Computation of surface displacements, tilt and gravity variations due to ocean tide loading

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ABSTRACT
This paper deals with the computation of surface displacements, tilt and gravity variations at the Earth's surface produced by ocean tide loading, through two different methods. The first method consists of the convolution between the ocean tide distribution and the corresponding Green's functions, following the procedure of Farrell (1972). To avoid the Gibbs effect associated to the truncation of the infinite harmonic series, various asymptotic expressions of the load Love numbers are given. Besides, successive improvements of the Farrell's method are also revised. The second method is based on the preliminary development of the ocean tides in spherical harmonics. Thereby, expressions for displacements, gravity and tilt variations in terms of the load Love numbers and the spectral amplitudes of the load are obtained.

Key words: Ocean tide loading, Green's functions

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1. INTRODUCTION

The ocean tide loading is the response of the Earth to the variation of the ocean tides, which are caused by the gravitational attraction of the Moon and Sun. It is well known that the loading effects are comprised of three main contributions. The first one is the elastic deformation of the solid Earth due to the weight of the water load. The second contribution, which appears in the gravity and tilt components, comes from the Newtonian (direct gravitational) attraction of the tidal water mass. Besides, is produced a redistribution of mass within the Earth that induced gravity changes.

The study of the deformation of the Earth associated with the loading effects caused by the oceanic tides is a classic problem in geophysics. All highly accurate observations in geodesy and geophysics are significantly affected by ocean tidal loading (Francis & Mazzega 1990). For instance, in addition to the tides of the solid Earth the gravity and tilt tide records contain the oceanic tidal effects. The typical tidal gravity signal due to ocean loading and attraction is usually between 1% and 10% of the total observed signal (Baker 1998) and the tilt loading can be larger than the body tide tilt, especially at stations very close to the coast (Pagiatakis 1990). The space geodetic measurement techniques, such as Global Positioning System (GPS) and very long baseline interferometry (VLBI), are also affected by the ocean tide loading, which causes vertical displacements of more than 10 cm in some areas (Baker et al. 1995). Therefore, geodetic and geophysics measurements should be carefully corrected for ocean tidal loading when high precision is required.

Studies of deformations of simple Earth models associated with surface loading were initiated by Slichter & Caputo (1960) and Jobert (1960). In 1960, Munk & Mac Donald introduced the concept of loading Love numbers, and Longman (1962) gave a theoretical solution to calculate these numbers. He used the equations of the free oscillations of the Earth derived by Pekeris & Jarosch (1958) and by Alterman et al. (1959), and calculated the load Love numbers to the 40th order. Typically, for computational convenience, a 'standard' Earth model has long been considered. This model assumed the Earth to be spherically symmetric, non rotating and perfectly elastic, such as the preliminary reference Earth model (PREM) (Dziewonski & Anderson 1981). In 1972, Farrell improved the Longman's theory. He obtained the load Love numbers to the 10,000th order for different Earth models, and he also calculated the loading Green's functions for the deformation of the Earth's surface. Thus, a quantitative study of the ocean tide is made possible through the convolution of the ocean tide distribution and the Green's functions. This Green's functions approach (also called convolution method) derived by Farrell have been traditionally used, and applied in various geodetic investigations (e.g., Mangiarotti et al. 2001). An alternative approach to compute the ocean tide loading effect is based on the initial spherical harmonic expansion of the load. This method reduces the convolution to a single product of the load Love numbers and the spectral amplitudes of the load for each harmonic degree (Le Meur & Hindmarsh 2000). This method has been used more frequently in recent years (e.g. Elósegui et al. 2003).

In this paper we briefly outline the main equations of the deformation of the Earth and the associated boundary conditions are introduced. The Load Love number, considering a given model for the internal structure of the Earth, are defined and subsequently the loading Green's functions are derived. Also, asymptotic expressions of various load Love numbers are given and some remarks are made concerning to different Earth models. The displacements and the gravity and tilt changes due to ocean tide loading effect are computed using the Farrell's (1972) formalism, hereafter called the Green's functions approach or convolution method. An alternative approach based on the spherical harmonics method is also reviewed. For both approaches, a brief revision on which methods have been used by authors for different geodetic applications is done.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS.

