Non-linear processes in shallow waters. Influence on local observed mean sea level

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ABSTRACT
A combined theoretical and empirical method is applied to study the nonlinear interactions occurring in shallow waters among astronomical, overtide and compound constituents. Although the isolated non-linear terms present a small amplitude, the global response of the oceans to the primary and secondary interactions can mean an important contribution to the sea level variability in continental shelf and coasts with a strong tidal signal. Thus, an accurate determination inside these areas is of considerably importance both in mean sea level studies and in tidal prediction. To this purpose, a formal theoretical development of the generating processes has been accomplished. At the same time, the tidal parameters of the most significant shallow water constituents have been empirically determined in six nearby stations located in the Cantabric Sea. The methodology developed for this determination and later elimination of over and compound tides is described in this work. Besides, the simultaneous determination allows us to study the dependence of these non primary waves on the properties of the emplacements. The effectivity of the method has been contrasted through an iterative process based on spectral analysis of the residuals. The application, component by component, of this technique, allows us to minimize the influence of tidal forces on mean sea level, as it is shown in the increased accuracy.

Keywords: Tide gauge observations, ocean tides, mean sea level, nonlinear interactions, shallow water constituents

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1. INTRODUCTION

Non linear interactions among astronomical tidal components is one of the problems that has limited, traditionally, the accuracy of local mean sea level determinations in marginal areas of shallow waters. In fact, in a realistic ocean it is necessary to extend the study to the second interactions involving astronomical, overtide, and compound-tide constituents, as every wave interacts and can affect significantly each other. A more complicated coupling occurs between tidal harmonics and various oscillations induced by thermal, wind and atmospheric pressure fluctuations (Militello and Kraus, 2001; Janssen, 2004) as well as oceanic currents (Qin et al., 1999). These complex processes impose that in a rigorous tidal analysis, shallow waters constituents should be studied separately. Thus, we consider a necessary task the correct modelization and subsequent elimination of the wave-wave non linear interactions, to obtain an accurate determination of local mean sea level.

When ocean tidal waves propagate in shallow waters, they are strongly distorted. The causes of this nearshore processes are, mainly, bottom friction and other hydrodynamic processes. Among them, generation of stationary waves and local resonance effects are included. These phenomena are closely related to water layer thickness, topography, seafloor composition and coastal configuration. As they alter the progression of tidal waves, variations in amplitude and phase of the astronomical constituents take place, changing the tidal range. But, at the same time, several subharmonic called overtides, even or compound tides, are originated. Additionally, as the frequency of these new generated components is close to the tidal harmonics derived directly from the Equilibrium Theory, modulation of tidal components can take place (Gil, 2002). Thus, the observed sea level in the area changes, as these distorted waves of non negligible amplitude normally modify and perturb the sea-level record.

The new generated components are responsible of singular tidal regimens in specific locations of the world coasts. One of the best known examples is the tidal variations at Southampton on the South coast of United Kingdom. There, the distortion takes the form of a double high and low water, due to the inference between the semidiurnal mode of oscillation, characteristic of the area, and secondary components of amplitude similar to the diurnal constituents. Other example can be found in the Gabes Gulf, a wide shallow water area located in the Mediterranean Sea, where this phenomenon is responsible of the observed strong semidiurnal regime and the amplification and generation of high-frequency constituents (Gil and De Toro, 2005).

In this work, the main tidal components originated by non-linear coupling and energy transfer among ocean waves are studied. In a first step, their theoretical parameters are obtained through a formal development of the generating processes, carried out within the framework of an inviscid shallow-water model. This determination is based on the fact that the distorted waves can be expressed as simple harmonic constituents with hourly angular speeds that are multiples, sums or differences of the frequency of primary tidal constituents (Pugh, 1987, 2004; Nayfeh, 2000; Maas and Doelman, 2002).

The following step is to perform a preliminary tidal analysis of the sea level record using a classical Least Square Method (De Toro et al., 1993; Venedikov et al., 1997). We have made use of an adequate group separation in order to determine the main astronomical constituents. Thus, when the tidal model obtained is subtracted, the residual curve is submitted to spectral analysis. This prior procedure allows the accurate discrimination of an elevated number of higher than diurnal frequency components. The identification of the most significant tides originated by nonlinear interactions is carried out at that time. These processes were applied to data series obtained in six nearby stations placed in the Cantabric Sea, which is an area of wide tidal variations and, thus, proper for this study. Five of them are located at Musel Port of Gijón, and are placed on the interior and exterior of the docks. The other one is placed inside Santander Harbour. Fig. 1 shows the emplacement of the six stations.

Figure 1. Stations placed at Musel Port of Gijón (left) and Santander Harbour (right).

Next, applying Least Squares Method to the residual curve, the tidal factors concerning to non linear shallow waters constituents are determined and, thus, the stability of these harmonic constants is contrasted simultaneously. The results show that, although the new generated components can have, separately, small amplitude, the combined effect of all of them can contribute to a non-negligible mean sea level variability. In the last step, those shallow water components with proved stability are apodized.

In the subsequent sections, a more detailed description of the aforementioned methodology is given. Its effectivity has been contrasted through an iterative process based on spectral analysis of the residuals.

Finally, the results obtained are discussed and it is shown that the influence of higher than diurnal tidal frequencies on sea level has been minimized, which means an increased accuracy to determine local mean sea level.

