Robust estimation in geodetic networks

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ABSTRACT
The application of robust estimation to geodetic networks is analysed versus the classical least-squares approach. In case of gross or systematic errors appearance either in the mathematical model or in the observations to be adjusted, least-squares estimation along with detection statistical tests over the results present considerable problems for isolating them and avoiding their influence. Conversely, robust estimation provides a maximum-resistance solution and therefore the capability of identifying and quantifying them. Finally, we show the advantages of dealing with robust estimation as a global optimization problem rather than as an iteratively reweighted least squares scheme.

Key words: Geodetic networks, robust estimation, global optimization, least-squares.

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1. INTRODUCTION

When computing and adjusting geodetic networks error diagnostics is a primary matter. It is conducted to ensure that the considered observations are free from gross and systematic errors and therefore the result is only affected by the inevitable random errors that are present in every measuring process.

Obviously, precaution in the observation process has to be the first measure to avoid undesirable error appearance. Moreover, an adequate data filtering previous to the adjustment, checking reciprocal observations, closures and even with more detailed schemes, see e.g. (Cen et al. 2003; Romero and Sevilla 1990), is a suitable routine to be adopted in order to detect and eliminate the wrong observations. However, in certain unfortunate occasions, some gross or systematic errors may slip into the adjustment process.

As usually considered (Berber and Hekimoglu 2003) there are two main fundamental approaches to deal with these undesirable errors: the application of detection statistical tests and the use of robust estimation.

Since Baarda's pioneer work (Baarda 1968) several statistical tests have been developed for the detection of systematic and gross errors starting from the adjustment results. Their performance is not always satisfactory, mainly because the adjustment they are based in has been yet contaminated. These tests perform especially ineffective if there is a considerable amount of gross errors; in this case they can remain undetected and, conversely, even correct observations may be wrongly declared as unacceptable.

On the contrary, robust estimation procedures were developed to oppose maximum resistance in the solution towards the influence of gross and systematic errors. Consequently, they are not only error detection tools but estimation procedures that provide the least-affected solution.

Though the term "robust" was first used in the work of Box (1953) many later contributions have given form to the present theory: (Andrews et al. 1972; Huber 1981), etc. In geodesy and related sciences it has been widely applied in the last decades: (Krarup et al. 1980; Fuchs 1982; Xu 1989; Krauss 1992; Harvey 1993; Yang 1999; Domingo 2001; Wieser and Brunner 2001, 2002), etc. Nevertheless there is also a place for new methodological developments such as the case that we will present later: solving the estimation problem by global optimization techniques.

Hence, the comparison between classic least-squares estimation and robust estimation may be stated as following:

– When the model is correct and the observations are affected only by random errors then the classic least-squares estimation yields the most likely solution. Robust estimation will provide a suboptimal result close to it.
– When systematic or gross errors affect the observations, or the model is not correct but only approximately correct or even incorrect, robust estimation will provide the best possible solution, whereas least-squares will yield a highly contaminated solution. However, as we will see, computing robust estimation by means of a global optimization technique rather

than by iteratively reweighted least-squares (which is the usual routine) will prove often to be essential.

Finally, examples of real geodetic networks will be given to illustrate those critical advantages of robust estimation versus least-squares estimation in case that undesired errors enter into the adjustment stage.

Therefore we will consider as the most suitable methodology for the geodetic network adjustment *the joint use of classic least-squares estimation and robust estimation*. Thus, in case there are only present random errors then least-squares will provide the best solution whereas robust estimation will give a result very close to it, so that it will support its validity. Conversely, if systematic and/or gross errors affect the observations, and/or there is a systematic error in the model, least-squares will prove to be inadmissible whereas robust estimation will provide the best possible solution so that the errors rest isolated and quantified.

2. LEAST-SQUARES ESTIMATION AND STATISTICAL ERROR TESTS

Let us summarize briefly the formulation in use when computing a classic geodetic network adjustment.