The response of the Earth to ocean tide loading is described by some equations of motion, hereafter called equations of deformation. If the spherically symmetrical, non-rotating, perfectly elastic and isotropic (SNREI) Earth model is adopted, to compute the load Love numbers (LLN) of degree n, the fundamental equations of the deformation of the Earth to be solved are obtained from the equations of equilibrium, the stress-strain relation and the Poisson's equation. The resulting system is a set of six linear differential equations of the first order given by (Alterman et al. 1959; Takeuchi & Saito 1972):

$$\left\{\begin{aligned}
\frac{dy_1}{dr} &= \frac{-2\lambda}{(\lambda+2\mu)r}y_1 + \frac{1}{\lambda+2\mu}y_2 + \frac{n(n+1)\lambda}{(\lambda+2\mu)r}y_3 \\
\frac{dy_2}{dr} &= \left[\frac{-4\rho_0 g_0}{r} + \frac{4\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}\right]y_1 - \frac{4\mu}{(\lambda+2\mu)r}y_2 + \\
&\quad \left[\frac{n(n+1)\rho_0 g_0}{r} - \frac{2\mu(3\lambda+2\mu)n(n+1)}{(\lambda+2\mu)r^2}\right]y_3 + \frac{n(n+1)}{r}y_4 - \rho_0 y_6 \\
\frac{dy_3}{dr} &= -\frac{1}{r}y_1 + \frac{1}{r}y_3 + \frac{1}{\mu}y_4 \\
\frac{dy_4}{dr} &= \left[\frac{\rho_0 g_0}{r} - \frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}\right]y_1 - \frac{\lambda}{(\lambda+2\mu)r}y_2 + \\
&\quad \frac{2\mu}{(\lambda+2\mu)r^2}\left[\lambda\left(2n^2+2n-1\right)+2\mu\left(n^2+n-1\right)\right]y_3 - \frac{3}{r}y_4 - \frac{\rho_0}{r}y_5 \\
\frac{dy_5}{dr} &= 4\pi G\rho_0 y_1 + y_6 \\
\frac{dy_6}{dr} &= \frac{-4\pi G\rho_0 n(n+1)}{r}y_3 + \frac{n(n+1)}{r^2}y_5 - \frac{2}{r}y_6
\end{aligned}\right. \qquad [1]$$

The unknowns y_1, y_2, y_3, y_4 and y_5 represent the radial displacement, the vertical component of stress, the tangential displacement, the horizontal component of stress and the gravitational potential change, respectively. The fifth equation in [1] can be understood as the definition of the variable y_6. The rheological parameters of the Earth λ, μ, and the density ϱ_0 are usually numerically given in function of the distance r from the center to the Earth's surface. G is the Newton's gravitational constant and g_0 denotes the initial gravitational acceleration of the Earth.

The SNREI model has a solid inner core, a liquid outer core and a solid (elastic) mantle. Therefore, the above equations system should be solved by appropriate boundary conditions at all internal boundaries and at the surface of the Earth. Longman (1962) stated the formulation of the boundary conditions associated to the equations of deformations system [1]. In the liquid core $\mu = 0$ and the system becomes a reduced set of 4 equations. At the solid-liquid boundaries the variables $y_1, \ldots, y_6$ are continuous and at solid-solid boundaries all $y_1, \ldots, y_6$ are continuous except for y_3. At the Earth's surface ($r = a$), the boundary conditions are given by:

$$\begin{cases} y_2 = -\dfrac{(2n+1)\, g_0^2(a)}{4\pi G} \\ y_4 = 0 \\ y_6 + \dfrac{n+1}{a} y_5 = (2n+1)\, g_0(a) \end{cases} \qquad [2]$$

where $g_0\,(a)$ is the acceleration due to gravity at the Earth's surface.

3. COMPUTATION OF THE LOAD LOVE NUMBERS

The dimensionless load Love numbers (LLN), originally defined by Love (1909) and Shida & Matsuyama (1912), define the response of the Earth model to a surface mass load forcing. The LLN of different orders are computed directly from the solution of the differential equations system [1] according to the related boundary conditions and integrated numerically from the center up to the Earth's surface. For a purely elastic Earth the LLN are essentially independent of the tidal frequency. They are defined by:

$$\begin{cases} h_n' = \dfrac{1}{a} y_1(a) \\ k_n' = \dfrac{1}{a g_0(a)} y_5(a) - 1 \\ l_n' = \dfrac{1}{a} y_3(a) \end{cases} \qquad [3]$$

These parameters are determined by the internal structure of the Earth and they are greatly affected by the structure of the crust and mantle, particularly in the region close to the load. Therefore, different values can be obtained according to various Earth models. The PREM model yields LLN almost identical to those determined by Farrell for the Gutenberg-Bullen A Earth model (Lambeck 1988). Generally, differences in the LLN due to different SNREI Earth models are assumed to be on the order of 1-1.5 percent (van Dam et al. 2003). Differences in the LLN can also appear due to how the equations in [1] are integrated and the application of different boundary conditions. Usually, the system have been solved by recurrent methods based on Runge-Kutta method. More recent, Guo et al. (2004) solved the fundamental equations of the deformation of the Earth using the Chebyshev-collocation method.

