Modelling of the surface flow in the Alboran Sea: Review of sensitivity studies of reduced gravity flows

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RESUMEN

Este trabajo es una revisión de unos estudios de modelización de la capa superficial en el Mar de Alborán. El objetivo de aquel proyecto cooperativo consistió en simular la circulación superficial observada y así profundizar en la comprensión de los procesos físicos que incurren en ella. Se utilizó primero un modelo de diferencias finitas en una cuenca rectangular para hacer un análisis de sensibilidad que permitiera estudiar la influencia relativa de los diferentes parámetros que representan las condiciones físicas del flujo. Basándonos en estos resultados se pasó a un modelo de gravedad reducida de elementos finitos para la simulación. El método de solución escogido fue la formulación de la Ecuación de Onda de Lynch y Gray (1979). Se halló que, independientemente del valor de la viscosidad turbulenta horizontal, la condición de no-deslizamiento (no-slip) genera soluciones que se corresponden con las características del flujo observado; no así con la condición de libre-deslizamiento (free-slip), de acuerdo con el estudio previo con diferencias finitas. Dado el papel fundamental que juega el tipo de condición de frontera en las paredes del modelo, se generalizó la condición de no-deslizamiento a uno de deslizamiento parcial o tracción, para eliminar así la necesidad de resolver las capas límite muy finas. De estos estudios se derivan cuestiones fundamentales sobre la formulación de modelos de flujos oceánicos y que conciernen a: (i) la correcta comprensión del efecto que tienen los esfuerzos laterales generados en los contornos de la costa sobre las características de los flujos en el seno del océano a escalas mayores; y (ii) la parametrización correcta de los valores frontera del esfuerzo para futuros estudios de modelos oceánicos.

El proyecto original en el que está basada esta revisión fue el resultado de un proyecto de colaboración entre Dartmouth College, el Instituto Español

de Oceanografía y AINCO-Interocean. El desarrollo de los modelos corrió a cargo de los Profesores Daniel Lynch y Francisco Werner; el modelo de diferencias finitas se puso a punto en el IEO y ambos modelos se pusieron a punto en AINCO-Interocean en ordenadores personales. Para su utilización y desarrollo, los modelos están disponibles en estos centros y se han publicado además sendos manuales técnicos (Escobar *et al.* 1988, Sánchez *et al.* 1989). Este trabajo es una revisión de los artículos publicados en Werner *et al.* 1988 y Lynch *et al.* 1989.

ABSTRACT

The article is a review of past modelling studies of the circulation of the upper layer of the Alboran Sea. The objetive of those studies was to simulate the observed circulation in the Alboran Sea and to understand the physical processes involved. A finite differences model was first used to carry out a sensitivity analysis that would allow the study of the relative influence of the parameters that represent the physical processes of the flow. Based on these results we moved on to the formulation of a reduced-gravity finite element model. The solution method of choice was the Wave Equation formulation of Lynch and Gray (1979). It was found that, independent of the value of the horizontal eddy viscosity, no-slip conditions yield solutions which agree with observed features of the flow, but free-slip conditions do not, in agreement with the finite differences formulation. Given the fundamental role played by the form of the boundary conditions on the model sidewalls, the no-slip condition was generalized to a condition of partial-slip or traction, thus removing the necessity of resolving very thin boundary layers. Those studies have raised fundamental questions in the modelling of oceanic flows concerning: (i) our understaing of the effect of lateral stresses generated at coastline boundaries on the larger-scale internal flow features; and (ii) the proper parameterization of boundary stresses for future modeling studies.

The original work, on which this review is based, was the result of a cooperative project between Dartmouth College, the Instituto Español de Oceanografía (IEO) and AINCO-Interocean. The development of the models was carried out by Prof. Daniel Lynch and Prof. Francisco E. Werner; the finite differences model was implemented at the IEO and both models were implemented at AINCO-Interocean on PC computers. They are available at those institutions for their use and further development. In addition, two technical manuals have been published to assists in this task (Escobar *et al.* 1988, Sanchez *et al.* 1989). This review article has been summarized from the work published in Werner *et al.* 1988 and Lynch *et al.* 1989.

1. INTRODUCTION

a) *The Alboran Sea Gyre*

The circulation in the western basin of the Alboran has been studied intensively in the past decade and is characterized, in its surface layers, by an almost permanent anti-cyclonic gyre east of the Strait of Gibraltar and west of the Alboran Island (fig. 1). The gyre has a circular shape with the strongest gradients and flows in the northern section; has a horizontal scale of 100 km and a vertical scale of 200 meters. Studies of the spatial and temporal variability of the gyre can be found in field measurements described by, among others, Lanoix (1974), Wannamaker (1979), Cheney and Doblar (1982) and Parrilla and Kinder (1985). Numerical modelling efforts studying the gyre have been carried out by Preller and Hurlburt (1982), Loth and Crepon (1984), Preller (1986) and Werner and Lynch (1986). Analytic studies are discussed by Nof (1978); and laboratory (hydraulic) simulations were implemented by Whintehead and Miller (1979) and Whitehead (1985).

Although all these theoretical and laboratory studies have reproduced the Alboran gyre in some form, some have been more successful than others at achieving the observed stability and dimensions. In particular, while the efforts of Preller and Hurlburt (on reduced-gravity and two-layer primitive equation models) are robust and reproduce the gross features of the surface layer circulation in the Alboran, the studies of Loth and Crepon (1984) and Werner and Lynch (1986) find the occurrence of the gyre elusive. The discussion presented herein provides some explanations of why the discrepancies arise even though the physical processes contained in the models listed above are similar at first glance. In particular, we focus on characteristics of the solution of the present model formulation when no-slip has been imposed on the sidewalls.

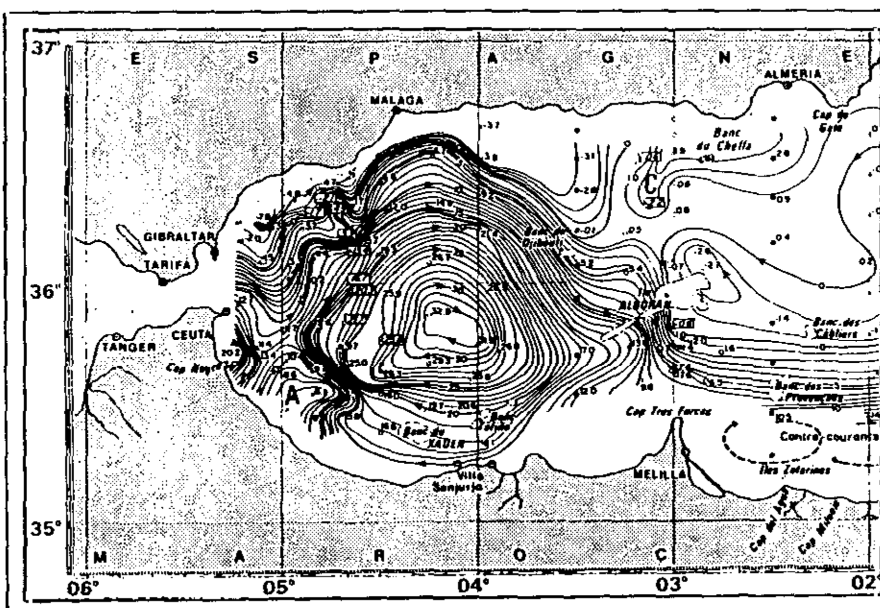

Figure 1.—Dynamic topography for the sea surface relative to 200 db for July-August, 1962. Taken from Lanoix (1974).

Certainly, since the no-slip condition depends on the inclusion of a non-zero horizontal eddy viscosity, the role of the horizontal eddy viscosity extends beyond that of sub-gridscale dissipation.

b) *Chronology of the Modelling Experiments:*

These studies of the Alboran Sea circulation began with the Wave Equation (finite element) formulation of Lynch and Gray (1979); hereafther LG79. This formulation seeks the solution of the inviscid, nonlinear, time-dependent, shallow water equations. Our first results were puzzling: although we reproduced the observed anticyclonic gyre, which characterizes the surface flow in the Alboran, our simulation included gyre-reversals on time scales of 1-2 weeks. No evidence supporting such behavior is available from field measurements. At the same time, in those cases where a steady, stable solution was computed, the model flow-field was uneventful, i.e., the model-inflow from the Atlantic would exit through the eastern open boundary without any evidence of a gyre. These findings are discussed in Werner and Lynch (1986).