The functional or mathematical model, which expresses the relationships between observed data and unknowns, can be defined by the system of equations

$$\mathbf{Ax} = \mathbf{l} + \mathbf{v} \tag{1}$$

where **A** is the coefficient matrix for the **x** vector of unknowns, **l** represents the vector of observed data, or differences between observed and approximate data, and **v** is the residuals vector.

Observations are commonly supposed to follow normal distributions and therefore their statistical behaviours are modelled by means of a variance-covariance matrix that is usually noted by $\mathbf{\Sigma_l}$. Its inverse provides the weighting matrix

$$\mathbf{P} = \mathbf{\Sigma_l}^{-1} \tag{2}$$

Thus, the most likely solution for the system (1) along with the weighting matrix (2) is obtained by the least-squares estimator

$$\min(\mathbf{v}^T \mathbf{P} \mathbf{v}) \tag{3}$$

which provides the unknowns vector

$$\mathbf{x} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P} \mathbf{l} \tag{4}$$

If the appearance of any gross error in the observations is suspected it is possible to apply the classic τ-test from the adjustment results. This test analyses the studentized residual

$$\tau_i = \frac{v_i}{\hat{\sigma}_{vi}} \sim \tau_{n-r} \tag{5}$$

where $\hat{\sigma}$ is the standard deviation obtained after adjustment for the residual v_i and τ_{n-r} denotes the τ-distribution with $n - r$ degrees of freedom, which is related to the Student's t-distribution by

$$\tau_{n-r} = \frac{\sqrt{n-r}\, t_{n-r-1}}{\sqrt{n-r-1+t^2_{n-r-1}}} \tag{6}$$

Observation *i* is rejected at a significance level of α (for instance α = 0.05) if the studentized residual τ_i exceeds a critical limit *c* being α = P [τ > c]. The significance level represents then the probability for a type I error to have taken place.

If $|\tau_i| < c$ then the zero hypothesis (no gross error is present at *i* observation) is admitted at a level of significance α. Otherwise the zero hypothesis is rejected and the alternative admitted at a level of significance α.

It has been demonstrated (Baselga 2003) that this procedure is affected by a critical limitation. For a particular network and a particular observation affected by a gross error, $|\tau_i|$ converges to a maximum value. Sometimes this value will be lower than any critical value *c* that can be sensibly chosen (for instance α = 0.05 yields *c* = 3.57). Therefore in some cases, which we should be aware of, the τ-test will not be significant.

Hence, if the observable *i* is affected by a gross error then

$$\lim_{\varepsilon \to \infty} \tau_i = constant \tag{7}$$

where this constant is of critical interest for each case in relation with the adopted *c* value. Denoting by e the gross error, a compact expression can be obtained for this limit in terms of matrices of the observing equations:

$$\lim_{\varepsilon \to \infty} \tau_i = \frac{\left[(\mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}-\mathbf{I})\mathbf{e}_i\right]_i}{\sqrt{\left[\frac{1}{n-r}\mathbf{Q}_{vv}\left[(\mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}-\mathbf{I})\mathbf{e}_i\right]^T\mathbf{P}\left[(\mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}-\mathbf{I})\mathbf{e}_i\right]\right]_{ii}}} \tag{8}$$

where $\mathbf{e_i}$ represents the null vector except by the *i*th-element which equals one. The subsets *ii* or *i* represent the corresponding elements of a matrix or vector.

In general, it is possible for statistical error tests applied over the least-squares solution to fail in the correct blunder detection just because the least-squares solution they rely on has been yet contaminated. Moreover, as it is well-known, the detection becomes much more difficult in case that not only but many undesired errors are present. Correlation between the residuals makes hard to detect the observables with error and however, it is possible for correct observations to have the largest residuals (Chen et al 1987, Núñez-García 1989, Berber and Hekimoglu 2003).

As a consequence, other reliable approach should be provided at least as a complement to these statistical tests applied over the least-squares solution.