4. GREEN'S FUNCTIONS

The Green's functions, which describe the deformation of the Earth induced by the ocean tide loading, depend on the properties of the Earth through the above LLN. For the displacements, gravity and tilt components at the Earth's surface the Green's functions have the following expressions (Farrell 1972):
The radial (i.e. vertical) displacement:

$$u(\psi) = \frac{a}{m_e} \sum_{n=0}^{\infty} h'_n P_n(\cos\psi) \qquad [4]$$

The horizontal displacement:

$$v(\psi) = \left(v_\theta(\psi), v_\lambda(\psi)\right) = \frac{a}{m_e} \sum_{n=1}^{\infty} l'_n \frac{\partial P_n(\cos\psi)}{\partial \psi} (-\cos\alpha, -\sin\alpha) \qquad [5]$$

where $v_\theta(\Psi)$ and $v_\lambda(\Psi)$ are the horizontal components of the displacement for the North-South (NS) and the East-West (EW) directions, respectively. Note that there is no horizontal displacement for $n = 0$.
The gravity (positive upwards):

$$g(\psi) = \frac{g_0(a)}{m_e} \sum_{n=0}^{\infty} \left[n + 2h'_n - (n+1)k'_n\right] P_n(\cos\psi) \qquad [6]$$

The tilt:

$$t(\psi) = \left(t_\theta(\psi), t_\lambda(\psi)\right) = -\frac{1}{m_e} \sum_{n=0}^{\infty} \left[1 - h'_n + k'_n\right] \frac{\partial P_n(\cos\psi)}{\partial \psi} (-\cos\alpha, -\sin\alpha) \qquad [7]$$

where $t_\theta(\Psi)$ and $t_\lambda(\Psi)$ are the NS and EW tilt components, respectively.

In equations [4] to [7], m_e, a and $g_0(a)$ are, respectively, the mass of the Earth, its mean radius and the gravity at the Earth's surface. P_n is the Legendre polynomial of degree n and, h_n', l_n' and k_n' are the load Love numbers of degree n defined by [3], with the prime distinguishing Love numbers used in loading theory from those used in tidal theory. At the Earth's surface, these functions depend on the angular distance Ψ between the load and the observation point, which is given by:

$$\cos(\psi) = \sin\theta\sin\theta' + \cos\theta\cos\theta'\cos(\lambda - \lambda') \qquad [8]$$

α is the azimuth of the load at the observation point, computed clockwise from North. Frequently, the Green's functions are written in a general form as:

$$G(\psi) = G_L(\psi)S_L(\alpha) \qquad [9]$$

being S_L the combination of trigonometric functions of the azimuth, which is necessary to compute a vector load as in equations [5] and [7].

As the Green's function are formed by weighted infinite sum of the load Love numbers, it is necessary to truncate n at some limited value. This truncation at finite n causes problems in the loading computation in coastal regions (so called Gibb's effect). This problem can be solved having into account that when n gets large enough h_n', nl_n' and k_n' become constant. Farrell (1972) derived the approximated expressions for these limits, called asymptotic values of the LLN:

$$\lim_{n\to\infty}\begin{bmatrix} h_n' \\ nl_n' \\ nk_n' \end{bmatrix} = \begin{bmatrix} h_\infty' \\ l_\infty' \\ k_\infty' \end{bmatrix} = \frac{g_0(a)m_e}{4\pi a^2(\lambda+\mu)}\begin{bmatrix} -\dfrac{\lambda+2\mu}{\mu} \\ 1 \\ -\dfrac{3\rho(\lambda+\mu)}{2\hat{\rho}\mu} \end{bmatrix} \qquad [10]$$

where ϱ and $\hat{\varrho}$ are the density at the Earth's surface and a mean value, respectively. If different asymptotic expressions are calculated differences in the LLN can appear. For example, Guo et al. (2004) computed the asymptotic values of the LLN by searching directly the asymptotic solutions of the fundamental equations of the deformation of the Earth [1]. They obtain more accurate LLN than those of Farrell (1972) by an order of $1/n$.