2. OCEAN TIDES INTERACTIONS IN SHALLOW WATERS. THEORETICAL MODEL

We start considering a specific tidal wave of amplitude *A*, frequency ω and wave length λ, propagating in undisturbed and inviscid shallow waters, of variable depth h(x). We assume the fundamental parametric condition for shallow water theory (h(x)/λ << 1), characterizing our global ocean circulation model according to the vertical and horizontal scale of the motion (Pedlosky, 1987). Working on these premises, their major deficiency is the absence of density stratification. Non-linear waves theory states that the hydrodynamic processes associated to a progressive wave, such as particle velocity and sea surface elevation, can be modelled with a non-linear system of differential equations. Their solutions can be expressed as a power series development of all quantities involved (Stoker, 1957). Thus, the instantaneous vertical displacement $\eta(x,t)$ of the ocean surface above the reference level corresponding to a specific tidal component can be expressed as a power series development

$$\eta(x,t) = \eta_0 + \eta_1\sigma + \eta_2\sigma^2 + \eta_3\sigma^3 + \eta_4\sigma^4 + \dots .$$

of a parameter σ that represents a pure harmonic with the same frequency that the original oscillation, but its amplitude and phase have been modified by the boundary conditions. Note that the non-linear terms mean the generation of new constituents. Although these non-linear effects can be considered negligible in pelagic stations, the amplitude of the resulting components is increased considerably when arriving to the continental shelf, mainly in coastal areas.

In a real ocean, the problem is complicated as many constituents, with different amplitude and angular velocity, are overlaid. Non-linear interaction among them take place, meaning coupling and energy transfer. These effects are added to stationary waves generation and local resonance phenomena, in connection with the small depth of the water layer and reflection along the coast line. However, the solutions of the resulting dynamical equations can be expressed as a power series development that depends on the parameter ξ,

$$\gamma(x,t) = \gamma_0 + \gamma_1\xi + \gamma_2\xi^2 + \gamma_3\xi^3 + \gamma_4\xi^4 + \dots .$$

In this case, that is sea level height itself (Pugh, 1987), meaning a linear superposition of harmonics in the signal

$$\xi = \sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i),$$

where A_i is the amplitude of shallow water constituents and ϕ_i the phase. Consequently, the total effect on the free sea surface elevation

$$H(x,t) = \eta(x,t) + \gamma(x,t)$$

can be expressed as a sum of simple harmonic constituents of angular velocity multiple, sum or difference of the astronomical constituents involved.

When studying the movement induced by tidal forces, other fields as pressure, temperature, salinity or density should be considered in order to build a complete theory describing the ocean behaviour in shallow waters. However, despite its deficiencies, this model can reveal important aspects of ocean dynamics in shallow waters. By other side, it allows a theoretical modelization of the harmonic components so generated that should be experimentally obtained through numerical analysis of the observations. With this aim, we have re-determined the main additional constituents. Later, they have been included explicitly in the tide generating potential of Tamura (1987).

Since M_2 and S_2 are generally the dominant semidiurnal constituents we are going to work, first we have taken into account their linear combination

$$I_0 = Z_0 + H_{M_2}\cos 2\tau + H_{S_2}\cos\left(2\tau + 2s - 2h\right)$$

that can be expressed as an equivalent wave describing, in absence of external perturbations, one of the main modulation of tidal range, associated to lunar variation. Among its different combinations, the quadratic interaction should be studied

$$I = K''\left[H_{M_2}\cos 2\tau + H_{S_2}\cos\left(2\tau + 2s - 2h\right)\right]^2 = K''[\frac{1}{2}\left(H^2{}_{M_2} + H^2{}_{S_2}\right) + \frac{1}{2}H^2{}_{M_2}\cos 4\tau +$$

$$+\frac{1}{2}H^2{}_{S_2}\cos 4\left(\tau + s - h\right) + H_{M_2}H_{S_2}\cos 2\left(2\tau + s - h\right) + H_{M_2}H_{S_2}\cos 2\left(s - h\right)]$$

where *K"* is the proportionality constant corresponding to quadratic interactions. Let us note that the distortions are proportional to the powers of the tide elevation. Theoretically, these coefficients rely only on the interaction order, but in a real ocean they depend, besides, on the frequencies of the involved harmonic components in connection with the boundary conditions, as will be shown in the harmonic analysis factors. So, the quadratic interaction between M_2 and S_2 gives two subharmonics with an amplitude proportional to the square of the over constituents M_4 and S_4, a forth-diurnal composed term, MS_4, and a long period component, M'_{sf}. Since this last component has the same frequency as the harmonic constituent M_{sf}, it just modifies its amplitude. Besides, the mean sea level is distorted by this interaction as there are also obtained two constant terms, denoted M'_0 and S'_0, modifying the constant solar and lunar terms

$$Z'_0 = \frac{1}{2}K''\left(H^2{}_{M_2} + H^2{}_{S_2}\right) = M'_0 + S'_0 .$$

Taking into account the tidal analysis results obtained in the studied stations (Gil, 2002), it is necessary to determinate the interaction involving M_2, S_2, K_2 and N_2. As an example, we show the triple order interaction of M_2, S_2 and K_2

$$\begin{aligned}
I &= K''\left[H_{M_2}\cos 2\tau + H_{S_2}\cos\left(2\tau+2s-2h\right) + H_{K_2}\cos\left(2\tau+2s\right)\right]^2 \\
&= K''[\frac{1}{2}\left(H^2{}_{M_2} + H^2{}_{S_2}H^2{}_{K_2}\right) + \frac{1}{2}H^2{}_{M_2}\cos 4\tau + \frac{1}{2}H^2{}_{S_2}\cos 4\left(\tau+s-h\right) + \frac{1}{2}H^2{}_{K_2}\cos 4\left(\tau+s\right) + \\
&+ H_{M_2}H_{S_2}\cos 2\left(2\tau+s-h\right) + H_{M_2}H_{S_2}\cos 2\left(s-h\right) + H_{S_2}H_{K_2}\cos 2\left(2\tau+2s-h\right) + \\
&+ H_{S_2}H_{K_2}\cos 2h + H_{M_2}H_{K_2}\cos 2\left(2\tau+s\right) + H_{M_2}H_{K_2}\cos 2s]
\end{aligned}$$

The subharmonic K_4 of K_2 is obtained, with argument 4(τ+s). Besides there are also generated, as in the interaction between M_2 and S_2, the waves M_4, S_4, MS_4 and M'_{sf}. The non-linear interaction between S_2 and K_2 can be noted in the term of argument 2(2τ + 2*s* - *h)*, shown in the component SK_4, and other of argument 2*h*, denoted as S'_{sa}, since it modifies S_{sa}. The components originated by the interaction between M_2 and K_2 have arguments 2(2τ + 2*s*) and 2*s* being, respectively, a fourth-diurnal harmonic MK_4 and a monthly period component, M'_f that is included in the tidal potential. Finally, it appears also a constant term, modifying the mean sea level

$$Z'_0 = \frac{1}{2}K''\left(H^2{}_{M_2} + H^2{}_{S_2}H^2{}_{K_2}\right) = M'_0 + S'_0 .$$

Note that the effect of the double interaction is included in the triple one.