3. ROBUST ESTIMATION

In every adjustment process of some observational data to a predefined model there are inevitably some *apriori* assumptions. First, the proposed model is a mere hypothesis establishing the way observations and unknowns are related. Also the observations underlying distribution and the independence of data are usually postulated.

All of these assumptions are not *exactly* correct, but simplifications of a much more complex reality, perhaps not capable of being perfectly represented by means of algebraic equations (much arguing could be devoted to this topic). However, a main principle underlies the adoption of the adjustment model (both in its mathematical and statistical parts) and the derived results: the minimum discrepancies versus the reality only cause minimum variations in the solution.

Unfortunately this is not always the case. In particular, the least-squares adjustment considerably amplifies the *apriori* existing differences. Therefore robust estimation should be used instead.

Let us remember that an estimator for the parameter x based on the variables $m_1, m_2, \ldots, m_n$ is a function which has been defined for every possible set of values $m_1, m_2, \ldots, m_n$

$$\hat{x} = \hat{x}\ (m_1, m_2, ..., m_n) \qquad (9)$$

The obtained value for the set m_1, m_2, m_n is known as estimation.

An estimator is *unbiased* if its expected value equals the estimated parameter

$$E\ (\hat{x}) = x \qquad (10)$$

Besides, an estimator is said to be *consistent* if it tends exactly to the estimated parameter as the sample becomes larger, i.e.

$$\lim_{n\to\infty} E(\hat{x}) = x \qquad \lim_{n\to\infty} \sigma_{\hat{x}} = 0 \qquad (11)$$

An unbiased estimator is said to be more *efficient* as its variance is smaller. In particular, if and are estimators of x with s_1^2 and s_2^2 variances then the first will be said to be more efficient than the second if

$$\sigma_1^2 < \sigma_2^2 \qquad (12)$$

Finally, an estimator is *sufficient* if it considers all the information contained in the sample, i.e. $\hat{x} = \hat{x}\ (m_1, m_2, ..., m_n)$ is sufficient if $P\ (m_1 = M_1, m_2 = M_2, ..., m_n = M_n \mid \hat{x} = a)$ does not depend on x.

Robustness can be added to these classical desirable properties for estimators.

An estimator is more *robust* the greater it is its insensitiveness towards the *apriori* assumption variations. These discrepancies may derive from the non-fulfilment of the statistical model (observations affected by gross and/or systematic errors) or the functional model (which is also a systematic error). Therefore, not only we are seeking the most insensitiveness towards possible gross errors in the observations, (Huber 1981) denotes this feature as *resistance*, but also the underlying distribution and the mathematical model are being questioned.

Notice that in robust estimation the main principle is "security" rather than "efficiency". As a consequence, an estimator that provides a non-optimal solution close to the correct value is preferred instead of a much efficient estimator that provides an absurd solution when any assumption does not fulfil.

The classic mathematical definition of a robust estimator is that provided by (Hampel 1971) which is summarized next.

Being the independent observations $m_1, m_2, \ldots, m_n$ with common distribution f and the sequence of estimations

$$\hat{x}_i = \hat{x}_i\ (m_1, m_2, ..., m_n) \qquad i = 1, 2, ..., t$$

This sequence is said to be robust for $f=f_o$ if the sequence of distribution applications $f \rightarrow \ell_f(\hat{x}_n)$ is equicontinuous in f_o; that is if for each $\varepsilon > 0$ exists $\delta > 0$ and $n_o > 0$, so that for any f and any $n > n_o$

$$d_*(f_o, f) \leq \delta \quad \Rightarrow \quad d_*\left(\ell_{f_o(\hat{x}_n)}, \ell_{f(\hat{x}_n)}\right) \leq \varepsilon$$

where d_* is any metric generating weak topology.