Taking into account the asymptotic values of the LLN in [10] and the called 'Kummer's transformation', the radial displacement in [4] can be written as:

$$u(\psi)=\frac{ah'_\infty}{m_e}\sum_{n=0}^{\infty}P_n(\cos\psi)+\frac{a}{m_e}\sum_{n=0}^{\infty}\left(h'_n-h'_\infty\right)P_n(\cos\psi)= \\ =\frac{ah'_\infty}{2m_e\sin\left(\frac{\psi}{2}\right)}+\frac{a}{m_e}\sum_{n=0}^{N}\left(h'_n-h'_\infty\right)P_n(\cos\psi) \quad [11]$$

Although the second sum terminates after a finite number of term, it still converges rather slowly. Therefore, Farrell (1972) introduced a 'converging disc factor' and applied the Euler's transformation technique for speeding the convergence of the series. In the same way that for the vertical displacement, the horizontal displacement can be expressed as a sum, where the first term can be determined exactly, and the second sum is evaluated as explained before.
Following the same technique for the gravity and tilt effects, they can be respectively written as:

$$g(\psi)=g^N(\psi)+g^E(\psi) \quad [12]$$

$$t(\psi)=t^N(\psi)+t^E(\psi) \quad [13]$$

where the first term (corresponding to the exact solution) in each equation, denoted with the superscript *N*, is the direct, or Newtonian, acceleration:

$$g^N(\psi)=\frac{-g_0(a)}{4m_e\sin\left(\frac{\psi}{2}\right)} \quad [14]$$

$$t^N(\psi)=\frac{\cos\left(\frac{\psi}{2}\right)}{4\sin^2\left(\frac{\psi}{2}\right)} \quad [15]$$

The second term in the equations [12] and [13] corresponds to the elastic effects arising from the Earth deformation:

$$g^E(\psi)=-\frac{g_0(a)}{m_e}\left[\frac{\left(k'_\infty-2h'_\infty\right)}{2\sin\left(\frac{\psi}{2}\right)}+\sum_{n=0}^{N}\left[\left((n+1)k'_n-k'_\infty\right)+2\left(h'_n-h'_\infty\right)\right]P_n(\cos\psi)\right] \quad [16]$$

$$t^E(\psi)=\frac{1}{m_e}\left[\frac{\left(h'_\infty+k'_\infty\right)\cos\left(\frac{\psi}{2}\right)}{4\sin^2\left(\frac{\psi}{2}\right)}-\sum_{n=0}^{N}\left(h'_n+k'_n\right)\frac{\partial P_n(\cos\psi)}{\partial\psi}\right] \quad [17]$$

where the last term of the expansion is usually taken for degree 10,000.

Finally, to avoid the singularity at the origin the Green's functions are normalized. For the radial and horizontal displacements the normalizing functions are given, respectively, by:

$$u^*(\psi) = \frac{-Gh'_\infty}{g_0(a\psi)} \qquad [18]$$

$$v^*(\psi) = \frac{-Gl'_\infty}{g_0(a\psi)} \qquad [19]$$

The gravity and tilt are normalized with respect to the direct attractions of the load, equations [14] and [15].

The Green's functions computed for different SNREI Earth models only show discrepancies for small angular distances between the load and the observation points. This is due to the fact that these Earth models differ essentially at the crust and upper mantle (Francis & Mazzega 1990). van Dam et al. (2003) carried out a comparison between Green's functions derived from 4 different SNREI Earth models. They obtained differences in radial deformations always less than 0.04 mm, indicating that the choice of LLN for an SNREI model will not have a significant influence on the estimated loading effects.

The classical results of Farrell have been modified to describe more realistically the response of the Earth to the tidal loading. For instance, Pagiatakis (1990) considered the Green's functions taking into account the viscoelastic behaviour of the Earth, which become in this case complex and frequency dependent. He found that viscoelastic Green's functions differ at a maximum of 1.5% in amplitude and 0.3º in phase in comparison with a purely elastic Earth. He extended also the theory to a self-gravitating, anisotropy and rotating Earth. These realistic Earth models and those that take into account the mantle heterogeneity and Maxwell rheology (e.g. Mitrovica 1994) have been used mainly in loading studies on very slow deformations, such as the post-glacial rebound.

5. THE GREEN'S FUNCTIONS APPROACH

The loading effects are computed by the convolution between a numerical global model of the ocean tides and the Green's functions as follows (e.g. Baker 1984):

$$L(\theta,\lambda) = \rho_w \int_{\lambda=0}^{2\pi} \int_{\theta=0}^{\pi} a^2 \xi(\theta',\lambda') G(\psi) \sin\theta' d\theta' d\lambda' \qquad [20]$$

where $L(\theta,\lambda)$ is the loading effect at the observation point on the Earth's surface, being θ and λ the colatitude and longitude, respectively. ϱ_w is the density of the ocean water and $\xi(\theta',\lambda')$ is the height of the ocean tide at the loading point. G is the appropriate Green's function (equations [4] to [7]) evaluated on the angular distance Ψ between the observation and the load points given by [8].