Following this procedure with N_2 and taking into account also the main diurnal constituents P_1, O_1 and K_1, the partial tide coefficients and phase of the composed harmonics, there have been obtained, among others, MSN_2, MN_4, MP_1, SO_1, MO_3 and MK_3.

Significant interactions among diurnal (or lower period components) and long period harmonics can be detected in shallow water areas. As an example, we show how the different terms of the quadratic interaction between M_2 and the annual component S_a, are obtained:

$$I = K''\left[H_{M_2}\cos 2\tau + H_{S_a}\cos\left(h-p_s\right)\right]^2 = K''[\frac{1}{2}\left(H^2{}_{M_2} + H^2{}_{S_a}\right) + \frac{1}{2}H^2{}_{M_2}\cos 4\tau +$$

$$+\frac{1}{2}H^2{}_{S_a}\cos 2\left(h-p_s\right) + H_{M_2}H_{S_a}\cos\left(2\tau-h+p_s\right) + H_{M_2}H_{S_a}\cos\left(2\tau+h-p_s\right)].$$

We get five components: a constant term, a component with the period of M_4, two semidiurnal components with arguments 2τ - (h - p_s) and 2τ + (h - p_s) and a long period constituent with period 2(h - p_s), modifying the term with the same frequency in the second order potential. The most interesting waves are the semidiurnal, the first one turns up in the third order tidal development and the second one in the second order one. These composed harmonics are very important in coastal observations, as they modulate M_2 with a period that corresponds to the anomalistic year. But, since they are perturbed by radiational and meteorological effects, p_s is not considered, generally, in the tidal analysis (Pugh, 1987). The results are the components MA_2 and MB_2. They modulate M_2 with a period of a tropic year, quite close to the one of S_{sa}. Consequently, in practice, a lot of additional constituents are required in order to represent shallow waters distortions. Thus, after determining the partial tide coefficients, argument numbers and hourly angular velocities of all the subharmonics and composed harmonics not explicitly present in Tamura's tidal development, we have included them. Many of these components, as M'_{sf}, S'_{sa}, M'_4 or $2MS_2$, are constituted also by a tidal constituent, derived directly from the tidal development. The amplitude obtained through the harmonic analysis is the amplitude of all the components, i.e., the amplitude of the pure constituent added to the amplitudes of the non-linear interactions generated ones. On the other hand, tidal analysis of tide gauge data confirms that in semidiurnal regime shallow waters, the forth-diurnal, sixth-diurnal and higher order even constituents have, generally, higher amplitude than the odd ones (Pugh, 1987). However, the importance of coastal terms and the number of significative constituents can vary from the area, depending on nature of the interactions and sea bottom configuration.

As noted in the obtention of MA_2 and MB_2, symmetric harmonics of M_2, it must also be taken into account shallow waters components in time variations of tidal astronomical constituents. Their nodal factors, $f_{M_4} = f_{M_2} f_{M_2}$ and $u_{M_4} = 2u_{M_2}$, depend on the factors f and u of the pure harmonics generating them. Consequently, their effect is greater in nodal variation than the originated by pure harmonics in close frequencies. The expression of nodal factors and angles also shows that amplitude and phase lag of the modulation depend, mainly, on the basin configuration, just like for the tidal constituents.

Table 1 lists the main over and compound tides, their arguments, the generated waves and angular velocity. The last column shows the group that they modulate in the long period, diurnal and semidiurnal bands, which are the main species considered in the tidal potential development.

Table 1. Main over and compound tides.

Component	Argument	Generated by	*w* (º/h)	Modulated group
S'_{sa}	2h	S_2, K_2	0.0821	S_{sa}
M'_{sf}	2(s-h)	M_2, S_2	1.1016	M_{sf}
M'_f	2s	M_2, K_2	1.0980	M_f
$MP_1 = \tau'_1$	τ-s-2h	M_2, P_1	14.0252	O_1
SO_1	τ+3s-2h	S_2, O_1	15.9748	OO_1
$\mu'_2 = 2MS_2$	2τ-2(s-h)	M_2, S_2	27.9682	μ_2
MSK_2	2τ-2h	M_2, S_2, K_2	28.9020	M_2
MA_2	2τ-h	M_2, S_a	28.9430	M_2
MB_2	2τ+h	M_2, S_a	29.0217	M_2
MKS_2	2τ+2h	M_2, S_2, K_2	29.0662	M_2
MSN_2	2τ+3s-2h-p	M_2, S_2, N_2	30.5444	S_2
$2SM_2$	2τ+4s-4h	M_2, S_2	31.1059	$2K_2$
MO_3	3τ-s	M_2, K_1	42.9271	
M_3	3τ	M_2, M_1	43.4762	
SO_3	3τ+s-2h	S_2, O_1	43.9430	
MK_3	3τ+s	M_2, O_1	44.0252	
S_3	3τ+3s-3h	S_2, S_1	45.0000	
SK_3	3τ+3s-2h	S_2, K_1	45.0411	
K_3	3τ+3s	M_2, K_1	45.1230	
MN_4	4τ-s+p	M_2, N_2	57.4238	
M_4	4τ	M_2	57.9682	
SN_4	4τ+s-2h+p	S_2, N_2	58.4397	
MS_4	4τ+2s-2h	M_2, S_2	58.9841	
MK_4	4τ-2s	M_2, K_2	59.0662	
S_4	4τ+4s-4h	S_2	60.0000	
SK_4	4τ+4s-2h	S_2, K_2	60.0824	
K_4	4τ+4s	K_2	60.1640	
$2MK_5$	5τ+s	M_2, K_1	73.0093	
$2SK_5$	5τ-s-4h	S_2, O_1	75.0411	
$2MN_6$	6τ-s+p	M_2, N_2	86.4079	
M_6	6τ	M_2	86.9523	
$2MS_6$	6τ+2s-2h	M_2, S_2	87.9682	
$2MK_6$	6τ+2s	M_2, K_2	88.0503	
$2SM_6$	6τ+4s-4h	M_2, S_2	88.9841	
MSK_6	6τ+4s-2h	M_2, S_2, K_2	89.0662	
M_8	8τ	M_2	115.9364	

