have made fruitful for Philosophy a concept encountered in Math textbooks[42]. Although it does not seem to occupy a prominent place in the *Critique* when taken literally, the analysis of the DN shows its importance for the work in progression, as what there was still in a faltering manner called Analogy of phenomena, Analogy of nature,

[42] Note that it would not have been the first time, as can be seen in the concept negative quantities; and, as at least Schulthess points out in the course of his thesis, the same would have happened with "function".

Analogy of understanding, Analogy of experience[43] would latter be embodied in the seemingly core section of the *Analytic of Principles*. This embodiment is also indicated by the way Kant characterizes retrospectively in the *Phaenomena and Noumena*, the superior rules of pure understanding expounded in the *System of Principles*, by attributing to them the same procedure encountered in DN, that of exposition: "Its principles [of understanding] are merely principles of exposition of appearances-[…]-" (KrV, A247/B303); and: "-[…]- what we have sustained so far: that our pure cognitions of the understanding in general are nothing more than principles of the exposition of appearances" (KrV, A250).

In this way, if principles of pure understanding provide us with "the concept that contains the condition and, so to say, the exponent of a rule in general -[…]-" (KrV, A159/B198), we do not deal with a closed system, which does not permit growth. It is much more the case that by means of including the exponents, whereupon we set the fundament of all relation in appearances, in the intension of phenomena we, therefore, acquire a ground by means of which we may encounter recognizable features (as a *Merkmal* by means of which we recognize the minds on activity in appearances[44]) that allow the particulars in nature to be brought under the extension of rules, given or still to be found, in a determine manner. In the same way, by the "-[…]- unity of nature in the interconnection of all phenomena under certain exponents -[…]-" (KrV, A216/B263) we could understand the positing of a common ground through which appearances are yet not determined, but determinable in an unending process of thinkable completion.

In exposition we refer the ground of all relation and concatenation of our representations to the sheer spontaneity of thinking. But, as such, the universal acts of the mind or inner actions are by no means restricted to working with sensible data. In the open field of mere thinking, the exponent shows more clearly its mathematical provenience. Paying here the price of not being able to denote given objects for what is thought or, as in Mathematics, to compose the infinite chain of cases drawn from the procedure implied in the given exponent, reason nonetheless possesses the same operating cog, albeit, if salutary, only a regulatory one. This operative concept, as the exponent of a rule, accounts for the

[43] Respectively, in the *Reflexionen* 4675, 648 e 4682, 669; R4675, 652; R4681, 667 e R4684, 672; R4602, 606 (1773-1775)

[44] "I have indeed a rule to search for it [the fourth member] in experience, and a characteristic sign [*Merkmal*] to find it therein (KrV, A179-180/B222).

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very act under which Kant thinks our chaining of judgments, in the *ratiocinatio polysyllogistica* in general, or in thinking the unconditioned in the series of conditions.

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