The convolution integral [20] is evaluated of over all the oceans. Usually, for its numerical computation, the oceans are divided into a set of cells for which tidal amplitudes and phases are obtained from an ocean tide model. Therefore, the convolution integral can be replaced by a sum over all cells as follows:

$$L(\theta,\lambda)=\rho_w\sum_{i=1}^{N}\xi_i(\theta',\lambda')G(\psi_i)\Delta S_i(\theta',\lambda') \qquad [21]$$

where N is the total number of oceanic cells of the tidal model, $\Delta S_i(\theta',\lambda')$ is the area of the cell centred at the load point of index i. The tabulated values of the Green's functions are interpolated for the corresponding angular distance Ψ_i.

This Farrell's method has successively been refined by other authors to simplify the computations. E.g. Goad (1980) and Agnew (1996) improved the method by using the so called integrated Green's functions and their normalizations. The Goad's technique uses azimuthal and geocentric angles rather than latitude and longitude, which is known as template method (Heiskanen & Moritz 1967). So, taking into account the Green's function expressed as [9] over surface ring sectors forming a template centred on the observing site, the convolution integral [20] can be written in terms of the angular distance Ψ and the azimuth α as follows:

$$L(\psi,\alpha)=\rho_w\int_{\psi=0}^{\pi}a^2\xi(\psi,\alpha)G_L(\psi)\sin\psi\left(\int_{\alpha=0}^{2\pi}S_L(\alpha)d\alpha\right)d\psi \qquad [22]$$

Thus, the Green's functions are replaced by their integral over a template element in which the tidal heights are assumed to be constant. So that, the integral [22] becomes a sum. Finally, in order to use the already tabulated Green's functions given by Farrell (1972) and to remove the singularity in $\Psi = 0$ in the Goad's method, Agnew (1996) normalized the tabulated Green's functions as follows:
For displacements and the elastic component of gravity:

$$\begin{cases} G_t(\psi)=Ka\psi G_L(\psi) \\ G_t^{'}(\psi)=a^2G_t(\psi)\left[2\sin(\psi/2)/\psi\right]/Ka \end{cases} \qquad [23]$$

For the elastic component of tilt:

$$\begin{cases} G_t(\psi) = Ka^2\psi^2 G_L(\psi) \\ G_t^{'}(\psi) = a^2 G_t(\psi)\left[2\sin(\psi/2)/\psi\right]^2 \Big/ Ka \end{cases} \qquad [24]$$

with $K = 10^{12}$ (SI units) and $K = 10^{18}$ for gravity.

The integrated Green's functions for the Newtonian part for a point at elevation h above the sea level are:

$$G_N \int_{\psi_i}^{\psi_{i+1}} \frac{\varepsilon + 2\sin^2(\psi/2)}{\left[4(1+\varepsilon)\sin^2(\psi/2)+\varepsilon^2\right]^{3/2}} \sin\psi d\psi \qquad [25]$$

$$\frac{G_N}{g_0(a)} \int_{\psi_i}^{\psi_{i+1}} \frac{\sin(\psi/2)\cos(\psi/2)}{\left[4(1+\varepsilon)\sin^2(\psi/2)+\varepsilon^2\right]^{3/2}} \sin\psi d\psi \qquad [26]$$

where $\varepsilon = h/a$ and G_N is the gravitational constant. Setting $\varepsilon = 0$ in this equations and taking into account that $g_0(a) = \frac{G_N m_e}{a^2}$, results the integrated Green's functions for the equations [14] and [15], respectively. In equations [25] and [26], for small distances Ψ, it is possible to approximate $\sin\Psi = \Psi$, so that they can be exactly integrated.

In the computation of the loading effects using the convolution method there are two main error sources. The first source arises mainly to uncertainties in the ocean tide model, more than errors to inaccuracy of the Earth model through the Green's functions. As we mentioned before, the results using Green's functions for recent Earth models show a discrepancy which probably never exceeds 2%. As the ocean tides are spatially variables, the ocean tide loading effects depend on the global spatial distribution of the ocean tides relative to the observation point (Baker 1984). It is known that the waters adjacent to the observing site have the largest influence on the displacements, gravity and tilt changes due to ocean loading. Scherneck (1991) states that about half of the tidal loading effect arises from tides within 2000 km of the site considered. Besides, in shallow waters such as along coastlines and on continental shelves the tides vary highly (Penna & Baker 2002). Therefore, accurate modelling of the tides in the surrounding area is crucial for near coastal sites. Because of the low resolution of the global ocean tide models, they are often supplemented with specific regional and local ocean models. Moreover, a fine subdivision of the grid for the sea cells adjacent to the observation point is needed to provide better accuracy in the convolution integral.