The theoretical determination of the constant terms originated by non-linear interactions is not, a priori, complex: it could be done determining the theoretical amplitude factor using a subharmonic with an unique origin. However, the practical determination is not so easy, since the estimated amplitude is modified by the interaction itself. Besides, for the same boundary conditions, the amplification is dependent on the implied frequencies.

3. TIDAL ANALYSIS OF THE NON LINEAR SHALLOW WATER CONSTITUENTS

Let us note that if there are several nearby stations, with similar global characteristics (for instance, tidal regime or belonging to the same regional basin), but different local bottom rheology, as different bottom composition or depth, it is possible to study and determine the influence of various factors on non-linear interactions, mainly boundary conditions. With this aim, we have selected five different locations within the Musel Port (Gijon, Spain) and a secondary station in Santander Harbour, all of them in the Cantabric Sea. This area is characterized by a strong semidiurnal tidal regime, with a tidal range up to 5 meters, proper for the study. The bathymetry presents, at the stations, a minimum of 7.5 meters, increasing to 50 meters depth at 5 kilometers and 100 meters depth at 9 kilometers. Besides, it shows both locally and regionally a variable sea bottom composition. As it will be proved, this facts originate different anomalies in connection with tidal waves propagation and with the complex distribution of non-linear bottom interactions at the main frequencies of all the phenomena. The stations in Musel Port, denoted by E1, E2, E3, B1 and B3, belong to a network established in 1989. Station B1 is placed in a breakwater of 1550 meters while station E1 is located at its inner side. In other main dock is placed station B3. The other two stations, E2 and E3, are located in the inner docks of the harbour. The bathymetry of the port is quite complex, since there are shoal patches and reefs. The depth of the inlet, with the exception of these structures, increases quite regularly. By other side, the basin bottom is formed by littoral sediments, rocks and sand, excepting the shoal patches and reefs. The sixth station is placed in the closed Santander bay, with low bathymetry and sandy bottom, opposite to regional structure of Gijon's basin.

The first step has been the determination of the power spectrum of the residuals obtained after the elimination of the standard tidal model in all the stations of Gijon. This step is only performed after apodizing the main astronomical constituents. The tidal model has been determined by the least square tidal analysis NSV method (Venedikov et al., 1997). Spectral analysis has confirmed the existence of significant components, practically in all the tidal bands. The frequencies of these components are very similar for the different stations, although their precision is limited by the length of the data and the noise level, that changes depending on the situation of the stations inside the docks. In many cases, the components are included in a wave group of close frequencies.

In order to get a correct interpretation of the results, it should be taken into account that when waves superposition takes place, the amplitude is increased when the frequency of a resonance is close to the frequency of the tidal constituent (Zahran, 2000). This effect has a strong local dependence. Fig. 2 presents, as an example, the power spectrum obtained in E2. Table 2 lists the periods (*T*, in hours) and amplitudes (*A*, in millimetres) of the main components detected by this analysis.

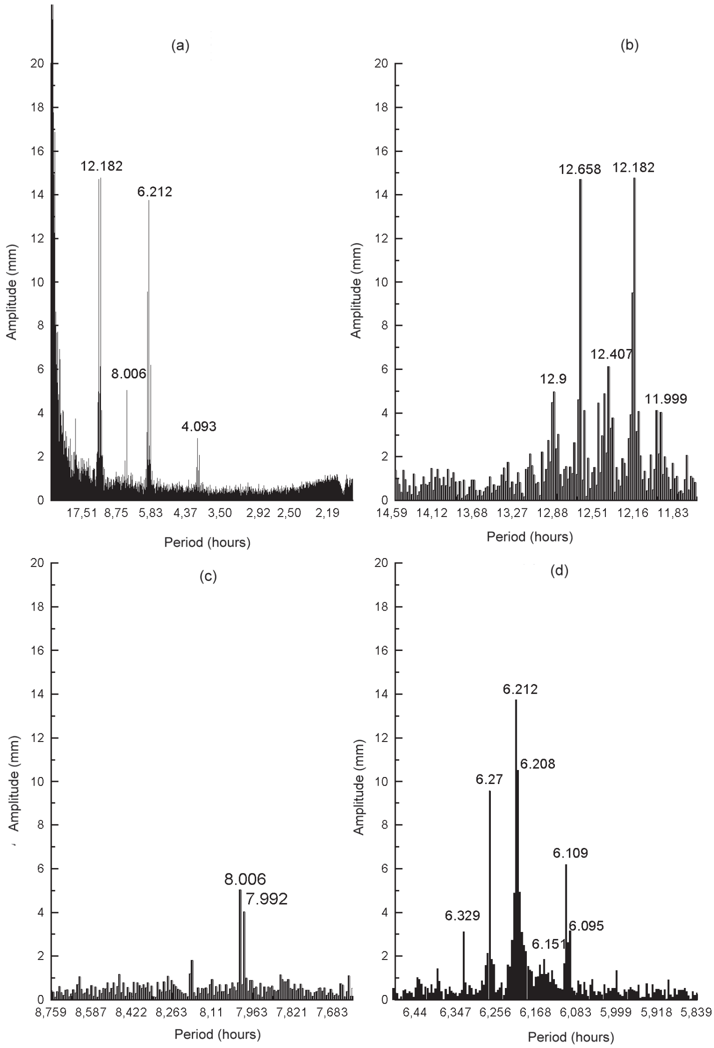

Figure 2. (a) Tidal spectrum of E2, (b) semi-diurnal, (c) third-diurnal and (d) fourth-diurnal components.

