EVALUATION OF SITE EFFECTS IN SEDIMENTARY BASINS.

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ABSTRACT

Different techniques to evaluate seismic site effects in sedimentary basins are reviewed. Broadly speaking, the methods used to analyse the seismic response of a certain place can be classified as empirical, semi-empirical and numerical. Empirical techniques basically estimate site effects with the data recorded by seismic stations, whereas semi-empirical methods use small earthquake records as empirical Green’s functions to simulate strong motion accelerograms. Among the numerical techniques we have distinguished three main groups of methods: a) Domain methods, b) Ray theories and gaussian beams expansions, and c) Boundary methods. A selection of some of the contributions using these methods, with a wide set of references, is also presented for the interested reader.

1. INTRODUCTION

The seismic motion that takes place in a certain site depends essentially on three factors (see figure 1): 1) the rupture characteristics of the fault (source), 2) the trajectories travelled by the seismic waves once emitted (path), and 3) local geology and topography conditions (site).
The source mechanism controls the distribution of the energy radiated in space and time. This energy spreads as seismic waves through the Earth, which are then affected by the differences in the mechanical properties of the media they pass through, causing reflection, refraction and diffraction phenomena. Moreover, when these waves travel near the surface, their spectral and temporal properties are likely to change significantly. These processes are not the same for different surface points and depend on the local and specific conditions of each place.

![Figure 1. Factors that contribute to seismic motion](image)

In seismic engineering, it is very important to study ground motion in the range of frequencies from 0.1 Hz to about 20 Hz. Also, it should be noticed that values of seismic wave propagation velocities near the terrestrial surface range from around 200 m/s to 2000 m/s. This means that the wavelengths associated to these motions vary from 10 m to about 20 km. When the dimensions of the uppermost geologic structures, (such as sedimentary basins, mountains, canyons, etc.) are comparable with these predominant wavelengths, important changes are likely in the seismic motion on the irregularities and on their proximities.

Local conditions, therefore, should be kept in mind in the studies of seismic risk, seismic microzonation, and in the seismic design of structures of social or industrial interest. In particular, these conditions play a very important role in the seismic response of big structures as bridges, dams, energy facilities, constructions of social interest, etc.

There follows a description of the most common techniques that have been used in this kind of studies, as well as a reminder of some of the contributions obtained using these methods.
2. EXAMPLES OF EARTHQUAKES WITH IMPORTANT LOCAL EFFECTS

Throughout history, many of the earthquakes known as destructive have been influenced by local effects. Some noticeable examples are: the Andalusian earthquake of 1884 in southern Spain; the San Francisco (USA) earthquakes in 1906 and 1957; those of Kanto, Tonankai, Niigata and Kobe (Japan) in 1923, 1944, 1964 and 1995, respectively; the earthquakes of 1957, 1975, 1985 and 1989 in Mexico; in Caracas (Venezuela) in 1967; in Blackish Hill (USA) in 1989, and in Luzon (Philippines) in 1990.

One of these examples was the earthquake of Michoacan (Mexico) on September 19th, 1985, with magnitude Ms = 8.1. Although the epicentre was located 400 km away from Mexico City, the earthquake took more than 10,000 lives and more than 1,000 constructions were destroyed (Beck and Hall, 1986). However, in the epicentral area only light damages appeared in engineering structures (Lomnitz and Castaños, 1985). The records of strong motion obtained during this earthquake in Mexico City showed considerably longer durations in stations located on lacustrine materials than those located on hills. Another observation concerns relative spectral amplifications of up to 50 times for the range of frequencies from 0.2 to 0.7 Hz (Singh et al., 1988). Similar effects have appeared in other earthquakes that have taken place in Mexico City before and after 1985.

As a result of these amplifications, great efforts had to be made to understand the seismic response of the Valley of Mexico in the following years (e.g. Flores et al., 1987; Bard et al., 1988; Sánchez-Sesma et al., 1988; 1993b; Singh et al., 1988; Kawase and Aki, 1989; Seligman et al., 1989; Pérez-Rocha et al., 1991; Chávez-Garcia, 1991; Singh and Ordaz, 1993; Mateos et al., 1993a; Chávez-Garcia and Bard, 1994; Fäh et al., 1994, Chávez-Garcia and Cuenca, 1996) as well as the effects related with the source and the path (e.g. Campillo et al., 1988; 1989; Ordaz and Singh, 1992; Chávez-Garcia et al., 1994; Fäh et al., 1994). Moreover, other authors have considered these site effects on the soil-structure interaction in the Valley of Mexico (Avilés and Pérez-Rocha, 1998).