The Green's functions approach has been traditionally used and applied in various geodetic investigations. For instance, Wang et al (2002) used this method to estimate the loading effects on GPS, gravity and tilt measurements. Their numerical results show that the three modelled quantities, displacements, gravity and tilt, vary dramatically as the water level increases. They showed also that loading effects would hinder the detection of signal associated with earthquakes and landslides from the observed data based on geodetic techniques in their area of study. Besides, Zerbini et al. (2002) estimated the seasonal oceanic loading effects on continuous GPS and gravity observations, following the Green's functions approach. For instance, they found amplitudes of the seasonal signals in the range of 1-2 mm for ocean loading effects on the height series and for gravity, amplitudes for the annual waves between 0.4 – 0.6 μGal (1 μGal = 10^{-8} ms^{-2}).

6. THE SPHERICAL HARMONIC APPROACH

The ocean load $\xi(\theta',\lambda')$ can be initially represented in terms of a spherical harmonics expansion (Le Meur & Hindmarsh 2000):

$$\xi(\theta',\lambda') = \sum_{n'=0}^{\infty} \sum_{l'=-n'}^{n'} \xi_{n'}^{l'} Y_{n'}^{l'}(\theta',\lambda') \qquad [27]$$

being the load completely defined by the spectral coefficients $\xi_{n'}^{l'}$. The spherical harmonic $Y_{n'}^{l'}(\theta',\lambda')$ of degree *n*' and order *l*' are fully normalized, as defined in Edmonds (1960), in the sense of:

$$\int_0^{2\pi}\int_0^{\pi} \overline{Y}_n^l(\theta',\lambda') Y_{n'}^{l'}(\theta',\lambda') \sin\theta' d\theta' d\lambda' = 4\pi\delta_{n'n}\delta_{l'l} \qquad [28]$$

Next, an equivalent convolution to the first method can be carried out. In order to simplificate the notation, following Mitrovica (1994), the Green's functions [4] to [7] can be expressed symbolically of two forms:

$$G(\psi) = \sum_{n=0}^{\infty} \left(A_n P_n(\cos\psi)\right) \qquad [29]$$

and:

$$G(\psi) = \sum_{n=0}^{\infty} \left(A_n P_n(\cos\psi)\right) \qquad [30]$$

where [4] and [6] are of the form [29], whereas the vector Green's function for the horizontal displacement [5] and tilt [7] are of the form [30]. The symbol ∇ denotes the gradient operator.

Let us consider the form [29]. With this approach, the equivalent of [20] by replacing [27], becomes:

$$L(\theta,\lambda)=\rho_w\sum_{n=0}^{\infty}a^2 A_n\sum_{n'=0}^{\infty}\sum_{l'=-n'}^{n'}\xi_{n'}^{l'}\int_0^{2\pi}\int_0^{\pi}Y_{n'}^{l'}(\theta',\lambda')P_n(\cos\psi)\sin\theta' d\theta' d\lambda' \qquad [31]$$

Thus, taking into account [28] and the following addition theorem of spherical harmonics (e.g., Jackson 1975):

$$P_n(\cos\psi)=\frac{1}{(2n+1)}\sum_{l=-n}^{n}\overline{Y}_n^l(\theta',\lambda')Y_n^l(\theta,\lambda) \qquad [32]$$

in [31] yields:

$$L(\theta,\lambda)=\rho_w\sum_{n=0}^{\infty}\sum_{l=-n}^{n}\frac{4\pi a^2}{(2n+1)}A_n\xi_n^l Y_n^l(\theta,\lambda) \qquad [33]$$

Therefore, the response of the Earth to the load can be expressed as:

$$L(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{l=-n}^{n}L_n^l Y_n^l(\theta,\lambda) \qquad [34]$$

where:

$$L_n^l=\rho_w\frac{4\pi a^2}{(2n+1)}A_n\xi_n^l \qquad [35]$$

represents the set of spherical harmonic coefficients.
An analogous procedure is applied to the Green's functions of the form [30], which gives the following expression for the Earth's response to the load:

$$L(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{l=-n}^{n}L_n^l\nabla Y_n^l(\theta,\lambda) \qquad [36]$$

with the spherical harmonics coefficients given by:

$$L_n^l=\rho_w\frac{4\pi a^2}{(2n+1)}B_n\xi_n^l \qquad [37]$$

The computation of the spherical harmonics derivatives ∇Y_n^l in [36] with respect to θ, can be easily done following the recursion relationships given by Edmonds (1960) for the associated Legendre's functions.