We next undertook a comparison of our results with the solutions of Preller (1986). At this point we adapted a finite difference model (kindly supplied by Dr. Christian Le Provost of the Institut de Mechanique de Grenoble, France, and Dr. Marianela Fornerino of la Universidad de Zulia, Venezuela) to the Alboran Sea. The physical processes included in the finite element and finite difference models were the same; the only difference was the inclusion in the finite difference model of horizontal eddy viscosity terms and the associated option to exercise stress conditions at lateral boundaries via no-slip. Both our initial experiments were aimed at establishing the internal role of the horizontal viscous terms in the overall model solution. We found results strongly resembling the steady inviscid finite element solution, i.e., the stable, steady solutions did not reproduce the gross features of the Alboran gyre and were also uneventful. No cases of unsteady flows with gyre-reversals were obtained, confirming that the dynamic reversals in Werner and Lynch (1986) arose simply from the lack of internal dissipation of arbitrary startup transients. At this point, the boundary conditions on the model solid sidewall boundaries were in all cases free-slip, i.e. no tangencial stress at the wall.

Having shown that the effect of horizontal viscosity per se was not enough to generate the gyre, we were left with only the form of the boundary condition on the model solid sidewalls differentiating our formulation from Preller's. To our surprise; we found that upon imposing no-slip conditions the solution changed radically, exhibiting a strong, persistent anticyclonic gyre on inflow (see Blandford, 1971, for related discussions of the effect to boundary conditions on large scale ocean modeling). All other parameters, such as geometry, f— versus bet-planes, and magnitude of the horizontal viscous terms, were found to play a secundary role (Werner *et al.*, 1988).

Having stablished the fundamental importance of the form of the boundary condition on model sidewalls, we returned to the Wave Equation (finite element) model. The main task at this point was to reformulate the governing equations to include the effect of horizontal stress terms, i.e., to allow for the presence of horizontal viscosity so that no-slip conditions could be imposed in the finite element model. Specifically, with the inviscid model we were free to impose only one boundary condition: no normal flow through the solid wall; but, by including horizontal viscous terms, we enabled the specification of a 2nd boundary condition, which will be: either the flow velocity tangential to the solid boundary, or the specification of the tangential stress on the fluid across the boundary layer, where resolution of the viscous boundary layer is undesireable.

2. SENSITIVITY STUDY OF SLIP vs. NO-SLIP (Werner *et al.* 1988)

For the purpose of studying the effects of various parameters on the circulation of the Alboran Sea, numerical experiments were carried out to study the effect of:

1) free-slip vs. no-slip conditions on the domain's sidewalls;
2) f-plane vs. beta-plane;
3) nonlinearities; and
4) horizontal eddy viscosity (A_H) and upstream relative vorticity.

In this section we focus on presenting the more significant result of the finite difference analysis; the discussion of all the different numerical experiments, contained in Werner *et al.* 1988, is not included here since they are of second order and will be considered later in the finite element model results. Unless otherwise stated, the results we show next were obtained after a 360 day simulation; only small quantitative changes are expected in longer runs. Plots of the pycnocline displacement relative to the initial 200 m depth of the interface are included. Positive values of the interface position indicate depressions of the interface relative to the initial 200 m while negative values indicate interface displacements above the initial 200 m layer thickness. The vector plots display vertically averaged velocities in m/sec.

3. FINITE DIFFERENCE MODEL FORMULATION:

The numerical model we use in this study is a slight variation of that described in Fornerino (1982) and Le Provost and Fornerino (1985). The hydrodynamic equations are the depth-integrated shallow water equations:

$$\frac{\partial U}{\partial t}+\nabla(H^{-1}\,\mathbf{UU})+f\mathbf{k}\times\mathbf{U}$$

$$=-g' H\nabla\zeta-\frac{\tau}{H^2}|\mathbf{U}|\mathbf{U}+A_H\nabla^2 U \qquad (1)$$

$$\frac{\partial \zeta}{\partial t}+\nabla\cdot\mathbf{U}=0 \qquad (2)$$

where

$$\mathbf{U} = \int_{-a}^{\zeta} \mathbf{u}\, dz,$$

$$\mathbf{u} = u\mathbf{i} + v\mathbf{j}$$

$$\mathbf{U} = U\mathbf{i} + V\mathbf{j}$$

$$H = h + \zeta.$$

Here, the following definitions hold:

i, j and **k** unit orthogonal vectors in the (x, y, z) reference system, with x eastward, y northward and z vertically upward;

t time;

H the total thickness of the upper layer at any time (the lower layer is at rest in this reduced-gravity study);

h the undisturbed upper layer thickness;

ζ the displacement of the interface from rest;

u, v the eastward and northward velocities respectively;

A_H the horizontal eddy viscosity;

f the Coriolis parameter $[=f_0 + \beta(y - y_0)]$, evaluated at a latitude of 36° N, where y_0 is the southernmost value of the y coordinate and $\beta = 2\ 10^{-11}\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$;

g' the reduced gravity $\left(= \frac{\Delta\rho}{\rho} g \geq 0.002g\right)$,

where $\Delta\rho$ represents the density difference between the upper and lower layers, ρ is a mean density and g is the gravitational acceleration;

ρ the interfacial stress coefficient and is dimensionless; and

∇ and ∇^2 the horizontal gradient $(\partial/\partial x,\ \partial/\partial y)$ and Laplacian $(\partial^2/\partial x^2,\ \partial^2/\partial y^2)$ operators respectively.

The model domain is a rectangle 600 km long (in x) and 160 km wide (in y) and reproduces the mesh used by Preller and Hurlburt (1982) and Preller (1986). Initially, the system is at rest and the surface layer thickness is 200 m [thus the Rossby radius of deformations is $(g' H/f^2)^{1/2} \approx 23$ km]. Instantaneously, at $t=0$, a transport of 1.0 Sv (Sv $\equiv 10^6\ \mathrm{m^3\,s^{-1}}$) is prescribed (Lacombe and Richez, 1982) through a 20 km wide port on the western wall and is kept constant thereafter. The spatial discretization is $\Delta x = 10$ km and $\Delta y = 5$ km and the differencing follows an Arakawa-C scheme. Explicit treatment of the temporal domain requires that the Courant-Friedrich-Levy (CFL) criterion be satisfied: $\Delta t \lesssim \Delta x/([2g'(H+\zeta)]^{1/2}+u)$ which in this case dictates a $\Delta t \Delta t \lesssim 2700$ sec. Herein all runs were made using a time step of 1800 sec. The run-time on a 32-bit precision VaxStation II (LINPACK rating of 0.17 MFLOPS; Dongarra, 1986) is approximately 16.5 hours of CPU per 360 day simulation, and the required memory is 0.82 Mbytes. The remaining details of the numerical solution can be found in Fornerino (1982) and Le Provost and Fornerino (1985).

On the eastern outflow-boundary a modified Orlanski-type radiation condition is imposed (Camerlengo and O'Brien, 1980; Chapman, 1985, see condition MOE therein). Experiments with both *free-slip* and *no-slip* conditions on the sidewalls of the system were carried out and are discussed below. In the case of *free-slip* we require the flow normal and the momentum flux through the solid boundaries to vanish, i.e. $\mathbf{u}\cdot\mathbf{n}=0$ and $\nabla\mathbf{u}\cdot\mathbf{n}=0$, where $\mathbf{n}$ is the unit vector normal to the boundary. On the other hand, when *no-slip* is imposed, we require the normal and the tangential velocities to vanish at the sidewalls, i.e. $\mathbf{u}\cdot\mathbf{n}=0$ and $\mathbf{u}\cdot\mathbf{s}=0$ where $\mathbf{s}$ is the unit vector tangential to the boundary.

Free-slip vs. no-slip:

Other parameters: fully nonlinear, beta-plane, A_H 100, m^2/sec.

Figures (2a, b) show the resulting interface position and with free-slip conditions and a constant interfacial stress term of 5×10^{-4} which was included for numerical stability. The entering flow immediately is deflected to the right (due to rotation) and hugs the southern coast as a coastally trapped jet. A basin-wide and relatively weak recirculation is set up in the northern portion of the domain. This results can be related to the channel-flow studies discussed by Gill (1976), Nof (1978), and Wang (1985). Although experiments with free-slip and $\tau=0$ became unstable in 2-3 months, the results up to the point of the simulation's termination resembled that in figures (2a, b) leading us to conclude that the inclusion of a nonzero should not detract from the basic features described above.