Table 2. Power spectrum after eliminating tides.

Station B1		**Station B3**		**Station E1**		**Station E2**		**Station E3**	
T	A	***T***	A	***T***	A	***T***	A	***T***	A
								12.897 12.846	10.064 8.885
12.658	12.574					12.658	14.710	12.647 12.598	15.850 13.078
12.430	5.059	12.463	3.438	12.444	4.793	12.442 12.407	4.891 6.127	12.454 12.407	19.820 39.462
12.209	10.595	12.162	2.000	12.183	3.174	12.182	14.771	12.175	11.964
11.997	6.651	11.916	3.044			11.999	4.130	11.952	15.845
8.007 7.988	3.311 4.076	8.016 7.980	2.163 2.700			8.006 7.992	5.039 4.036	7.998	3.516
6.271	10.061	6.272 6.265	5.594 6.844	6.272	8.601	6.270	9.570	6.275	8.027
6.215	15.172	6.210	21.041	6.205	18.181	6.212 6.208	13.737 10.52	6.215 6.203	15.118 10.650
6.105	6.212	6.102	8.061	6.108	8.238	6.104	6.186	6.099	6.462

Only in station E3, in the semidiurnal band, there are clearly detected two components with periods 12.897 and 12.846 h and amplitude close to 1 cm. The presence of these waves, included in the groups $2N_2$ (233.-236.) and μ_2 (237.-23X.), is due to the fact that they are affected by long period astronomical modulations and they cannot be separated by tidal analysis using a data length of 321 days. More specifically, component $2N_2$ is affected by modulations of period between 18.613 and 91.311 days, which are associated to the variables (*N'*, *p*, *h*, *2h*, *4h*). In the second group, the component 238.554 and the constituent μ'_2, also denoted as $2MS_2$, are the main responsible of the modulations.

There is a component with amplitude higher than 1 cm in practically all the stations whose period is 12.658 h, that was included in the group N_2 (243.-245.). However, successive analysis performed on 180 days intervals, moving the central epoch 7 days, show variations of 1 cm of the amplitude of this main component. These variations are consequence of its modulation with other waves included in the group. As it is affected by these strong modulations, with periods associated to (*N'*, *p*, *h*, *2h*, *4h*), its correct elimination is complex.

Special attention must be paid to the next group. Periods close to 12.4 h have been eliminated in the M_2 (252.-258.) band, theoretically, and that is the higher amplitude component in the area. Apart from the characteristic astronomical modulations, there are four non-linear components included in this group, that are symmetric two by two with respect to M_2. These components are MA_2 and MB_2, and MSK_2 and MKS_2. These last components are the result of the interaction of M_2, K_2 and S_2, but they do not come from their addition, but from M_2+S_2-

K_2 and M_2-S_2+K_2, respectively. Their arguments are 2τ-2h and 2τ+2h . They are also the result of the quadratic interaction of M_2 and S_{sa}:

The result of this interaction is, again, a constant term, a component with argument 4τ, coinciding with M_4, other two symmetric components to M_2, with

$$I = K''\left[H_{M_2}\cos 2\tau + H_{S_{sa}}\cos 2h\right]^2 = \frac{1}{2}K''\left(H^2{}_{M_2} + H^2{}_{S_{sa}}\right) + \frac{1}{2}K''H^2{}_{M_2}\cos 4\tau +$$
$$+K''H_{M_2}H_{S_{sa}}\cos\left(2\tau - 2h\right) + K''H_{M_2}H_{S_{sa}}\cos\left(2\tau + 2h\right) + \frac{1}{2}K''H^2{}_{S_{sa}}\cos 4h.$$

arguments 2τ-2h and 2τ+2h, denoted as MSK_2 and MKS_2, which modulates M_2, and has a period of 182.621 days. The last component, with argument *2h*, is denoted as S'_{sa}. The components MA_2, MB_2, MSK_2 and MKS_2, despite being also separated by least square analysis, could not be eliminated after this process, as will be justified in this work.

When obtaining the tidal model, the component with period 12.18 h was included in the group L_2 (265.435-265.675), which had an amplitude factor slightly greater than other semidiurnal components. This can be due either to the presence in the group of other constituents (265.555, 265.655, 265.665) with significant amplitude or to astronomical modulations with periods 8.847 years and 182.621 days, induced by the variables *p* and *2h*, respectively. Their amplitude should be taken into account, except in stations B3 and E1.

In the group of the semidiurnal components has been detected, finally, a significative harmonic component with period close to 12 h, corresponding to the wave MSN_2, with argument 2τ+3s-2h+p . It is the result of the interaction of M_2, S_2 and N_2, and modulates S_2 with a period of 8.847 years. Its amplitude is small, except for station E3. We would like to point out that the waves generated by non-linear interactions among pure tidal constituents of even species has, normally, higher amplitude that those generated by waves belonging to odd species. The exception to that is when the implied constituents have solar origin, where atmospheric and radiational terms with coincident frequencies are added to the Newtonian terms. An example is S_5, component with small amplitude, generally, that is undetected in Earth tide records, since thermal and pressure variations are not so explicit as in ocean tidal records.

The greater amplitude components in the area are M_2, S_2 and N_2 and, thus, it seems obvious that the non-linear interactions among them should also be taken into account. Consequently, some of the most interesting results have been found when considering the fourth-diurnal components, as components M_4, MN_4 and MS_4. Other non-linear interaction of fourth-diurnal or smaller period have not been detected by spectral analysis. The reason is that, due to the small resolution of this kind of analysis, they are hidden by components of higher amplitude in the tidal species.