Many of the models proposed to explain the seismic response of the valley have considered elastic properties and have been unable to reproduce the long durations starting from the accelerations registered in the hill area. There is only one exception, namely in the model proposed by Flores et al., (1987), Seligman et al., (1989) and Mateos et al., (1993a). These authors proposed that the trapping of horizontal P waves inside the silts was the cause of the monochromaticity and the long duration of the motion observed on the
lacustrine materials. However, neither Flores et al., (1987) nor Seligman et al., (1989) gave a quantitative explanation on how it happened. Mateos et al. (1993a) proposed an argument consisting essentially of three parts: 1) On a layer overlying a halfspace, for certain frequencies and under the incidence of $SV$ plane waves beyond the critical angle, great amplifications of the horizontal motion occur in the free surface, 2) This amplification is due to the generation of horizontal $P$ waves in the layer, and 3) When lateral boundaries are added to the flat layer, forming a closed basin, the previous horizontal $P$ waves resonated, causing great amplifications inside the basin. However, Sánchez-Sesma and Luzón (1996,1997) in an attempt to understand the mechanism of generation and propagation of horizontal $P$ waves in a sedimentary basin showed that: 1) although the first part is a very well-known fact (Burridge et al., 1980; Shearer and Orcutt, 1987; Kawase et al., 1987; Mateos et al., 1993b), 2) the contribution of horizontal $P$ waves to the horizontal motion at the free surface is small, and 3) the third part of the argument is not certain. Sánchez-Sesma and Luzón (1996,1997) concluded that the only stable waves that can produce lateral resonances in a rather shallow basin are the surface waves. This theoretical study has been confirmed by Shapiro et al. (2001) who, working with real records, observed that the wavefield in the lake-bed zone in Mexico City is dominated by higher-mode surface waves.

Another earthquake in which local effects were observed was that of Spitak (Armenia) in December 7th, 1989, of magnitude $Ms = 6.8$. In this case, the city of Leninakan (located on an extensive alluvial basin 32 km away from the epicentre) suffered significant damage, whereas other cities such as Kirovakan (located in a mountainous region on compact rock 25 km from the rupture surface) suffered far less damage (Borcherdt et al., 1989). The studies performed with the records of the aftershocks of this earthquake showed relative amplifications of up to 30 times in the interval from 0.4 to 2 Hz (Borcherdt et al., 1989).

Another earthquake where important local effects have been observed occurred at Loma Prieta (USA) in October 17th, 1989 with magnitude $Ms = 7.1$. In the epicentral region, the accelerations were relatively independent of the surface geology, and starting 50 km from the epicentre, a strong dependence was observed between the local geology and the amplitudes of ground motion. This difference was clear enough in the Marina district and more specifically in the driveways between San Francisco and Oakland, where differences of more than 200% in the value of the acceleration were obtained, in comparison with those in areas of more compact materials in the city (Borcherdt, 1990). The azimuthal dependence of the source was also
evident in sites of similar geologic conditions: to the north-west of the epicentre (San Francisco and Oakland) bigger accelerations were observed than those recorded towards the north (San Jose, Hayward and the valley of Livermore).

Another example is the January 17th, 1994 Northridge (California) earthquake (e.g., Teng and Aki, 1996). This was a magnitude 6.7 earthquake that heavily shook the communities throughout the San Fernando Valley and the Simi Valley. This earthquake proved to be the most costly earthquake in United States’ history to this date (with estimated losses of 20 billion dollars). The same day one year later, the Hyogo-ken Nambu earthquake, with magnitude Ms = 6.8, struck the city of Kobe, Japan. This was the most destructive earthquake in the last 80 years in Japan, since the 1923 Kanto earthquake. The earthquake caused over 5000 deaths and estimated losses in the city of nearly 200 billion USA dollars (e.g., Somerville, 1995). The peak velocity of the near field ground motions from the main shock recorded at two sediment sites was far larger than that recorded at the adjacent rock sites (Toki et al., 1995). Pitarka et al. (1997) suggest, by modelling the ground motion for an aftershock, that the observed spatial amplitude variation may be attributed mainly to basin edge effects.