On the other hand, figures (3a, b) show that the requirement of no-slip significantly alters the circulation, creating an anti-cyclonic gyre upon entering the main basin and then meandering until the flow exist through the eastern open boundary. The sense of the gyre's circulation and this location roughly agree with the observed flow in the Alboran Sea; the case show in figure 3 reproduces results discussed in Preller (1986). Since the result with no-slip prescribed on the boundaries was obtained with $\tau=0$, it may be inferred that the prescribed value of the horizontal eddy viscosity is «large enough» to prevent the unbounded acceleration of the flow. Henceforth all experiments are carried out with $\tau=0$ reducing in this way the number of adjustable parameters.

The inclusion of nonlinear advection of momentum was found to be a necessary condition (but not sufficient in view of the above discussion) for the

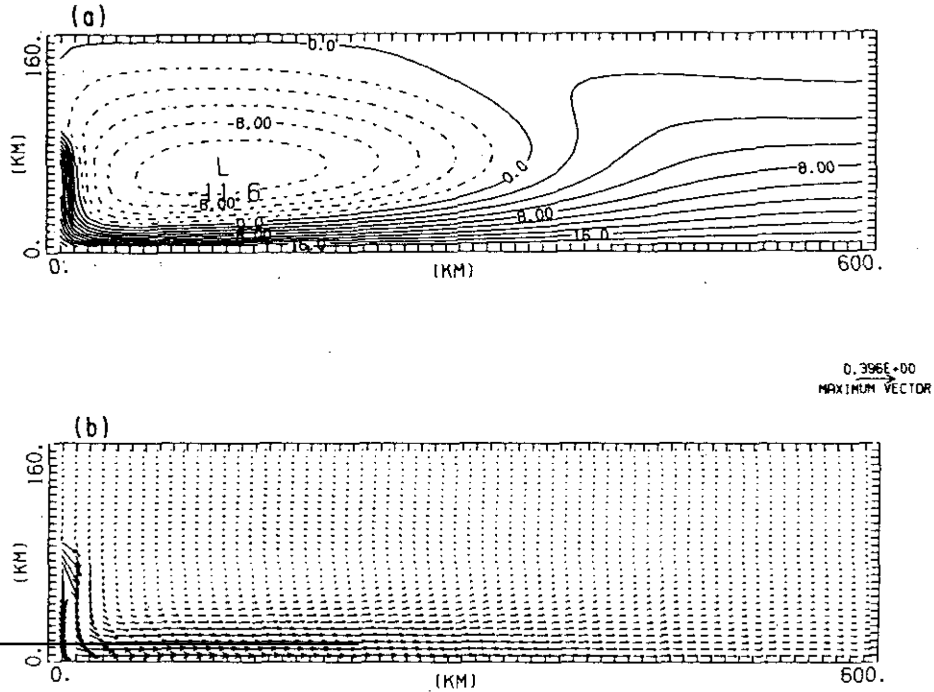

Figure 2.—Solution after 360 days with: free-slip, fully nonlinear, beta, $A_H = 100$, m/s, centered port, eastward flow, $\tau = 10$: (a) pycnocline displacement in meters relative to 200 meters and (b) vertically averaged velocities in m/s.

appearance of the gyre (see fig. 6 a, b and 7 a, b in Werner *et al.*, 1988).

Moving the inflow-port 20 km north of the center of the western wall and imposing an inflow angle of 21° from the horizontal (to the northeast based on observed geometric features of the Strait of Gibraltar produces a result which is closer to the observed flow-field (figures 4a and b). This result reproduces one by Preller (1986) and is included for completeness.

4. DISCUSSION OF THE MODEL RESULTS (Werner *et al.* 1988):

The numerical experiments described above, and others discussed in the main paper, show that the existence of the Alboran Gyre may be attributed to the non-linear response of a viscous reduced-gravity flow subject to no-slip boundary conditions. Although the importance of including non-linear effects, particulary in the vicinity of the model's entry port (where the Rossby number is at least of order 0.1) is predictable, the finding that the no-slip condition is neccessary deserves further attention. In fact, it appears that the absence of the no-slip condition may account for the elusiveness of the gyre

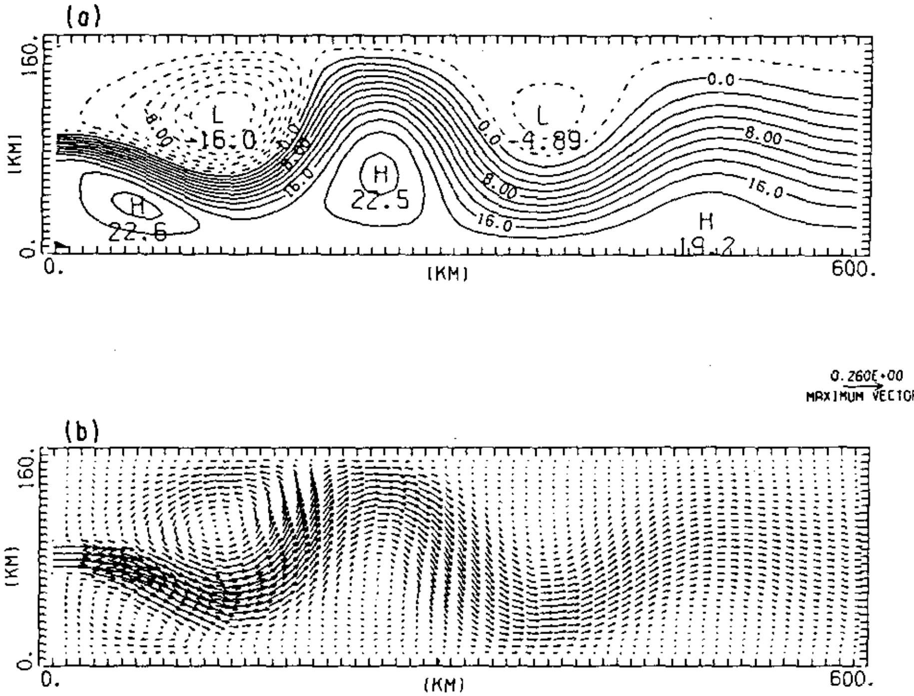

Figure 3.—As in figure 2 but with no-slip and $\tau = 0$.

described in the studies of Loth and Crepon (1984) and Werner and Lynch (1986).

a) *The Alboran Sea Gyre:*

With the previous remarks in mind, a simplified account of the Alboran gyre as contained in the present formulation follows. Consider the evolution of the flow over the first 15 days with free-slip and no-slip shown in figures 5a-d and 6a-d respectively and focus on the gyre-region (since its occurrence appears to the unrelated to the meanders downstream). Initially the flows are similar: a Kelvin wave front propagates along the southern coast (figures 5a and 6a) and the pressure gradients are balanced by Coriolis. The main difference at this stage is the slower propagation of the no-slip case (see Hsieh *et al.*, 1983). By the 6th day (figures 5b and 6b), a qualitatively different picture has emerged: a «high pressure cell» inducing clockwise rotating gyre) located south of the entry port is established in the no-slip case. Its evolution is quick, overshooting by the 9th day the strength of the gyre observed after 360 days (compare figures 6c and 3a). The high pressure cell in the no-slip