In the study of a robust estimator performance it is essential the influence curve analysis. This function, also following (Hampel 1971), is

$$IC(f, \hat{x}) = \lim_{t \to 0} \frac{\hat{x}(f_t) - \hat{x}(f)}{t} \qquad (13)$$

Other derived functions (Tukey sensibility curve, jacknife, etc.) are beyond the scope of the present paper. For further explanations refer to specific bibliography, for instance: (Huber 1981), (Hampel 1971), (Andrews et al. 1972) and especially (Davies 1993) who provides a thorough and updated view.

Robust estimators can be classified as:

– M-estimators or Maximum likelihood estimators.
– L-estimators or Linear combination of order statistics estimators.
– R-estimators or Rank-test derived estimators.

Among them, M-estimators are the most flexible and the easiest to generalize to multiparameter cases. This is why they are the most commonly used and the ones we shall refer to. Some of them are:

– L_p-norms: as for instance the L_1-norm which yet Laplace proposed to use long ago and the L_2-norm that in fact is very little robust.
– The Huber estimator
– The Andrews estimator
– The Danish method
– Other: Geman-McClure, Tukey o Welsch estimators.

The general scheme for the use of an M-estimator is the following.

Let O_1, O_2,... , O_m be independent observations to be adjusted following certain established model (1). The residuals of the adjustment are obtained by means of

$$\min \rho(v) \tag{14}$$

where the score function ρ(v) is an estimator characteristic. For the case of least-squares $\rho(v) = \Sigma v_i^2$.

Then the influence curve (13) can be represented as:

$$IC(v) = \frac{d\rho(v)}{dv} \tag{15}$$

3.1. Solution by Iteratively Reweighted Least-Squares (IRLS)

In order to solve the mathematical model (1) along with the minimum condition (14) for the selected robust estimator, the iteratively reweighted least-squares (IRLS) method is the most usual choice. The score function ρ(v) is then the used for classic least-squares considering the following weights

$$\omega(v) = \frac{IC(v)}{v} = \frac{1}{v}\frac{d\rho(v)}{dv} \tag{16}$$

Some widely used robust estimators are presented in Table 1 with its corresponding score functions ρ and equivalent weight functions ω.

Table 1. Some M-estimators.

Estimator		ρ(v)	ω(v)
Huber	If $\|v\| \le k\sigma$	$v^2/2$	1
	If $\|v\| > k\sigma$	$k\sigma(\|v\| - k\sigma/2)$	$k\sigma/\|v\|$
L_1		$\|v\|$	$1/\|v\|$
$L_1 - L_2$			
Fair		$c^2[\|v\|/c - \log(1+\|v\|/c)]$	$1/(1+\|v\|/c)$
Geman-McClure			
Tukey	If $\|v\| \le c$	$c^2/6.(1 - [1 - (v/c)^2]^3)$	$[1 - (v/c)^2]^2$
	If $\|v\| > c$	$c^2/6$	0
Welsch		$c^2/2.[1 - \exp(-(v/c)^2)]$	$\exp(-(v/c)^2)$

The procedure can be summarized as follows

1) First a classic least-squares adjustment using unit weight (or *apriori* estimated weights) is performed.
2) From the adjustment residuals new weights are computed by means of the corresponding estimator weight function (or the *apriori* estimated weights are rescaled by that value)
3) A new least-squares adjustment is performed along with the new weights. Then one must return to step 2 until two consecutive solutions are considered to be identical.

In the case of correlated observations two different approaches can be followed. One (Baselga 2003) is to first diagonalize the weight matrix and accordingly transform the system of equations, then solve as an independent observations problem and finally undo the transformation. Another possibility is to adopt a convenient procedure that accounts for correlated data, see the studies by (Xu 1989), (Yang 1994), etc.

This is the common procedure to calculate the robust estimation. However, one must notice that solving the robust estimator by means of different weighted least-squares adjustments, although easy to compute, undermines robust estimation capabilities.

Hence, to a certain extent, one is recovering the least-squares main problem: its easiness in spreading undesirable errors all along the solution. In fact, the weighting process accounts for eliminating the influence of gross or system errors but the weights are always computed from a previous and possibly highly contaminated least-squares adjustment.