Once these kind of seismic local effects were identified as the causes of great damages in many earthquakes around the world, the International Association of Seismology and Physics of the Earth’s Interior (IASPEI) and the International Association for Earthquake Engineering (IAEE) formed a Joint Working Group on the Effects of Surface Geology. This group proposed that a set of international “blind” prediction experiments were the best way to assess the various methods used to predict local effects. The first experiment was performed using weak motion data recorded at Turkey Flat in California (USA). The second one was conducted using weak and strong motion data from the Ashigara Valley (Japan). The results of both experiments were presented at the International Symposium on the effects of surface geology on seismic motion, held in Odawara (Japan) in 1992 (IASPEI-IAEE Joint Working Group on ESG et al., 1992). The second, and latest ESG Symposium was celebrated in Yokohama (Japan) in 1998. At this Symposium, recent studies taking into account advances in this field since 1992 were summarized (IASPEI-IAEE Joint Working Group on ESG et al., 1998). The meeting also focused on systematically comparing prediction methods and results through the simultaneous simulation of ground motion for the Kobe earthquake using common data distributed prior to the Symposium.
3. METHODS FOR THE ESTIMATION OF SEISMIC SITE EFFECTS

Although analytic solutions have been obtained for the surface motion of alluvial valleys with simplified geometries (e.g. Trifunac, 1971; Wong and Trifunac, 1974, Lee, 1984), more realistic problems require a different approach in order to obtain useful results. Therefore, non-analytical methods have been developed to analyse the seismic response of a certain place. Broadly speaking, the methods used can be classified as empirical, semi-empirical and numerical.

Empirical Methods

In this kind of methods the basic material used in the estimation of site effects are the empirical data recorded by seismic stations. Usually two types of data are used: 1) earthquakes and/or explosions, and 2) microtremors.

The method of the spectral ratios of earthquakes and/or explosions is one of the most reliable experimental methods for characterising the effects of local geology on seismic motion. The method was described by Borcherdt and Gibbs (1970). The technique consists of dividing the Fourier amplitude spectra (usually the horizontal component), of a record in an area of silts by the corresponding one recorded on hard rock. Let us assume that \( B(f) \) is the Fourier amplitude spectra of the ground motion that occurred during an earthquake at a place located in a sedimentary inclusion, which can be expressed as \( B(f) = S(f) \cdot P(f) \cdot G(f) \), where \( S(f) \), \( P(f) \) and \( G(f) \) are the source, path and site factors, respectively, and that \( R(f) \) is the corresponding spectra of the motion at hard rock for the same earthquake, which can be represented as \( R(f) = S(f) \cdot P'(f) \), where \( P'(f) \) is the path factor in the motion. If the distance between the sites of motions \( B(f) \) and \( R(f) \) is small enough compared with the epicentral distance, it can be assumed that the path’s factors, \( P'(f) \) and \( P(f) \), are almost equal. Then the site influence can be estimated by the following spectral ratio:

\[
\frac{B(f)}{R(f)} = \frac{S(f) \cdot P(f) \cdot G(f)}{S(f) \cdot P'(f)} \approx G(f) \quad (1)
\]

The results are relevant when the average of spectral ratio is calculated over many events (Field and Jacob, 1995). The relative amplification of an area regarding another can be obtained with this technique. This quotient is also called Empirical Transfer Function, and it provides the resonant
frequencies of the place and an estimation of the amplification levels at these frequencies. The technique has been used in many areas around the world with records of nuclear explosions and/or earthquakes: Chávez-García et al., (1990) near Thessaloniki (Greece), Morales (1991) in the Granada basin (Spain), Frankel et al. (2001) in San José (Santa Clara Valley, California), Satoh et al. (2001b) in and around the Sendai basin (Japan).

Furthermore, the horizontal to vertical spectral ratio of the seismic record (earthquake or explosion), provides information about the geological structure beneath the seismic station. This quotient (known as Receiver Function), when computed in a station located in a sedimentary inclusion, informs about the influence of the local geology of the site. This method was used by Shapiro et al (2001) to obtain the most relevant resonant frequencies of the lake-bed zone in Mexico City. Satoh et al. (2001b) also used it in the Sendai basin (Japan) and compared the results with other empirical techniques.

The regression analysis methods apply statistical techniques to records obtained during earthquakes in order to determine the correlation degree that exists for a dependent variable with respect to one or more independent variables. Philips (1985) and Philips and Aki (1986) with an analysis of regression from 90 microearthquakes (magnitudes between 1.5 and 3) recorded in a seismic array of 150 stations located in Central California, determined the response of different types of materials (granite, sediments, marble, etc.). For an earlier review of this kind of studies, see Aki (1988).