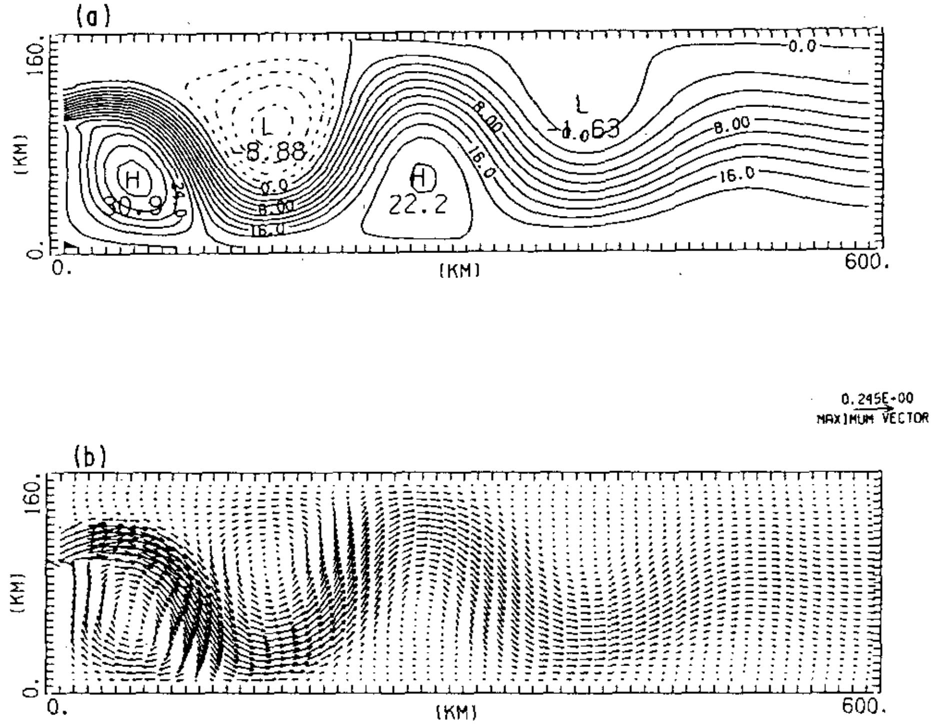

Figure 4.— As in figure 3 but uncentered port and northeastward flow.

case inhibits the inflow to turn southward (slipping along the coast) and forming a boundary current as it is permitted in the free-slip case.

In terms of vorticity, the negative vorticity (clockwise shear) acquired by the southern edge of the inflowing current at the corner of the entry port, is balanced by the positive wallinduced relative vorticity (counterclockwise shear) of the return flow closing the gyre. Similary, the counterclockwise shear acquired on the northern edge of the inflowing current is balanced by the negative wall-induced vorticity on the portion of the cyclonic gyre in contact with the northern wall. The fluid not entrained in the gyres adjusts by changes in the curvature along its trajectory: injection of negative vorticity causes a northward deflection and southward in the case of positive vorticity injection. In this manner, the current displays a meandering trajectory even on an f-plane. The shorter wavelength meanders of the current downstream of the gyre observed in the beta-plane case (not shown here) due to the background/ambient potential vorticity gradient and may be interpreted as a stationary Rossby wave in the presence of a zonal current (see Pedlosky (1979), pp. 108-111).

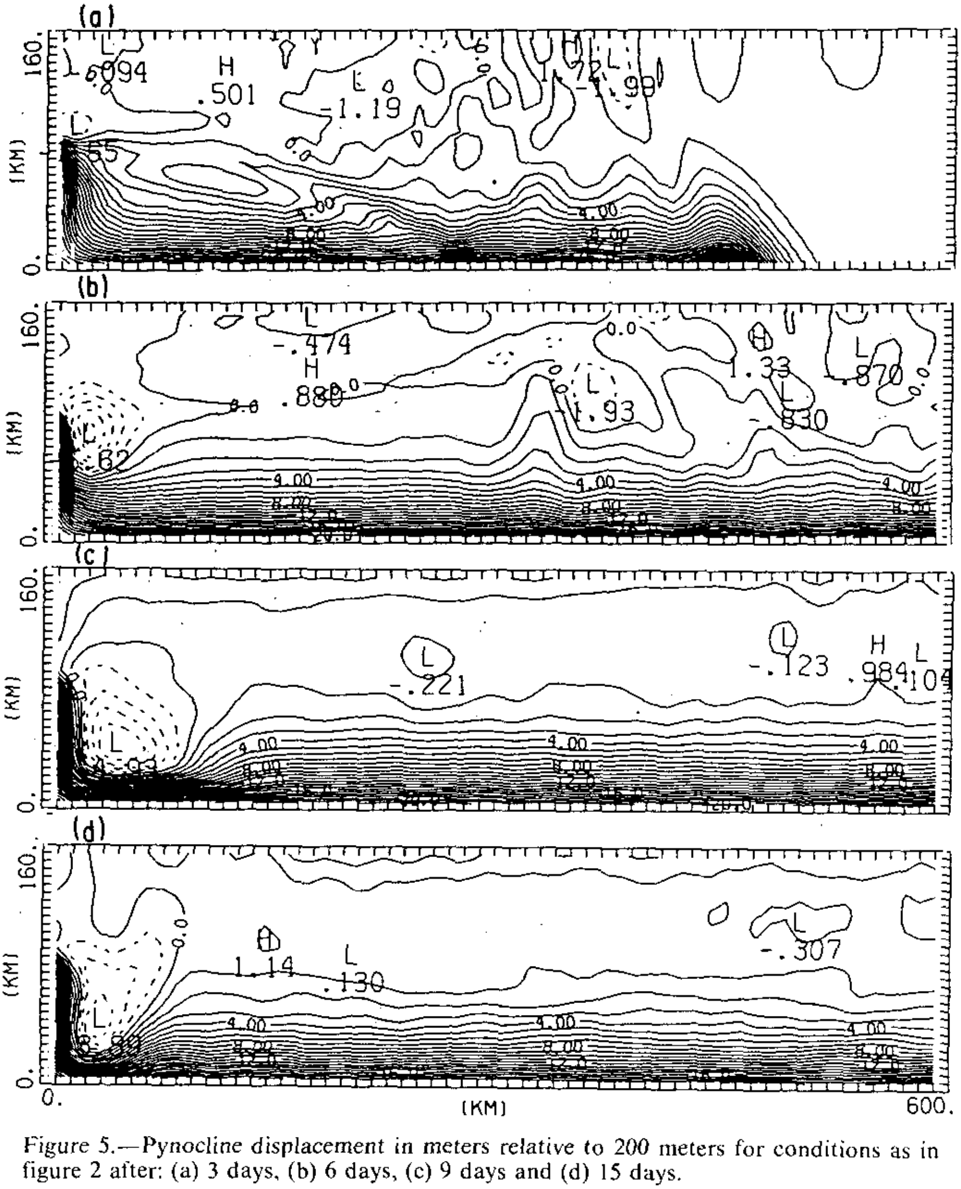

Figure 5.—Pynocline displacement in meters relative to 200 meters for conditions as in figure 2 after: (a) 3 days, (b) 6 days, (c) 9 days and (d) 15 days.

b) *Sensitivity study of no-slip vs. Partial slip (Lynch* et al., *1989)*

The results discussed next, after describing the finite element model formulation, included some form of lateral stress condition: either complete no-slip or partial-slip/traction. The case with complete free-slip becomes