Therefore, IRLS though easy to compute may not be the best option to perform robust estimation when the real solution lies *far away* from the initial solution of the perturbed system and observations. As we will see in the examples, in certain cases robust estimation is not that robust unless considered as a Global Optimization (GO) problem rather than as an IRLS problem, which only seeks the local optimum. Therefore we have concentrated on applying GO techniques to robust estimation and testing how far it is an improvement on the IRLS approach.

3.2. Solution by Global Optimization techniques (GO)

Instead of following the classical procedure for computing robust estimation by means of an iterative least-squares process with variable weights, we propose an improvement in the process robustness: the application of global optimization techniques (GO) to the robust estimation computing process.

GO techniques can be divided in three major categories: simulated annealing, genetic algorithms and interval arithmetic based techniques.

Simulated annealing was first proposed by (Metropolis et al. 1953) as an iterative heuristic method analogous to the process of crystalline network self-construction. This Monte Carlo nature method was then exploited by (Kirkpatrick et al. 1983), who established the present form of the algorithm. Since then it has been widely applied in a variety of GO problems, see e.g. (Berné and Baselga 2004) in which formulation is explained and later applied to the First-order design problem.

Genetic Algorithms were proposed by (Holland 1975) to emulate the natural evolution of species. Natural selection rules are applied in this iterative and heuristic method to search for the global optimum. Thus, this more adaptive individual is found after the evolution of a system in which heredity, crossing, mutation and survival rules have been implemented. For further details consult the specific book by (Man et al. 1999).

Finally, interval arithmetic based techniques, whose foundations lie in (Moore 1966), are very reliable although sometimes very inefficient deterministic methods for finding the global optimum. A comprehensive explanation is given in the book by (Floudas 2000).

Recently Xu (2002, 2003) proposed a new successful hybrid global optimization method consisting in finding first one point in each nonconvex feasible region and then applying a local optimization procedure.

Any of these GO methods will prove to be useful in solving the functional model defined by the system (1) along with the considered robust estimation function (Table 1), which reads

$$\begin{cases} \mathbf{Ax} = \mathbf{l} + \mathbf{v} \\ \min \rho(\mathbf{v}) \end{cases} \tag{17}$$

In the following example we will see the clear advantages of solving the system of equations (1) by robust estimation rather than by least squares estimation. Besides, once robust estimation has been established as the most convenient alternative, we will contrast the improvement of solving robust estimation system (17) by a GO method rather than by IRLS techniques as usually proposed.

4. EXAMPLE

The following network has been observed partly with GPS techniques (Northern part) and partly with classical terrestrial observations (Southern part). Its average distance is about 1 km and can be considered as a least-constrained network for its point E belonging to the regional 4th order Comunidad Valenciana network is the only fixed point (Fig. 1).

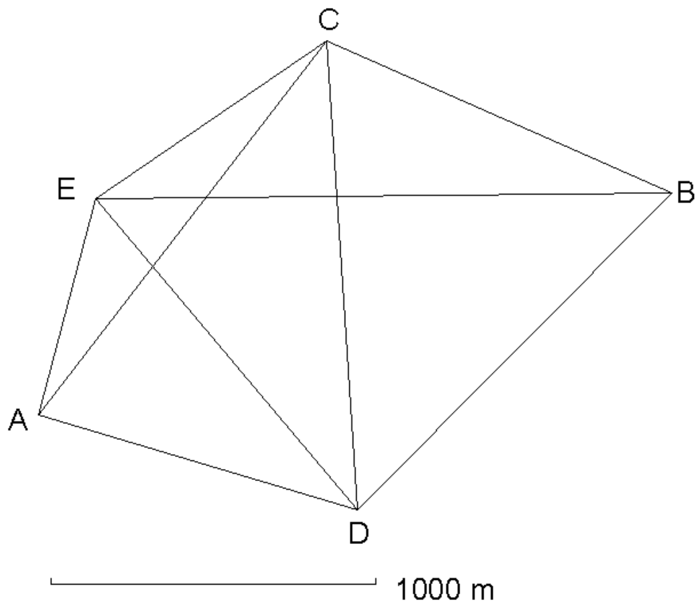

Figure 1. Network plot

Firstly, let us consider the network adjustment for observations only affected by random errors. As known, the classic least squares adjustment provides the most likely solution whereas any robust estimator yields a sub-optimal solution close to the least-squares estimation. We will consider along with the least-squares estimator the least sum of residuals estimator (i.e. the L_1 norm) and the Huber estimator as representing robust estimators.