Microtremors represent the vibrations generated by natural phenomena such as atmospheric fronts, geothermal reactions, sea waves, and so on, or due to human activities such as those produced by traffic, heavy machinery, etc. One of the main advantages that the use of microtremors offers is the ease with which the registration process is carried out, since it is not necessary to wait for the occurrence of events. There are usually two ways of proceeding with microtremors: a) Using their spectra to determine dominant periods or changes in the spectral amplitude for certain periods, and b) computing the spectral ratios of microtremors in a similar way to the one used with earthquakes and/or explosions. Moreover, it has been possible to estimate S wave velocity profiles using the dispersion characteristics of Rayleigh waves from microtremors. In this way Kagawa et al. (1996), presented various profiles in several zones of the Granada basin (Spain), and Satoh et al. (2001a) the S-wave velocity structure in and around the Sendai basin (Japan). A well known problem of this technique is the fact that there may be not any similarity between the spectra of recorded earthquakes and those of measured microtremors at a site. This might occur because the recorded waves: 1) are of different type, and 2) have different propagation paths. (e.g., Udwadia and
Trifunac, 1973). An in-depth study of the use of the analysis of microtremors and several of its applications can be found in Bard (1999).

Nogoshi and Igarashi (1971) considered that an indicator of the underground structure can be found by evaluating the horizontal to vertical spectral ratio (HVSR) of the microtremors recorded with a single station. This technique implies (Lermo and Chávez-García, 1994; Dravinski et al., 1996) that microtremors are primarily composed of Rayleigh waves, produced by local sources, which propagate in a surface layer over a half-space. Nakamura (1989) considered that the HVSR was a reliable estimation of the site transfer function for $S$ waves. Konno and Ohmachi (1998) conducted a fuller study of the technique and extended the problem to consider a multilayered system as well. These authors improved the technique, which up until then had had some theoretical gaps. However, several studies have shown that the HVSR can reveal, with some guarantee, the fundamental peak resonant frequency at a site (e.g., Lermo and Chávez-García, 1994; Field and Jacob, 1995; Lachet et al., 1996; Seekins et al., 1996; Coutel and Mora, 1998). In a recent work, Luzón et al. (2001) by means of a numerical study in flat sedimentary basins, concluded that the HVSR is reasonably good at predicting the fundamental local frequency when there is a high-impedance contrast between the sedimentary basin and the bedrock, but it cannot accurately predict the amplification levels for each period. The low cost-efficiency ratio of the HVSR is the main reason for its widespread use around the world, for example, in the Sendai basin (Japan) (Satoh et al., 2001b); in Almería (Spain) (Navarro et al., 2001); in Caracas (Venezuela) (Duval et al., 2001); in Memphis (Tennessee, USA) (Bodin et al., 2001).

Semi-Empirical Methods

The use of small earthquake records as empirical Green’s functions to simulate strong motion accelerograms can serve to estimate the level of the ground motion in a sedimentary basin. In this method, which was first proposed by Hartzell (1978), the source function of the main earthquake to simulate, is calculated with its focal mechanism, the length and the rupture history of the fault. Then, the convolution between the Green’s function and the source function leads to the strong earthquake ground motion. Morales et al. (1996) used one small event, with magnitude $M_w = 5$, as an empirical Green’s function to obtain the synthetic records of a possible magnitude $M_w = 7.0$ earthquake at various locations in the Granada basin (Spain). The $S_{max}$ of the simulated seismograms, around 0.8 g, underscored the importance of site effects in the Granada basin in the ground motion induced by moderate
earthquakes. Kamae et al. (1998) used a hybrid Green’s function to simulate the near ground motions from the 1995 Hyogo-ken Nanbu earthquake. The method has been applied in many places around the world, for example in Grenoble (France) by LeBrun et al. (2001), or in Loma Prieta (U.S.A.) by Frankel (1995).

**Numerical Methods**

In the description of the propagation of elastic waves in a continuous, elastic, homogeneous and isotropic media, one should consider the Navier equation:

\[
\mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (\lambda + \mu) \frac{\partial^2 u_j}{\partial x_i \partial x_j} + \rho \frac{\partial f_i}{\partial t} = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

where \( \lambda \) and \( \mu \) are the Lamé constants, \( \rho \) the mass density, \( f \) is the volume force, and \( u \) is the displacement field. Analytic solutions for the wave propagation, although limited because of the condition that only regular geometries can be solved, have been very helpful for understanding the physical processes involved in the problem. Moreover, these kinds of results have been very useful in validating solutions obtained with numerical techniques that can deal with more realistic geometries. For further details about analytic solutions see e.g. Sánchez-Sesma (1987).