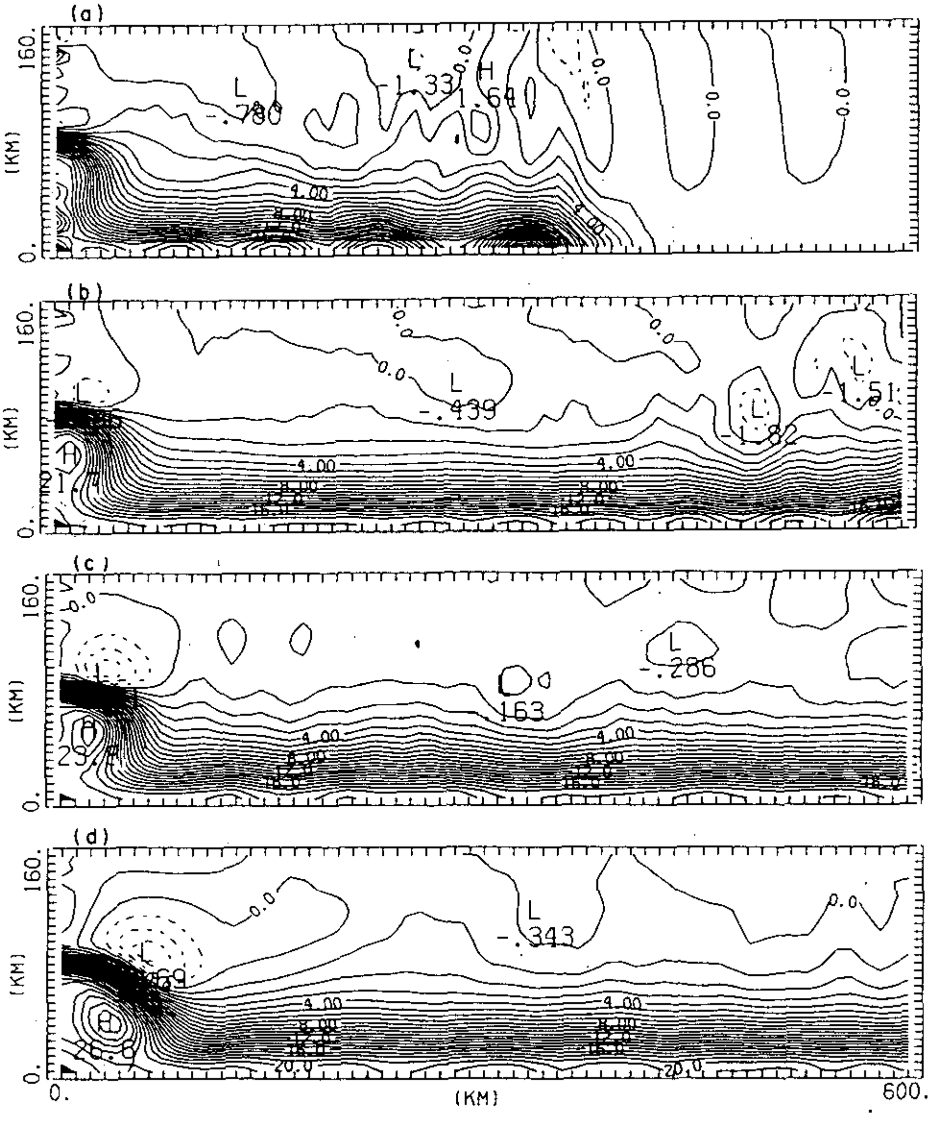

Figure 6.—As in figure 5 but for conditions as in figure 3.

easily unstable due to the absence dissipation mechanisms and id described in figure 2. Note that a small amount of bottom/interfacial stress was needed to stabilize the free-slip simulation. The purpose was to explore the model response to and role of lateral stresses when the no-slip condition is relaxed to one of traction or partial slip.

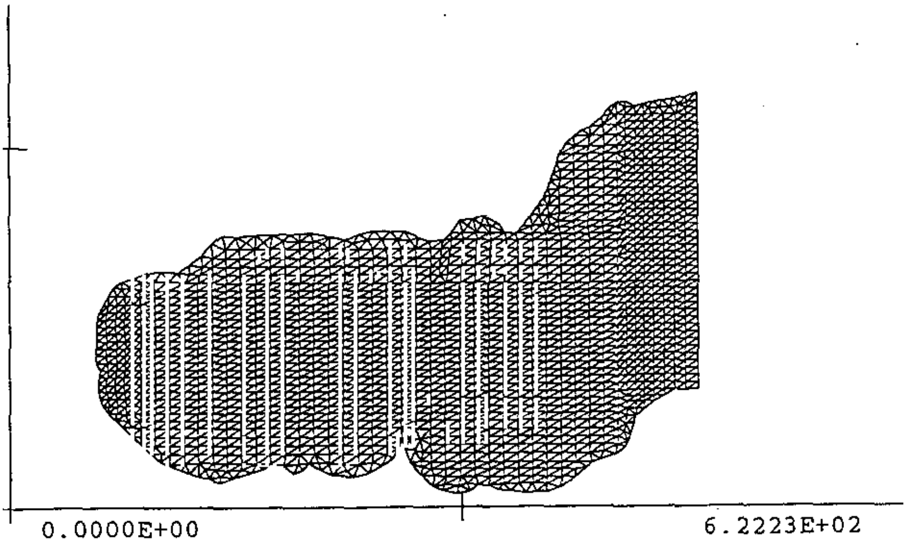

Figure 7.—Finite element mesh used in the computations. The mesh has 1672 nodes and 3080 elements.

5. FINITE ELEMENT MODEL FORMULATION:

The basis of the generalized Wave Equation formulation (LG79; and Kinmark and Gray, 1985) is:

$$\frac{\partial^2 H}{\partial t^2} + \tau_0 \frac{\partial H}{\partial t} - \nabla \cdot [\nabla \cdot (H\mathbf{v}\mathbf{v}) + gH\nabla\zeta + \mathbf{f} \times H\mathbf{v} + (\tau - \tau_0) H\mathbf{v} - H\Psi] = 0 \qquad (1)$$

The variables are: $H(x, y, t)$, the total fluid depth; $\zeta(x, y, t)$, the free surface elevation; the bathymetric variation is $h(x, y) = H - \zeta$; $\mathbf{v}(x, y, t)$, the vertically averaged velocity; g, the acceleration due to gravity; $\mathbf{f}$, the Coriolis parameter (we included a β-plane approximation); $\tau(x, y, t)$ is the bottom friction parameter (or interfacial stress in a reduced-gravity calculation and is identically zero herein), ψ represents atmospheric **and/or viscous effects;** (x, y) are the horizontal spatial coordinates; t is time and ∇ is the horizontal gradient operator. The reduced-gravity form is conventional, e.g., Werner *et al.* (1988) and is not reproduced herein. The arbitrary constant τ_0 introduced by Kinnmark and Gray (1985) renders the depth (H) matrix stationary in an implicit scheme, and stabilizes certain nonlinear instabilities. The value of τ_0 is held fixed at $2 \times 10^{-4} s^{-1}$ for all cases considered herein. Briefly, Equation (1) is solved for the fluid depth; this solution is then used in the calculation of the velocity:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + g \nabla \zeta + \mathbf{f} \times \mathbf{v} + \tau \mathbf{v} = \Psi \qquad (2)$$

and the process in repeated allowing the solution to evolve in time (LG79).

The inclusion of horizontal shear stress is achieved by adding a Galerkin form of:

$$\Psi = A_H \nabla^2 \mathbf{v} \qquad (2)$$

everywhere in the interior, and by exercising either no-slip ($\mathbf{v}=0$), or the more general traction condition:

$$A_H \frac{\partial(\mathbf{v} \cdot \hat{\mathbf{t}})}{\partial n} = -k \mathbf{v} \cdot \hat{\mathbf{t}} \qquad (4)$$

on all sidewall boundaries. Here $\hat{\mathbf{t}}$ is the unit vector tangent to the boundary; A_H is the horizontal eddy viscosity, set to 100 $m^2 s^{-1}$ in our computations; and, k is a slip coefficient (units of length/time) whereby $k=0$ corresponds to free-slip (no stress) and $k=\infty$ to no-slip (maximum stress).

The mesh employed in the calculations is shown in figure 7. In all cases shown below, we imposed 1.0 Sverdrup (1 Sverdrup $= 10^6$ $m^3 s^{-1}$) on inflow and required the same on outflow; no radiation outflow conditions were considered. Similarly, all results are shown 26 weeks after start; initially all velocities are at rest and the surface layer thickness is 200 meters. The forcing is increased linearly from zero to its final value over the first 5 days. A β—plane ($\beta = 2 \times 10^{-11}$ $m^{-1} s^{-1}$) approximation is used, and the reduced gravity term $g'[=(\Delta\rho/\rho)g]$ is fixed at $0.002g$. The time-stepping is explicit, with a time-step of 15 minutes, requiring approximately 3.2 hours of CPU on a Vax-Station III for the full 26 week simulation.

a) *Results of the No-slip vs. Partial-slip Formulation:*

In addition to successfully reproducing the Alboran gyre with the «viscous» Wave Equation formulation, it was established that the no-slip conditions on model sidewalls may be relaxed to partial-slip (or traction) conditions and still obtain realistic flow fields (compare figures 8 through 11). The inflow angle of 21° to the north of the horizontal is an attempt to include the inflow direction from the Straits of Gibraltar due to the inclination of the Straits' axis relative to the Alboran basin (see Preller, 1986 and Werner *et al.*, 1988). Figures 9 and 11 were computed with k 0.1 m/s; similar results were obtained with k = 0.05 and with k = 0.025 m/s; the result with k = 0.025 m/s