The process described above has been followed also in the secondary station of Santander. Its aim is to contrast the dependence of non-linear interactions from geographical position and local bathymetric characteristics. The data of this station are divided in two main blocks: the first one from January 1st to September 30th, 1988 and the second one from October 1st , 1988 to January 2nd ,1990. As spectral analysis does not allow to obtain the power spectrum of observations with gaps, two different power spectra have been determined. This allows us to contrast the time dependence of the interactions. The results are represented in Fig. 3. It can be noted a generalized amplification of the components with respect to the stations of Puerto del Musel, which is a consequence of the morphology of the site. On the one hand, the results also show large differences among the amplitudes determined in certain frequencies, characterized by an amplification in the second observational interval. This amplification is specially important in the period close to MSK_2, MA_2, MB_2 and MKS_2, due to the fact that when apodizing M_2 group, it was also eliminated part of the modulation originated by them on the group. As the length of the second interval is greater, the effect of the modulation is not so important as a consequence of the analysis procedure and the fact that a finer group is allowed by the analysis method. On the other hand, the sixth-diurnal components show a considerable amplitude for periods of 4.168, 4.140 and 4.095 hours, corresponding to $2MN_6$, M_6, $2MK_6$ and $2MS_6$, respectively. These components have not been detected in Puerto del Musel. The differences are due to the importance of regional bathymetry and bottom morphology in composed harmonics generation. With respect to these components, $2MN_6$ is originated by the cubic interaction of M_2 and N_2, M_6 by M_2 and $2MK_6$ and $2MS_6$ by the interaction between M_2 and K_2 and M_2 and S_2, respectively.

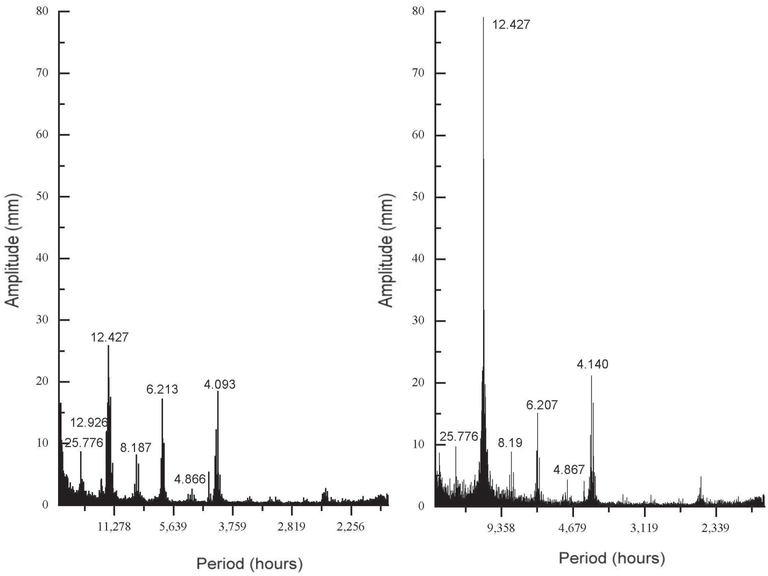

Figure 3. Tidal spectrum of the residuals at Santander station.

After studying the periodic components in mean tide level using spectral analysis, we have obtained, in a second step, the harmonic constants using numerical filters built using the aforementioned least square method. After the determination of the frequencies of the interactions, this method is more suitable, since it can be analyzed any data length, with interruptions. Thus, is possible to obtain more discriminated results. Besides, it is selective and the transitory effects are eliminated. Finally, a last and very important advantage for our purposes is the fact that for tidal frequencies is possible to obtain amplitude factors and phase lags with respect to the equilibrium tide of the components included in a tidal potential. With this aim, we have modified the standard tidal potential developments, including the composed harmonic and subharmonic components detected in the previous step (Tables 3 and 4). Using this technique and with a group separation that only takes into account the pure component, the results presented in Table 3 were obtained. The amplitudes and mean square errors (m.s.e.) are given in millimetres. Note the small magnitude of the mean square error of the determination that implies that most of the components are significant, though they are under the precision of the observations. The validity of this determination has been tested studying the temporal variability of the harmonic parameter of the corresponding waves, that must stay stable. In this study, we have performed analysis of 180 days, moving the central epoch 7 days. This procedure has been done iteratively for each component in each station, trying to separate the effect of modulation from a real interaction. Generally, the fourth-diurnal components present small variability, specially MN_4 and M_4, that are those having bigger amplitude (Fig. 4). This fact is obvious from the mean μ (in mm), the standard deviation σ (mm), the variation coefficient (%) and the maximal relative variation obtained for the different components (Table 4). The values for the fourth-diurnal waves are smaller (despite of the fact that the normalized amplification factor is greater) than those of the semidiurnal components. Let us remark the values obtained for M_4, showing that the variations are, generally, quite smooth, despite their mean variations induced by other effects. Thus, the two main fourth-diurnal components can be correctly determined and eliminated.

