# "On the History and The Mathematics of the Wold-Juréen (1953) Utility Function, And Its Basis for the Modeling of Giffen Behavior" revisited

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**Abstract.** Robert Sproule (2022a) proves that the Wold-Juréen utility function is characterised by Giffenity and under a particular condition, the Giffen good is an inferior good. We demonstrate that only a change of sign in the utility function is sufficient for the commodity being a Giffen good as well an inferior good without any restriction imposed upon the parameters of the problem.
**Keywords:** Giffen goods; inferior goods.

## [es] "On the History and the Mathematics of the Wold-Juréen (1953) Utility Function, And Its Basis for the Modeling of Giffen Behavior" revisado

**Resumen.** Robert Sproule (2022A) demuestra que la función de utilidad de Wold-Juréen se caracteriza por la Giffenity y bajo condiciones particulares el bien Giffen es un bien inferior. Demostramos que solo un cambio de signo en la función de utilidad es suficiente para que la mercancía sea un bien Giffen y un bien inferior sin ninguna restricción impuesta sobre los parámetros del problema.
**Palabras clave:** Bienes Giffen; bienes inferiores.

## [pt] "On the History and The Mathematics of the Wold-Juréen (1953) Utility Function, And Its Basis for the Modeling of Giffen Behavior" revisitado

**Resumo.** Robert Sproule (2022a) prova que a função de utilidade Wold-Juréen é caracterizada por Giffenity e sob uma condição particular, o bem de Giffen é um bem inferior. Demostramos que apenas uma mudança de sinal na função de utilidade é suficiente para que a mercadoria seja um bem de Giffen e também um bem inferior sem qualquer restrição imposta aos parâmetros do problema.
**Palavras-chave:** Bens de Giffen; bens inferiores.

**Sumario:** 1. Introduction. 2. Giffenity and Inferiority in the Wold- Juréen (1953) Utility Function. 3. Discussion. 4. Conclusion. References.

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## 1. Introduction

The durable challenge posed by a long list of illustrious scholars is to reconcile Giffen behaviour (an increase in the demand for a commodity with an increase in its price) with the axioms of consumer theory. The demand curve for the Giffen good slopes upwards. Sproule operates with the Wold-Juréen utility function $U = (x_1 - 1)(x_2 - 2)^{-2}$ where $x_1$ is the Giffen/inferior good, $x_2$ is the normal good, $x_1 > 1, 0 < x_2 < 2$. We observe that the utility function is not concave and, to that extent, the search for consistency with the axioms of consumer theory does not emerge naturally. At the same time, the connection between the convex function and the desired concavity of the utility function is trivial. Only a change of sign differentiates them. We proceed to rewrite the problem accordingly.

## 2. Giffenity and Inferiority in the Wold- Juréen (1953) Utility Function

The well-known problem is to maximise the negative of the utility function given above subject to the budget constraint $p_1 x_1 + p_2 x_2 = m$, where $p_1$, $p_2$, and $m$ are the prices of commodities 1 and 2 and income respectively. Setting up the problem in the standard manner, we derive the following Marshallian demand functions for $x_1$ and $x_2$ respectively.

$$x_1^* = \frac{1}{p_1}\left[2p_2 - m\right] + 2 \qquad (1)$$

$$x_2^* = \frac{1}{p_2}\left[2m - 2p_1\right] - 2 \qquad (2)$$

The paper by Sproul contains an involved discussion on the sign of the expression in square brackets in equation (1). Working with the indirect utility function, $v(p_1, p_2; m)$, we are able to establish the following result.

**Lemma**. $m \geq 2p_2$

Proof. On occasion we will use the vector notation, **p** for $(p_1, p_2)$ and **x** for $(x_1, x_2)$. We need only a single property of the indirect utility function. The indirect utility function is decreasing in prices, $v(2\mathbf{p}; m) \leq v(\mathbf{p}; m)$. Therefore, $(2p_1, 2p_2; m) \leq (p_1, p_2; m)$.
We have the following inequality.

$$(2p_1, 2p_2; m).\mathbf{x} \leq (p_1, p_2; m).\mathbf{x} \leq m \qquad (3)$$

What remains is to take the particular value of the column **x** = (0, 1; 0).

$$(2p_1, 2p_2; m). (0, 1; 0) \leq (p_1, p_2; m). (0, 1; 0) \leq m \quad (4)$$

We find that commodity 1 is a Giffen good as well as an inferior good (an increase in income lowers the demand for the commodity).

$$\frac{\partial x_1^*}{\partial p_1} \geq 0 \ \text{ and } \frac{\partial x_1^*}{\partial m} \leq 0 \qquad (5)$$

Commodity 2, on the other hand, is a normal good. In sum, we have the following

**Proposition**. With the Wold-Juréen utility function, $U = - (x_1 - 1)(x_2 - 2)^{-2}$, good 1 is a Giffen good and an inferior good. Good 2 is a normal good.

We do not require heavy technology like the distinction between global quasi-concavity and local concavity that permeates the exegesis by Sproule. Earlier efforts were also laboured, with numerical analysis (Sproule, 2022B), and the incorporation of the second-order conditions of utility maximisation (Sproule, 2022C). We claim that none of the axioms of consumer choice theory are challenged.

## 3. Discussion

The origins of the Giffen paradox are worth recalling. As a result of the devastating potato famine in Ireland in the 19th century, the demand for potatoes followed the price of potatoes upward. Robert Giffen generalised the observation to the conjecture that under similar conditions, the demand for and prices of necessities would move in the same direction. Clearly, context matters and in the state of a recession, for instance, when all attention is focused on procuring basic goods, an increase in their demand under conditions of constrained supply will cause prices to rise which will not reduce the demand for food and other necessaries. Recent evidence for Giffen behaviour is the market for meat and other essentials in China under the Covid-19 pandemic (Zhou, 2022). The pandemic also meant lockdowns, entailing layoffs and reduced incomes, the $m$ in the microeconomics above. The task for governments is clear; increase incomes by creating jobs directly in hard and soft infrastructure, repair and create supply lines. Normal goods remain so through recessions and upturns, but lock in and hysteresis might characterise the market in wage goods for the working class.

## 4. Conclusion

Sproule's intuitions about the Wold-Juréen utility function are borne out. All the same, a lot of the labour expended by him (and coauthors) is uncalled for. We show that his basic insights can be captured easily using duality theory in effectively a one-line proof of an inequality. The history of the subject is fascinating and we can expect extensions and elaborations. Work on Giffen inputs has begun to be undertaken and a consequent downward-sloping supply curve could complement an upward-sloping demand curve in basic goods. The remark on local and global supply chains for food and critical inputs made in the previous section is apposite here. The divide between Marshall's industrial economics and his public policy will need to be bridged.

# References

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