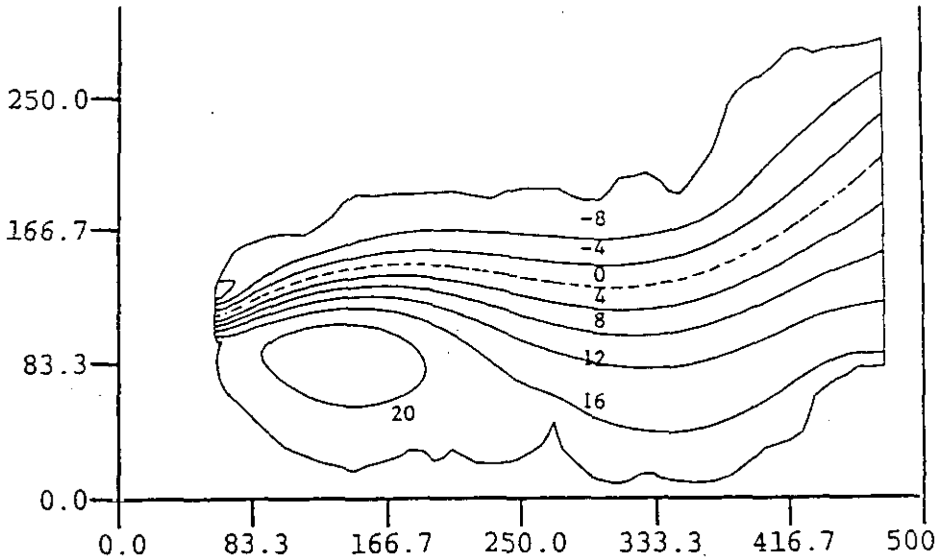

Figure 8.—Pynocline displacement (meters) relative to 200 meters; positive values indicate depressions. *No-slip boundary condition; axes in km.*

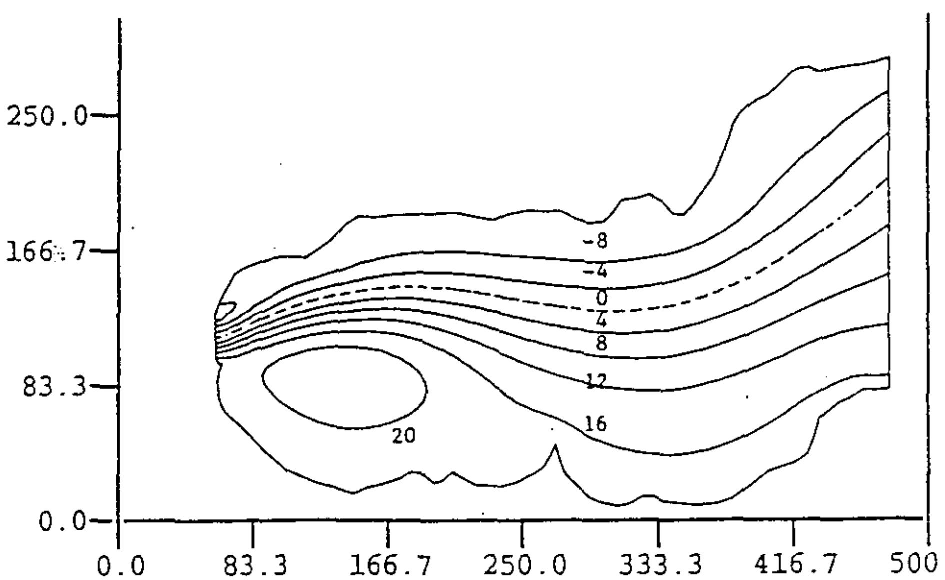

Figure 9.—Same as figure 3 but with traction boundary condition, k = 0.1 m/s.

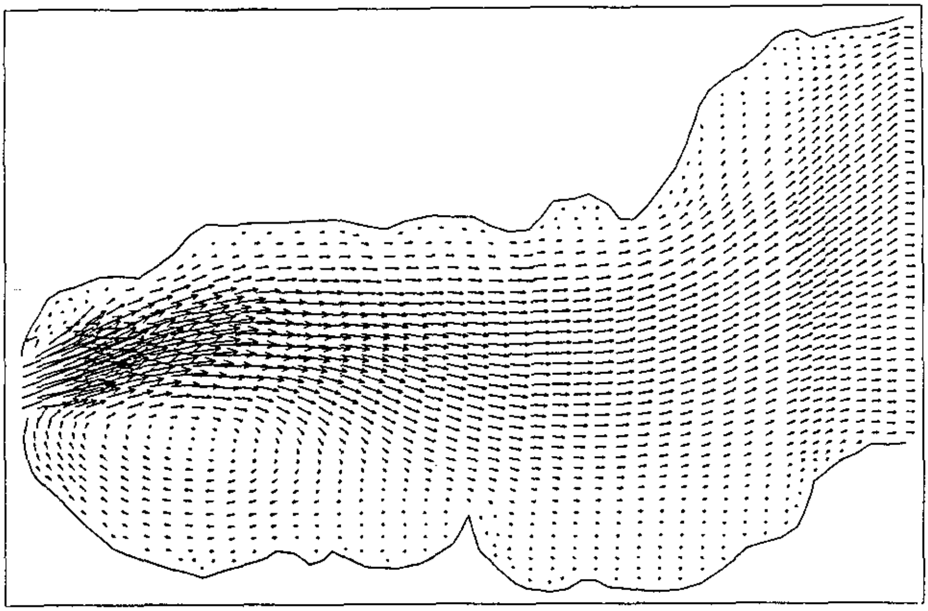

Figure 10.—Vertically averaged velocities with no-slip boundary conditions. The inflow is angled 21° north from the horizontal. Maximum velocity shown is 0.29 m/s.

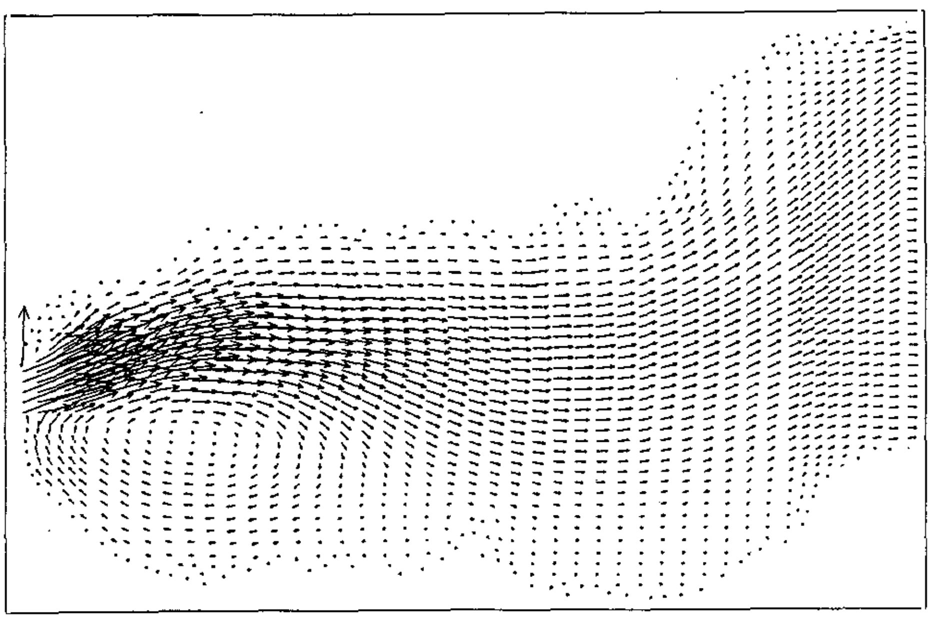

Figure 11.—Same as figure 5 but with traction boundary condition, k = 0.1 m/s. Maximum velocity shown is 0.29 m/s.