Table 3. Tidal analysis considering the composed and subharmonic constituents

Component			**Station B1**		Station B3		Station E1		Station E2		Station E3		Santander	
Arg.	*n*	Wave	**A**	**mse**	*A*	**mse**	*A*	**mse**	*A*	**mse**	*A*	**mse**	*A*	**Mse**
147-147	1	MP1	1.476	.192	1.918	.214	2.247	.265	.675	.083	1.692	.20	2.979	1.110
183-183	1	SO1	1.681	.191	.587	.213	1.872	.264	.495	.082	1.201	.20	1.502	1.106
237-237	1	MU2	1.270	.199	2.319	.179	.455	.261	3.884	.162	9.376	1.53	2.130	4.239
253-253	2	MSK2	7.498	.187	3.486	.214	1.928	.264	2.527	.152	40.608	1.43	28.077	3.922
254-254	2	MA2	13.106	.188	.302	.211	5.073	.263	7.538	.152	39.685	1.47	39.173	3.934
256-256	3	MB2	3.070	.163	.290	.182	3.754	.231	2.910	.132	37.313	1.27	19.809	3.402
257-257	2	MKS2	2.214	.192	2.048	.218	1.417	.271	1.622	.156	42.701	1.47	28.511	4.012
283-283	1	MSN2	1.160	.179	.860	.160	1.765	.233	.508	.145	10.193	1.37	6.866	3.798
291-291	1	2SM2	1.113	.182	1.444	.163	2.625	.236	1.632	.148	10.490	1.39	1.510	3.861
345-345	2	MO3	1.162	.092	1.176	.132	1.061	.145	.219	.058	1.675	.13	1.229	.777
363-363	1	SO3	1.475	.090	.728	.129	.182	.142	.702	.056	.404	.13	.607	.754
355-355	4	M3	.179	.093	.867	.135	.799	.150	.187	.058	.771	.13	.893	.819
365-365	1	MK3	1.504	.089	1.052	.128	1.776	.140	.681	.056	.706	.13	2.090	.751
382-382	1	S3	2.236	.086	1.717	.124	1.382	.135	1.064	.054	1.495	.12	2.506	.744
445-445	2	MN4	10.882	.064	10.913	.107	8.782	.114	10.463	.135	8.898	.11	10.047	.651
455-455	2	M4	19.661	.060	20.777	.099	1.509	.117	20.262	.115	20.342	.11	20.254	.640
463-463	1	SN4	.587	.057	.745	.094	1.133	.107	.805	.131	1.395	.10	2.149	.605
473-473	1	MS4	6.007	.054	7.570	.090	7.003	.103	6.504	.127	6.105	.10	10.604	.582
475-475	2	MK4	1.408	.054	2.419	.074	2.234	.079	1.941	.095	1.914	.07	1.881	.404
491-491	1	S4	.619	.052	1.286	.087	.499	.099	.337	.128	.642	.09	2.308	.555
555-555	1	M5	.0597	.047	.991	.096	.1198	.091	.219	.251	.983	.080	.9596	.5144
565-565	1	2MK5	.4124	.043	.906	.089	.4754	.085	.577	.233	.887	.074	1.1845	.4768
5X0-5X0	1	S5	.7735	.038	.419	.078	.2542	.075	.599	.205	.548	.066	1.7033	.4272
645-645	1	2MN6	2.406	.327	1.933	.707	2.2474	.991	1.362	.277	2.214	.506	6.8756	.9130
655-655	1	M6	3.873	.291	2.939	.629	3.0284	.882	3.471	.246	2.953	.449	14.142	.8092
673-673	1	2MS6	1.509	.248	1.828	.536	1.0497	.753	1.171	.210	1.688	.389	15.222	.6898
675-675	1	2MK6	.950	.245	1.546	.530	1.7777	.745	.709	.208	1.192	.385	6.5056	.6829
691-691	1	2SM6	.979	.225	1.058	.487	1.1969	.685	.509	.191	.602	.354	3.6549	.6280
693-693	1	MSK6	.535	.224	1.052	.485	1.3483	.682	.629	.190	.191	.352	3.8173	.6250
6IA-6IA	1	S6	1.235	.216	1.286	.469	1.4598	.655	.703	.185	.320	.336	1.2350	.6043

Table 4. Statistics on the subharmonic components.

Component	μ (mm)	σ (mm)	Variation Coefficient (%)	Relative variation
MSK_2	76.74	7.06	9.21	2.41
MA_2	126.35	11.88	9.40	2.47
MB_2	228.91	21.88	9.60	2.37
MKS_2	70.95	6.76	9.52	2.36
MN_4	1392.34	59.59	4.28	1.35
M_4	1678.77	33.19	1.98	0.61

The diurnal components have very similar significant amplitudes, though they are under the precision of the recorded data. However, the study of their temporal variability has allowed us to state that they were not correctly determined, mainly due to their small amplitude and to their inclusion in separated groups when performing the tidal analysis, since they could not be discriminated from the other components.

Concerning the semidiurnal components (see Table 3), the effect of MSK_2, MA_2, MB_2 and MKS_2 was apodized in previous steps, except for station E3 in Santander. In the selected groups separation, the group of MA_2 includes the components of argument $2\tau - h$ as $2\tau - h + p_s$. The same is applied to MB_2. There are several reasons for that. Firstly, these harmonic components are associated, since they are symmetric with respect to M_2. Secondly, in the stations where they have a greater amplitude, all the semidiurnal components have a standard deviation higher than the one of other frequency bands, which are closely related to large errors in the global analysis of the recorded data. Besides, these components were included in M_2 group and so eliminated. Thus, they cannot be considered «real» components, but «remains» of M_2. This fact has been contrasted with the study of the temporal variability of the amplitude of the semidiurnal components. This variability is quite irregular and should be taken into account, as can be seen in Fig. 4, where the changes of the composed components in Santander are represented. Note that the variations of the (normalized) amplitude factor are quite similar for MSK_2, MA_2, MB_2 and MKS_2, supposedly indicating that what is represented is the trace of the components of groups whose main component (M_2) has been eliminated. Consequently, these components were not apodized, since this process could produce more noise in the signal than gain in accuracy. This fact was confirmed when comparing the amplitudes obtained in the analysis after their theoretical elimination.

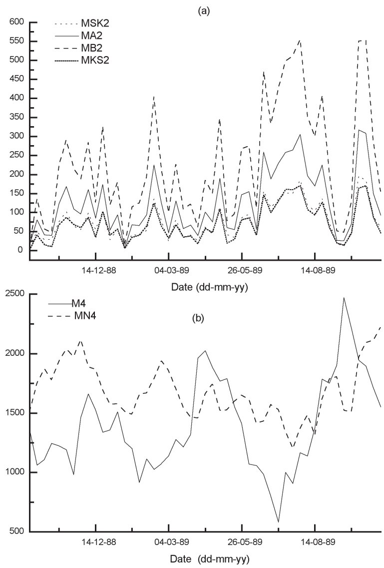

Figure 4. Normalized amplitude factor of semi- and fourth-diurnal components.