When a certain area consists of overlapping layers with different elastic properties, local effects under the incidence of seismic waves can be considered by means of 1D Earth models. These models usually represent a stack of flat layers overlaying a halfspace. The method of Thompson-Haskell is of special interest (Thompson, 1950; Haskell, 1953). This technique, which was initially developed to study the propagation of surface waves in stratified mediums, can be included within the techniques that make use of the propagator matrix introduced to seismology by Gilbert and Backus (1966) and is useful for any type of wave. In the Thompson-Haskell method, conditions of continuity of tractions and displacements in each interface between strata and the condition of null tractions at the free surface are imposed. The technique has been later reformulated in terms of transmission-reflection coefficients (Kennett, 1983).

Consider a layer with thickness \( H \) over a half-space. Let us represent, in the frequency domain, the incident displacement to the stratum by \( v_0(\omega) \) and that in the free surface by \( v(\omega) \). Then the quotient \( |v/v_0| \) shows the relative
amplifications in the layer, that is to say, this quotient is the transfer function of the layer. It can be shown that this quotient is:

\[
\frac{v(\omega)}{v_0(\omega)} = \frac{2}{\sqrt{\cos^2\left(\frac{\omega H}{\beta_l}\right) + \left(\frac{\rho_l \beta_l}{\rho_E \beta_E} \sin\left(\frac{\omega H}{\beta_l}\right)\right)^2}}, \tag{3}
\]

where \(\beta_l\) is the velocity of \(S\) waves in the layer, \(\beta_E\) is the velocity in the half space, and \(\rho_l\) and \(\rho_E\) are the mass densities for the layer and the half space, respectively. It can be observed that the displacement will be maximum for the frequencies that satisfy:

\[
f = \frac{\beta_l}{4H} (2n - 1) \quad n = 1, 2, 3, \ldots \tag{4}
\]

and that the amplification levels at these frequencies correspond to:

\[
\left|\frac{v(\omega)}{v_0(\omega)}\right|_{\text{max}} = 2 \frac{\rho_E \beta_E}{\rho_l \beta_l} \tag{5}
\]

Figure 2 displays the response, for vertical incident \(SH\) waves, of a layer with \(H = 100\) meters, \(\beta_l = 400\) m/s, \(\beta_E = 2000\) m/s for the half space, and \(\rho_l = \rho_E\).

1D models have been used as a first approach to the seismic response of the site in many places (e.g. Ordaz et al., 1992 in the Valley of Mexico; Luzón et al., 1998 in Almería, Spain). However, in many cases of interest the surface structures contain lateral discontinuities that cause important spatial variations of ground motion, which in turn drastically modify the expected 1D seismic response of the place.

The cases of 2D and 3D geometries have been addressed as a problem of diffraction of elastic waves. Diffraction is a phenomenon that characterizes the wave motion that appears when the incident wave propagates around obstacles or amid openings with dimensions comparable to its wave-length. In accordance with Huygens’ Principle, the boundaries of the obstacle act as sources of secondary waves spreading in all directions. Several numerical techniques for the estimation of seismic response in complex geological
configurations have been developed. These techniques have become more flexible and versatile as computational systems have improved, making it possible to study an increasingly larger set of more realistic structures.

The numerical techniques used in these problems may be divided into three main groups of methods: a) Domain methods, b) Ray theories and gaussian beams expansions, and c) Boundary methods. Good reviews of the topic can be found, for example, in the works of Luco et al. (1990) and Sánchez-Sesma (1996).

![Amplitude of displacement for the incidence of vertical SH waves in a system consisting of an elastic layer in a half space](image)

**Figure 2.** Amplitude of displacement for the incidence of vertical \(SH\) waves in a system consisting of an elastic layer in a half space

**Domain methods**

In this type of numeric outlines, also called direct methods, a simulation can be performed for the propagation of complete wave fields through a
medium by solving the wave equation or the approximate elastodynamic equations, usually in successive intervals of time. These techniques can be classified into three types: a) finite differences methods, b) pseudospectral method, and c) finite elements methods.