As in the Green functions approach, the problem of computing the infinite harmonic series [34] and [36] may be considered. Le Meur & Hindmarsh (2000) showed the power of the spherical harmonic approach, which can exhibits a very fast converge depending on the geometrical properties of the loading function.

The spherical harmonic approach has been frequently used in recent years in many geophysical applications. Elósegui et al. (2003) investigated the Earth's crust displacements associated to localized loads, such as the Great Salt Lake, through the spherical harmonic formalism. More recently, Blewitt (2003) recommend the spherical harmonics approach to study the interaction between loading dynamics and the terrestrial frame, although the Green's functions approach would be also applied. An interesting and recent application of the spherical harmonic approach on gravitational consistency and mass conservation on loading models can be found in Clarke et al. (2005).

7. DISCUSSION

This paper deals with two ways for computing the Earth crust displacements and the change in gravity and tilt at the Earth's surface produced by ocean loading effect. The first one, consist of the classical convolution between the ocean tides and the Green's functions (Farrell 1972). The second one, it is based on the preliminary development of the ocean tide in spherical harmonics (Mitrovica 1994; Le Meur & Hindmarsh 2000). There are specific advantages and disadvantages for the two approaches, which in principle seems difficult to determine which method provides the best solution.

The first approach leads very sensitive to the total number of harmonic degrees used to obtain the Green's functions. In practice, this problem can be solved by taking into account the asymptotic expressions of the LLN and the Kummer's transformation. The second method is less sensitive to the problem associated with the truncation in the infinite harmonic series, where the truncation level is chosen depending on the loading function. The main disadvantage of this method is that its spatial resolution is independent of the position (Mitrovica 1994). As a consequence, a high spatial resolution at one location forces the same resolution for other regions, which increases the computational time. Particularly, in the ocean tide loading problem some authors (e.g. Houze & Weijian 1987; van Dam et al. 2003) propose a hybrid method. Thus, one may divide the surface of the Earth into two regions for every station: a local region containing the station and a distant region. When computing the contribution from the local region, the Green's function approach should be applied. For distant regions the loading effect should be computed using the spherical harmonic approach. This hybrid method increased the speed of the computations and the main disadvantages of each method are compensated by using the other one.

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9. REFERENCES

AGNEW, D.C., 1996. SPOTL: Some Programs for Ocean-Tide Loading. SIO Ref. Ser. 96-8, 35 pp,. Scripps Institution of Oceanography, La Jolla, CA.

ALTERMAN, Z., JAROSCH, H. & C.L. PEKERIS, 1959. Oscillations of the Earth. *Proc. R. Soc. Lond.*, A252, 80-95.

BAKER, T., 1984. Tidal Deformations of the Earth. *Sci. Prog. Oxf.*, 69, 197-233.

BAKER, T.F., CURTIS, D.J. & A.H. DODSON, 1995. Ocen Tide Loading and GPS. *GPS World*, March 1995, 54-59.

BAKER T.F., 1998: Tidal gravity observations and Earth tide models. *Série Géphysique*, Observatoire Royal de Belgique 287-294.

BLEWITT, G., 2003. Self-consistency in reference frames, geocenter definition, and surface loading of the solid Earth. *J. Geophys. Res.*, 108 (B2), 2103, doi: 10.1029/2002JB002082.

CLARKE, P.J., LAVALLÉE, D.A., BLEWWIT, G., VAN DAM, T.M. & J.M. WAHR, (2005). Effect of the gravitational consistency and mass conservation on seasonal surface mass loading models. Accepted by *Geophys. Res. Lett.* March 2005, 2005GL0224411.

DZIEWONSKI, A.M. & D.L. ANDERSON, 1981. Preliminary Reference Earth Model. *Phys. Earth Planet Int.*, 25, 297-356.

EDMONDS, A.R., 1960. *Angular Momentum and Quantum Mechanics*. Princeton University Press, Princeton, N.J.

ELÓSEGUI, P., DAVIS, J.L., MITROVICA, J.X., BENNET, R.A. & B.P. WERNICKE, 2003. Crustal loading near Great Salt Lake, Utah. *Geophys. Res. Lett.*, 30 (3), 1111, doi: 10.1029/2002GL016579.

FARRELL, W.E., 1972. Deformation of the Earth by surface loads. *Rev. Geophys. Space. Phys.*,Francis, O. & P. Mazzega, 1990. Global charts of ocean tide loading effects.