Secondly we investigate the case when a gross observing error has slipped into the adjustment phase. The least-squares estimation may then be more or less sensitive to this error depending on its magnitude, network redundancy and the particular observation that is affected. If a single error provides a whole *contaminated* least-squares solution and the classical detection tests over this solution fail to detect the gross error (remember there is a problem with the variable convergence) then robust estimators will be the only successful alternative.

Finally, if there are many observing errors or they are located in observations with very low redundancy, the correct solution may lie *far away* from the initial least squares solution and therefore robust estimation based on IRLS may not result in the global optimum but only in a local one. In this case robust estimation will only be truly robust if considered as a GO problem.

4.1. Initial case

If the observations are only affected by random errors, the least squares estimation is the most likely solution. In Table 2 adjusted coordinates are given for the least squares solution and some robust estimates: the Huber estimator obtained by IRLS, the L_1 norm obtained by IRLS and the L_1 norm computed by simulated annealing.

Table 2. Adjustment results with the Least Squares estimator, Huber estimator, L_1 estimator and L_1 estimator solved by a Global Optimization method.

	LS estimator	Huber estimator	L_1 estimator	L_1 estimator (GO)
X_A	4918540.874	4918540.874	4918540.870	4918540.864
Y_A	-22596.126	-22596.126	-22596.125	-22596.126
Z_A	4047070.304	4047070.304	4047070.301	4047070.296
X_B	4918149.238	4918149.238	4918149.240	4918149.242
Y_B	-20624.838	-20624.838	-20624.840	-20624.838
Z_B	4047551.407	4047551.407	4047551.411	4047551.407
X_C	4917831.564	4917831.564	4917831.563	4917831.565
Y_C	-21669.275	-21669.275	-21669.278	-21669.275
Z_C	4047939.889	4047939.889	4047939.893	4047939.890
X_D	4918746.721	4918746.721	4918746.741	4918746.737
Y_D	-21622.251	-21622.251	-21622.250	-21622.251
Z_D	4046821.884	4046821.884	4046821.898	4046821.895

Least square results are satisfactory, with sensible residuals and an acceptable reference factor variance of $\sigma_0^2 = 0.70$. However, robust estimation procedures yield sub-optimal solutions, i.e. they are only relatively close to the least squares result, which is the most likely and therefore the *correct* one.

4.2. Synthetic case: a single gross error

Let us simulate a gross error in a single observable, for example the horizontal direction from D to A, which has medium redundancy ($r_i = 0.43$). We will show that this single error spreads through all the least-squares solution and remains undetected by the τ-test even for an infinite error value. Conversely, robust estimators will find a solution least affected by the presence of this error, which will be confined to its residual.

Table 3 shows the different residuals for the observation affected by a gross error of + 100 centesimal seconds, +500 centesimal seconds and finally +5 centesimal degrees. The least-squares residuals are quite far from the simulated errors. Discrepancies between simulated values and adjusted values are expected as representing the random error of the observation; however, the observed discrepancies between least-squares residuals and simulated values are high and therefore unacceptable as representing random errors in the observations. Furthermore, residuals for the remaining observations increase significantly making the whole solution unacceptable. Moreover, the τ-test is unable to detect any gross error in this observation, even for an infinity error value, which according to (8) will yield $\tau = 2.6458$, far from any sensible critical value that could be set (for instance, $\alpha = 0.05$ gives $c = 3.57$).