4. CONCLUSIONS

Elimination of nonlinear interactions has been performed following an iterative process, that has been contrasted using both specific software programs that we have developed and standard analysis methods. Only the fourth-diurnal components have been apodized, after proving the time stability of the elimination, as the elimination of other significant components could introduce noise in the residuals.

As an example, Table 5 lists the eliminated components, together with their amplitude *A* and phase lag *G*, with respect to Greenwich equilibrium tide. There are also listed the amplitudes obtained using harmonic analysis after the elimination of the components, A_e, and the standard deviation of the determination. These values are of some tenth of millimetre (for all the stations) and, thus, we can conclude that the elimination of non-linear interactions generated has been accurately done. Besides, the global amplitude of the apodized waves is of 3 centimetres.

Table 5. Eliminated components in station B1.

		MN4	***M4***	MS4
B1	**A**	10.882	19.661	6.007
	G	92º	191º	133º
	A_e	0.347	0.253	0.212
	eqm	0.066	0.059	0.054

With respect to the determination of the dependence of the factor K (that we have denoted as K'' or K''' to emphasize its dependence on the order of the interaction) on the frequency of the harmonic constituents. Thus, we have empirically estimated them for the higher amplitude components. In MA_2 and MB_2, the factor is multiplied by S_a amplitude, as it is also an unknown. In the same way, the determined factor for MSK_2 and MKS_2 is itself multiplied by S_a amplitude again. The results are shown in Table 6. Note that the empirical factor is very similar for all the stations, though it shows a dependence on frequencies that can also be appreciated in Fig. 5. Although in the upper part of the Figure is remarkable the factor that corresponds to MSK_2, MA_2, MB_2 and MKS_2, this result is artificial, since the represented factor is not K but it is multiplied by the amplitude of S_a. Besides, although the effect of these components was partially apodized as included in M_2 group, the results obtained can be considered optimal. With respect to the fourth-diurnal components, in Fig. 5b can be noted that the variations of factor K are quite similar for all the stations, although is slightly higher in Santander than in Gijon.

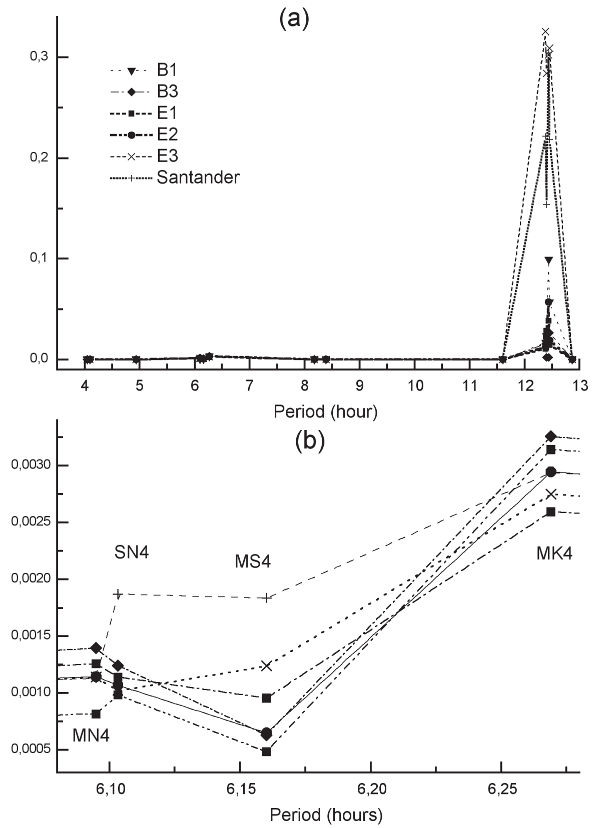

Figure 5. Variation of K with period for the different species (a) and fourth diurnal tides (b).

Table 6. Factor K of compound tides.

Comp.	B1	B3	E1	E2	E3	Santander
MU2	.0000021	.0000039	.0000007	.0000064	.0000159	.0000039
MSK2	.0567850	.0265188	.0145618	.0191249	.3094278	.2182569
MA2	.0992563	.0022974	.0383154	.0570492	.3023946	.3045117
MB2	.0232502	.0022061	.0283533	.0220235	.2843203	.1539855
MKS2	.0167674	.0155796	.0107023	.0122756	.3253762	.2216306
2SM2	.0000052	.0000068	.0000123	.0000078	.0000512	.0000081
MO3	.0000130	.0000209	.0000122	.0000025	.0000186	.0000166
MK3	.0000163	.0000078	.0000192	.0000009	.0000079	.0000240
MN4	.0031410	.0032550	.0025940	.0029440	.0027486	.0029389
SN4	.0004833	.0006297	.0009550	.0006494	.0012394	.0018351
MS4	.0009827	.0012414	.0011399	.0010681	.0010195	.0018707
MK4	.0008162	.0013937	.0012566	.0011451	.0011377	.0011310
2MK5	.0000046	.0000113	.0000055	.0000064	.0000098	.0000160
2MS6	.0000025	.0000030	.0000017	.0000019	.0000029	.0000278
2SM6	.0000046	.0000050	.0000056	.0000024	.0000029	.0000195

Finally, for a suitable local mean sea level determination, we have eliminated the shallow-water non-linear interactions from the recorded signal. Let us point out that the main achievement of this elimination is to increase 3 centimetres the accuracy and, thus, the possibility of a more precise empirical determination of the influence of other perturbing effects, as the inverted barometer effect or the wind influence. Likewise, we have shown the validity of the process in the determination of the interactions, though their amplitudes are under the precision of the records.

5. ACKNOWLEDGEMENTS

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