FRANCIS, O. & P. MAZZEGA, 1990. Global charts of ocean tide loading effects. *J. Geophys. Res.*, 95 (C7), 11,411-11,424.

GOAD, C., 1980. Gravimetric Tidal Loading Computed from Integrated Green's Functions. *J. Geophys. Res.*, 85 (B5), 2679-2683.

GUO, J.Y., LI, Y.B., HUANG, Y., DENG, H.T., XU, S.Q. & J.S. NING, 2004. Green's functions of the deformation of the Earth as a result of atmospheric loading. *Geophys. J. Int.*, 159, 53-68.

HEISKANEN, W.A. & H. MORITZ, 1967. *Physical Geodesy.* W.H. Freeman, San Francisco, 364 pp.

HOUZE, X. & M. WEIJIAN,1987. Effect of different Earth models on loading tidal corrections. *Earthquake Research in China*, 1 (Nº1), 123-134.

JACKSON, J.D., 1975. *Electrodynamics*. Wiley & Sons, New York.

JOBERT, G., 1960. Perturbation des marées terrestres. *Ann. Geophys.*, 16, 1.

LAMBECK, K., 1988. *Geophysical Geodesy: The Slow Deformations of the Earth.* Oxford University Press, New York.

LE MEUR, E. & HINDMARSH, C.A., 2000. A Comparison of two spectral approaches for computing the Earth response to surface loads. *Geophys. J. Int.*, 141, 282-298.

LONGMAN, I.M., 1962. A Green's Function for Determining the Deformation of the Earth Under surface Mass Loads. 1. Theory. *J. Geophys. Res.*, 67 (2), 845-850.

LOVE, A.E.H., 1909. The yielding of the Earth to disturbances forces. *Proc. R. Soc. London*, 82, 73-88.

MANGIAROTTI, S., CAZENAVE, A., SOUDARIN, L & J.F. CRÉTAUX, 2001. Annual vertical crustal motions predicted from surface mass redistributions and observed by space geodesy. *J. Geophys. Res.*, 106, 4277-4291.

MITROVICA, J.X., 1994. A spectral formalism for computing three-dimensional deformations due to surface loads. 1. Theory. *J. Geophys. Res.*, 99 (B4), 7057-7073.

MUNK, W.H. & G.J.F. MACDONALD, 1960. *The Rotation of the Earth*. Cambridge University Press.

PAGIATAKIS, S.D., 1990. The response of a realistic Earth to ocean tide loading. *Geophys. J. Int.*, 103, 541-560.

PEKERIS, C.L. & H. JAROSCH, 1958. The free oscillations of the Earth. *in: Contributions in Geophysics: In Honor of Beno Gutenberg*, Pergamont, New York. 171-192.

PENNA, N.T. & T.F. BAKER, 2002. Ocean tide loading considerations for GPS processing around Australia. *Geomatics Reseach Australalasia*, 77, 1-26.

SCHERNECK, H.G., 1991. A Parameterized Solid Earth Tide Model and Ocean Tide Loading Effects for Global Geodetic Baseline Measurements. *Geophys. J. Int.*, 106, 677-694.

SHIDA, T. & M. MATSUYAMA, 1912. Note on Hecker's observations of horizontal pendulums. *Memoirs of the college of Science and Engineering*, Kyoto Imperial Univ., 4, 187-224.

SLICHTER, L.B. & M. CAPUTO, 1960. Deformation of an Earth model by surface pressures. *J. Geophys. Res.*, 65, 4151-4156.

TAKEUCHI, H. & M. SAITO, 1972. Seismic surface waves. *Methods in Computational Physics*, Academic Press, New York, 11, 217-295.

VAN DAM T., PLAG H.-P., FRANCIS O. & P. GEGOUT, 2003. GGFC Special Bureau for Loading: Current status and Plans. *In*: B. *Richter, W. Schwegmann, and W. R. Dick. (Eds.)* Proceedings of the IERS Workshop on Combination Research and Global Geophysical Fluids, Bavarian Academy of Sciences, Munich, Germany, 18-21 November 2002. *IERS Technical Note 30*, 180-198.

WANG, H., HSU, H.T. & Y.Z. ZHU, 2002. Prediction of surface horizontal displacements, and gravity and tilt changes caused by filling the Three Gorges Reservoir. *J. Geodesy*, 76, 105-104.

ZERBINI, S., NEGUSINI, M., ROMAGNOLI, C., DOMENICHINI, F., RICHTER, B. & D. SIMON, 2002. Multi-parameter continuous observations to detect ground deformation and to study environmental variability impacts. *Global and Planetary Change*, 34, 37-58.