Approaches of this kind are very powerful since they allow the treatment of very complex structures. However, they have the very important drawback of being excessively demanding on computing memory and time resources.

**Finite difference methods**

Techniques based on the finite difference method have been successfully used in recent decades to study wave propagation in elastic media (see e.g. Alterman and Karal, 1968; Boore, 1972; Kelly et al., 1976). In these techniques, the differential equations that govern the motion are replaced by a group of recursive equations expressed as finite differences. Once the space is discretized forming a regular grid, the solution in each node of the grid in successive intervals of time is computed.

Harmsen and Harding (1981) calculated the response of a sedimentary basin for the incidence of $P$ and $SV$ waves and found very interesting results on the generation of surface waves in the lateral irregularities. In these examples, the authors worked with 2D geometry structures. The most realistic simulations of elastic wave propagation to date have been performed using the finite difference method, as illustrated by the recent work of Olsen (2000), who calculated the 3D response of the Los Angeles basin. However, although this technique is very reliable and has a high capacity, the high cost in computing time and the need for large-memory supercomputers make it impractical on many occasions, especially when large-dimensioned structures are required to study for a wide range of frequencies and times.

**Pseudospectral method**

In this technique, also called the Fourier method (Kosloff and Baysal, 1982), the spatial derivatives of the differential equations are evaluated along a row or a column of the spatial grid at one given time using the Fast Fourier Transform, whereas the time derivatives are calculated with the finite differences method. In order to derive with respect to the spatial coordinates, the row or column is transformed to the wave number domain $k$ (or spatial frequency); then the multiplication by $ik$, where $i$ is the imaginary unit, and its inverse transformation produce the corresponding spatial derivative.
Few points per wave-length are required in the grid to obtain good results (in the order of 4 or 5, Fäh, 1992). The method has allowed Ávila et al. (1993) to perform the analysis of two-dimensional sedimentary basins. These authors made an analytic multi-modal expansion to control the vertical variation of the field associated to the wave equation, and calculated synthetic seismograms in the surface of an alluvial valley under the incidence of SH elastic waves. For a clear and comprehensive review of this technique, see Faccioli (1991).

**Finite elements methods**

In these methods, the medium under study is divided into elements and the displacement is calculated in the nodes of the same elements where the forces involved in the problem are also applied. One advantage of this technique is that the elements do not have to be equal either in size or in form, so there can be as many different regions as required in every particular problem. Once the medium has been divided, the displacement in each node is related to the applied forces by the differential equations system. This system can be integrated in several different ways. Serón et al. (1990) studied five different ways in terms of the accuracy, operation, and needs of each technique using scalar or vectorial processors with a high storage capacity. These authors concluded that when the mass and reduction matrices that appear in the system of equations are diagonal, one of the most appropriate integration methods was that of centered differences (Newmark, 1959).

Another advantage of the finite elements method is the ability to calculate the response in configurations with non linear behavior (Joyner and Chen, 1975; Joyner, 1975). Recently, this method has been used to model three-dimensional structures (Toshinawa and Ohmachi, 1992; Li et al., 1992; Rial et al., 1992).

However, the technique does have certain limitations. If the response is required for higher frequencies, the medium must be divided into more elements, thus increasing time and computer memory requirements. This problem is usually solved by imposing artificial boundaries to reduce the medium under study, although in these cases, the time analysis must be shortened to avoid artificial reflections of the waves on these boundaries.

In recent years, an extension of the Finite Elements Method (FEM), known as the Spectral Elements Method (SEM), has been widely used to simulate the propagation of elastic waves. This is a high-order variational method for the spatial approximation of elastic-wave equations, which is highly efficient in terms of computer resources. The SEM is particularly well-suited for
managing complex geometries. A remarkable difference between FEM and SEM is that the former is an \( h \) method, while the latter is an \( h-p \) one, based on the use of high order piecewise polynomial functions (Canuto et al., 1988). This means that the FEM can only act on the refinement of the computational grid to increase the quality of the numerical solutions, while the SEM can indifferently exploit mesh refinement or increasing of the polynomial functions used to build the numerical solution. In seismology, the SEM has been recently applied by Komatitsch and Vilotte (1998) to simulate the seismic response of 2D and 3D geological structures.