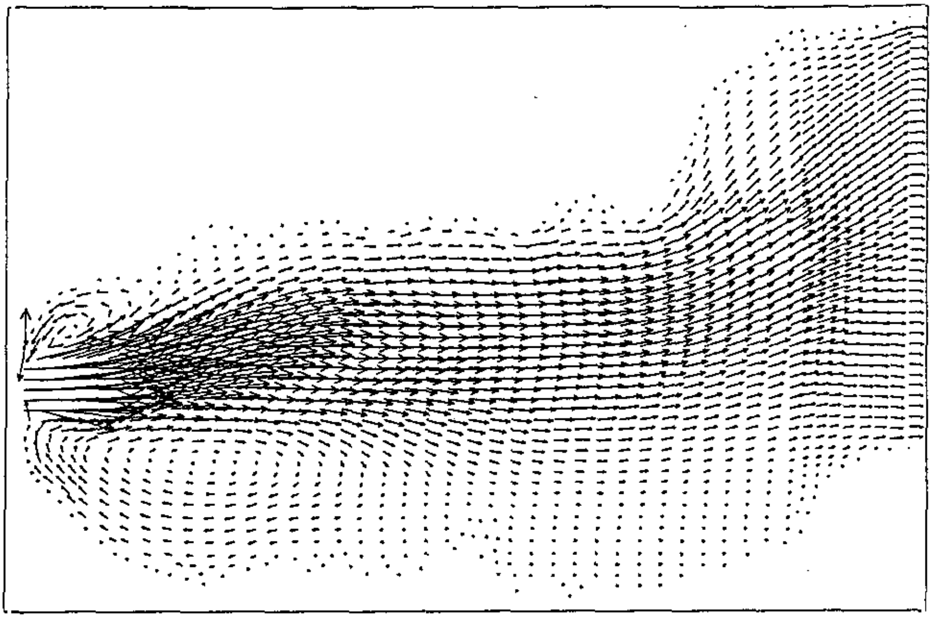

Figure 12.—Vertically averaged velocities with traction boundary condition, k = 0.1 m/s eastward inflow. Maximum velocity shown is 0.29 m/s.

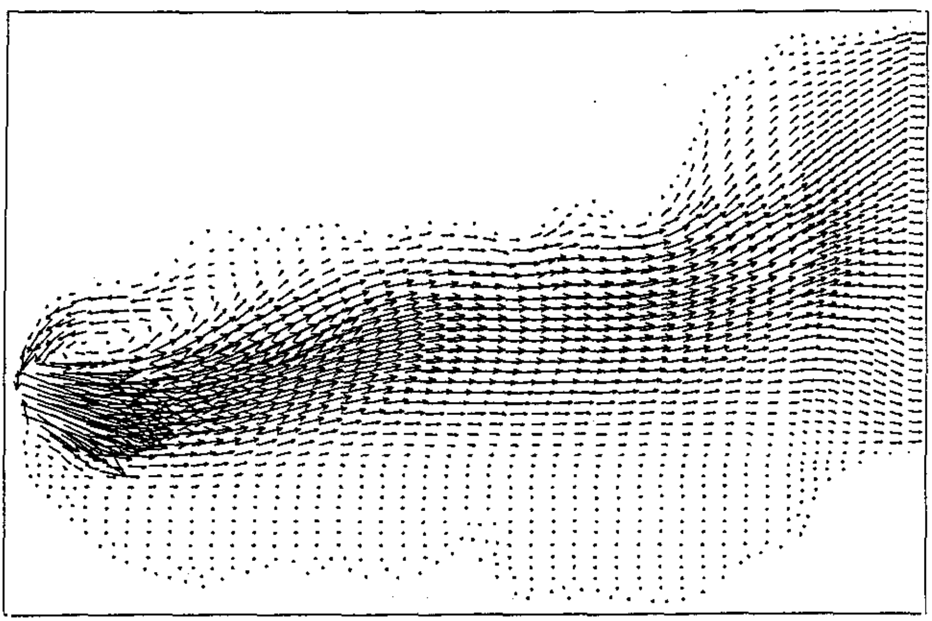

Figure 13.—Vertically averaged velocities with traction boundary condition, k = 0.1 m/s. The inflow is angled 21° south from the horizontal. Maximum velocity shown is 0.32 m/s.

is shown in figures 16 and 17. However, the computations with k = 0.015 m/s became unstable.

In other words, it was found that the key to establishing the observed flow features in the model lies not in the imposition of no-slip conditions per se, but in the concomitant generation of lateral stress at the solid boundaries. This is important in that many oceanic flows will exhibit viscous boundary layers which are very small compared to the length scales of the main flow features, and their proper resolution in order to bring the flow to rest can be prohibitive numerically.

b) *Other Results:*

Additional experiments with the formulation developed herein agreed with the findings described in Werner *et al.*, (1988). For example, the inclusion of beta is only a secondary effect to the existence of the gyre (not shown). We include below a select number of these experiments showing the sensitivity of the model's response to changes in specified boundary conditions. Only results with traction boundary conditions are shown. Note that the coastline is not plotted in the remaining figures to show the non-zero velocity vectors at the boundary.

Eastward and southeastward inflow:

Results with inflow directed eastward and southeastward are show in figures 12 and 13 respectively. The findings are not contrary to intuition, i.e., increasing the southward inflow component weakens the «main» anticyclonic Alboran gyre and strengthens the cyclonic gyre immediately north of the incoming jet. Note in figure 13 that when the inflow is directed 21° south of the horizontal, the main Alboran gyre is practically absent.

Cyclonic and anticyclonic incoming vorticity:

Results with eastward directed inflow possessing cyclonic and anticyclonic vorticity are shown in figures 14 and 15 respectively. While cyclonic inflow vorticity (fig. 14) shows an enhancement of the smaller gyre north of the inflow port relative to figure 12, the case with anticyclonic inflow vorticity (fig. 15) shows the incoming flow to turn in a more northward direction and to increase the main Alboran gyre's dimension (this latter result is in agreement with Preller, 1986).

Weak(er) traction:

A case as the one shown in figure 12 but with k = 0.025 m/s is shown in figure 16. Overall the two results do not differ qualitatively. The principal difference is in the immediate vicinity of the inflow port where larger velocity

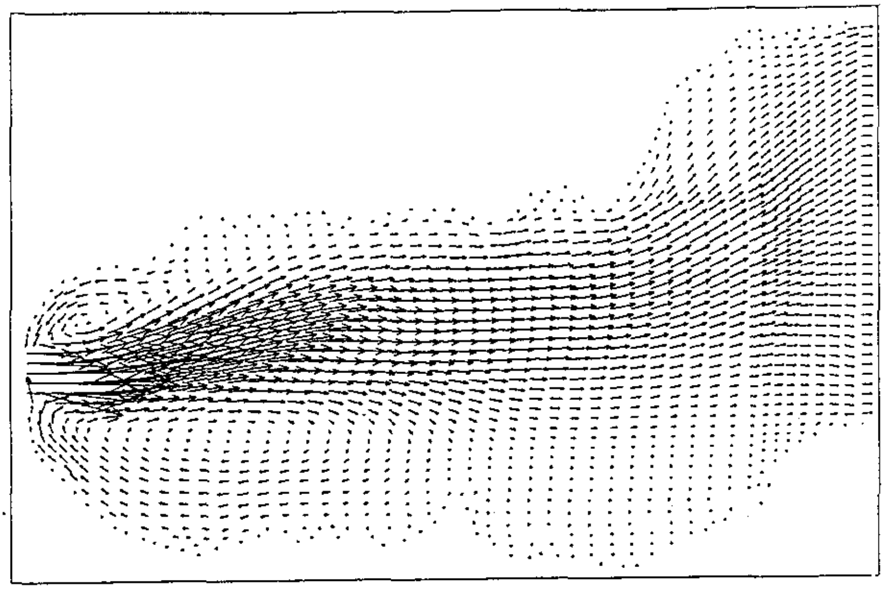

Figure 14.—Vertically averaged velocities with traction boundary condition, k = 0.1 m/s. The inflow is eastward with imposed cyclonic relative vorticity of magnitude 0.5×10^{-4}/s. Maximum velocity shown is 0.40 m/s.