Table 3. Adjustment results with the Least Squares estimator, Huber estimator, L_1 estimator and L_1 estimator solved by a Global Optimization method, for three different simulated errors in the same observation. Values in centesimal seconds.

Simulated gross error in D-A direction	LS estimator		Huber estimator	L_1 estimator	L_1 estimator (GO)
Residual	τ	Residual	Residual	Residual	
+100	-43	-2.3703	-72	-89	-99
+500	-216	-2.6325	-472	-489	-501
+50000	-21560	-2.6457	-49972	-49989	-50001

Conversely, the Huber estimator performs well and even much better the L_1 norm in its two calculations.

4.3. Synthetic case: two gross errors, one with low observing redundancy

Let us add a second gross error to the network and evaluate the different solutions. If we choose an observable with very little redundancy the network will be much less influenced by this wrong observation, but the error will remain undetectable... at least by classical IRLS robust estimation and obviously by statistical tests over the least-squares adjustment.

Zenithal angle from E to A has a very low redundancy ($r_i = 0.08$) in the network adjustment. Along with the former +100 centesimal seconds gross error in D-A horizontal direction let us simulate another error of +500 centesimal seconds in E-A zenithal angle. This second error will prove to be totally undetectable for the τ-test over the least-squares estimation and even for robust estimation performed by IRLS due to the fact it has little influence on the network inner consistence. However, results in terms of parameters can significantly vary and therefore the most reliable adjustment has to be ensured. Such an adjustment has to search for the score function global optimum and not only for a local optimum in the neighborhood of the initial most perturbed least-squares solution provided by IRLS.

If we compare the results of Table 4 we can conclude that the values for the horizontal direction are almost the same as in the previous example, i.e. unacceptable results for least-squares estimation and acceptable for IRLS Huber and especially for IRLS L_1-norm and L_1-norm with GO methods. However, for a very low-redundant observable such as E-A zenithal angle, least-squares solution is very far from the correct value and therefore Huber and L_1-norm estimators based on this first solution are not able to find the global optimum (indeed attained by GO), which is much farther away than the local optimum they obtain.

Table 4. Adjustment results with the Least Squares estimator, Huber estimator, L_1 estimator and L_1 estimator solved by a Global Optimization method, for two simulated errors. Values in centesimal seconds.

Observation	**Simulated gross error**	LS estimator	Huber estimator	L1 estimator	L1 estimator (GO)
		residual	residual	residual	residual
D-A hor. direction	**+100**	-44	-73	-90	-101
E-A zenithal angle	**+500**	-31	-13	0	-516

5. CONCLUSIONS

Least-squares adjustment of a geodetic network is the best alternative when neither gross nor systematic errors affect the observations nor the mathematical model. The least-squares adjustment provides then the most likely solution for the network.

On the contrary, i.e. if any systematic and/or gross error has occurred, the least-squares solution will prove to be very perturbed by them and provide thus unacceptable results. In that case, statistical tests for error detection can be applied over this solution but its efficiency is often limited, whereas robust estimation will offer maximum resistance against these undesired errors in the correct solution determination.

Moreover, it has been experienced that the usual procedure for computing the robust estimator by means of a iteratively reweighted least-squares adjustment (IRLS) becomes less effective in problematic cases due to the fact that actually it relies on the contaminating least-squares adjustment process. Instead, we concluded that robust estimation is only truly robust if dealt with as a global optimization problem (GO) and consequently proposed to directly solve the robust estimation by means of global optimization techniques.

Finally, as a summary, we propose the joint use of least-squares and robust estimators (by GO) for the geodetic network adjustment problem. When only random error occurs the least-squares adjustment will yield acceptable and sensible results, as being the most likely solution, whereas robust estimators will give suboptimal values very close to that of least-squares thus reinforcing their validity. If, conversely, any system or gross errors have affected either the mathematical model or the observations, robust estimation will be able to obtain the correct solution and also to identify this errors.

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