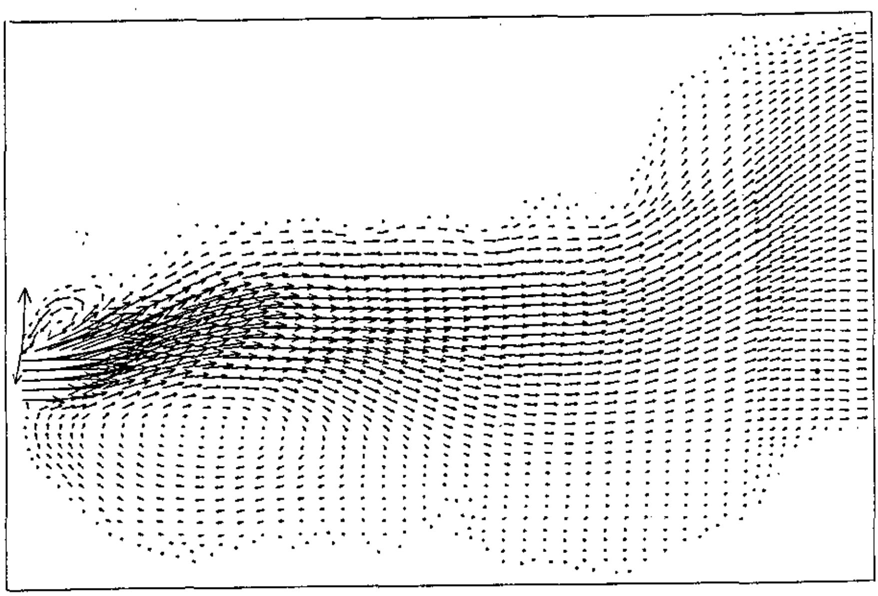

Figure 15.—Vertically averaged velocities with traction boundary condition, k = 0.1 m/s. The inflow is eastward with imposed anticyclonic relative vorticity of magnitude 0.5×10^{-4}/s. Maximum velocity shown is 0.40 m/s.

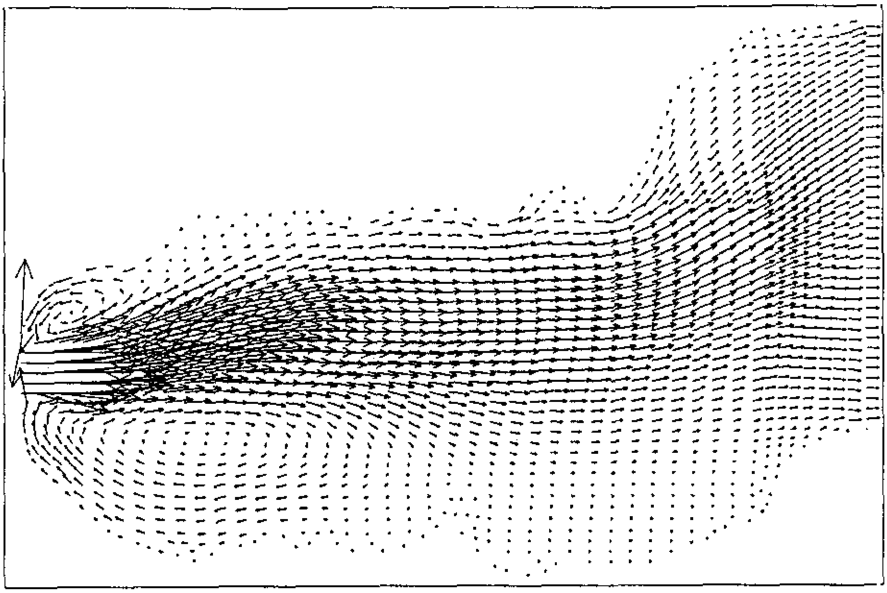

Figure 16.—Vertically averaged velocities with traction boundary condition, k = 0.025 m/s and eastward inflow. Maximum velocity shown is 0.30 m/s.

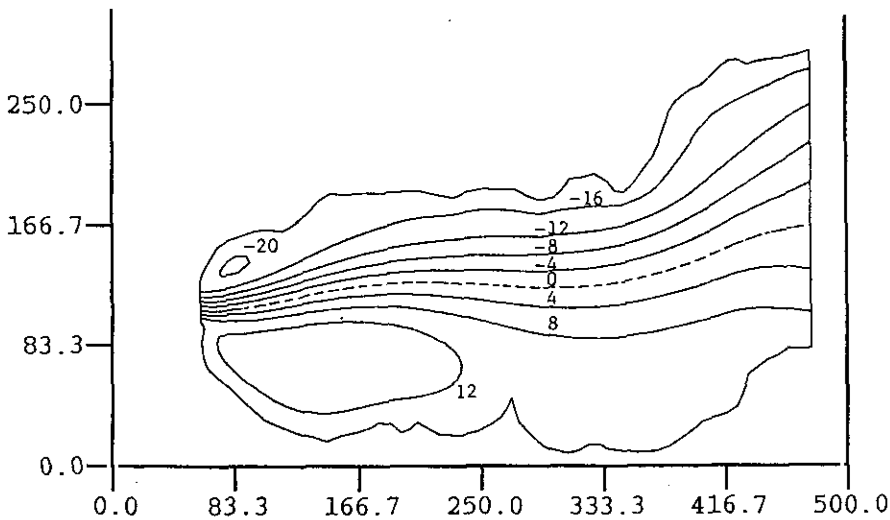

Figure 17.—Pycnocline displacement (meters) relative to 200 meters corresponding to figure 11; positive values indicate depressions. Traction boundary condition, k = 0.025 m/s; axes in km.

vectors are observed on the model's coastline in the case of k = 0.025 m/s (as one would expect). The resulting pycnocline displacement is shown in figure 17.

6. CONCLUSIONS AND DISCUSSION OF THE MODELLING EXPERIMENTS:

A review has been presented of sensitivity studies of the circulation in a channel. These studies were carried out with two reduced-gravity numerical models and an application to the circulation in the Alboran Sea was investigated. We sought to explore and evaluate the effects of different mechanisms such as: the imposition of no-slip conditions, the f vs. beta-plane, the magnitude of the horizontal viscosity, and the effect of nonlinearities. Based on these experiments, it became clear that to reproduce the observations: (i) the flow needed to be brougth to rest at the sidewalls, and (ii) the advective nonlinearities of momentum had to be included. All other processes (within reasonable excursions) have smaller quantitative effects on the circulation.

Although the results are satisfying in that they yield the desired circulation, they cannot fully address the physical mechanisms responsible for the observed flow, and as such, the present calculations must be considered as diagnostic. The dependence of the model solution on the no-slip sidewall boundary condition (and thus on the the inclusion of a non-zero horizontal eddy viscosity) calls for cautious interpretation of the results. A systematic study of the boundary conditions and their relation to physical processes influencing the solution still needs to be undertaken. However, the present reduced-gravity formulation is limited and a different approach is required.

The research has shown that in order to study the relevant dynamics of the Alboran Sea the effect of the lateral boundaries on the interior flow is essential. The lateral stress and the boundary-generated vorticity dominate the large-scale features of the solution in the interior. A reformulation of the finite element model of LG79 to include viscous effects successfully reproduced the gross features ot the field observations, and allowed the implementation of an alternative, more attractive sidewall boundary condition: the partial-slip (or traction) condition. Use of this condition eliminates errors arising from the improper resolution of viscous boundary layers implied by no-slip.

The results described in this review, reveal once again, the critical and unintuitive role that boundary conditions can play in the study of certain geophysical fluid dynamics problems. Discussions given by Blandford (1971) show that the no-slip constraint (in general circulation models) can change steady flow characterized by western, northern and eastern boundary currents to an unsteady flow with a western boundary layer and eddies generated in the northwest corner of the basin. Subsequent studies of general circulation models have, for the most part, included free-slip conditions most likely because of the difficulties encountered resolving the structure of boundary layers on no-slip walls (Haidvogel, 1979). These difficulties are compounded further if details of the vertical structure (in the pursuit of baroclinic effects) are also part of the problem. In the particular case of the Alboran Sea, the treatment of stratification effects is essential since the circulation in this basin of the Mediterranean is affected by at least 3 different water masses. More detailed descriptions of the trajectory and interaction of the Atlantic and Mediterranean waters (e.g. Tintore *et al.*, 1988) are likely to contribute to a revision of our understanding of the Alboran Sea's circulation. Nevertheless, the issues associated with boundary layers, e.g., such as the possible generation of barotropic and baroclinic instabilities, are a matter of interest. Evidence for the ocurrence of such hydrodynamic instabilities in the Algeria Current, east of the Alboran Sea, can be found in the study of Millot (1985).

Similar uncertainties may be found in the role of bottom friction. In the present study, a non-zero interfacial stress (for practical purposes a form of bottom friction) was needed to stabilize the flow in the free-slip case. At the same time, no interfacial/bottom stress was needed for an eddy viscosity of 100 m^2/s. The determination of the appropriate dissipative mechanism must also be considered in future studies.

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