**Theories of rays and gaussian beams**

In some cases, the problem of wave propagation can be simplified and the use of geometric techniques is appropriate. In these methods, asymptotic expansions are carried out for the wavelength \( \lambda =0 \), that is, for very high frequencies. The analysis is done in two steps (see e.g. Hanyga et al., 1985): a) find the paths for the propagation, that is, the rays, and b) find the intensity of the propagation along each ray.

This technique has been used to study wave propagation in layered media with lateral variation (Pao et al., 1984; Cerveny, 1985a; Chen, 1992) and 2D sedimentary basins (Jackson, 1971; Hong and Helmberger, 1977; Rial, 1984). Sánchez-Sesma et al. (1988) presented a technique which takes account of the complete family of rays in valleys with 2D geometry. The case of a 3D circular basin has been treated for incidence of \( P \) and \( S \) waves by Lee and Langston (1983).

An extension of the theory of rays has been used by combining this technique with a parabolic approximation of the wave equation (Nowack and Aki, 1984). In this approach, the focus-receiving system of rays serves to support the construction of the wave field. The wave equation is solved in centered coordinates and the parabolic approximation is locally satisfied in a vicinity of each ray. In this solution, the wave width is gaussian shaped around the central ray and the final solution is to overlap the different gaussian beams of each ray.

Several applications of this technique have been presented for the study of wave propagation for 2D structures (Cerveny and Psencik, 1983; Madariaga, 1984; George et al., 1987; Yomogida, 1985; Álvarez et al., 1991; Rodríguez-Zúñiga, 1992). Kato et al. (1993) present 3D simulations of the propagation of surface waves in the Kanto sedimentary basin (Japan).

Ray techniques and their extensions usually involve lower computing costs of than in other methods. However, these approaches pose other limitations.
(Cerveny, 1985b). The solutions obtained do not take account of the diffraction phenomenon, the boundaries should be soft and the dimensions of the irregularities have to be much greater than the characteristic length wave of the incident field. Ray methods cannot deal with the cases of incident inhomogeneous waves, such as Rayleigh waves, or waves reflected beyond the critical angle.

**Boundary methods**

In recent years, boundary methods have become highly popular and accepted. This technique avoids having to insert fictitious boundaries in the medium, as occurs with domain methods, thus reducing the dimensions of the problem and affording many computational advantages. Boundary methods can be combined with the use of finite elements (Zienkiewics et al., 1977; Khair et al., 1989; Takemiya and Tomono, 1992; Mossessian and Dravinski, 1992) thus allowing the modeled region to be smaller. Boundary methods mainly include three techniques based on: a) the use of complete systems of solutions (Herrera, 1981), b) the representation of the discrete wave number (Aki and Larner, 1970; Bouchon and Aki, 1977), and c) the use of integral equations (Brebbia, 1978; Cole et al., 1978).

**Methods based on complete systems of solutions**

These methods are based on the fact that the solution of a linear problem can be obtained from the lineal combination of independent functions, each of which satisfy the differential equation that represents the problem. The coefficients in the combination are calculated by imposing the boundary conditions for each case under study. The boundary conditions imply null tractions in free surfaces and the continuity of displacements and tractions in the surfaces separating two different media, the interfaces. These requirements help to find the coefficients for the lineal combination of simple solutions, which can be calculated with a minimal squares system.

The technique has been used to calculate the seismic response of sedimentary basins (Sánchez-Sesma and Esquivel, 1979; Dravinski, 1982; Sánchez-Sesma et al., 1988) with several 2D geometries and with different types of incident waves. This approach has also been used to study 3D structures with azimuthal symmetry for the incidence of P, SV and Rayleigh waves (Sánchez-Sesma, 1983; Sánchez-Sesma et al., 1989; Pérez-Rocha and Sánchez-Sesma, 1991).
**Discrete wave-number method**

In this technique (Aki and Larner, 1970) the wave field is expressed as the overlapping of plane waves, including inhomogeneous ones, spreading in all directions from the boundaries of the elastic space with unknown complex amplitudes. The total motion is obtained by integration in the horizontal wave-number. This integral is replaced by an infinite sum with the hypothesis of horizontal periodicity of the irregularity. Truncating this to a finite sum and applying the boundary conditions in the wave-number domain gives a system of linear equations for the unknown amplitudes.

This method has been extended to the time domain (Bouchon, 1973; Bard and Bouchon, 1980a, b) and has been used to study sedimentary deposits (Bard and Bouchon, 1980a,b; Bard and Bouchon, 1985; Bard and Gariel, 1986) with 2D geometry under the incidence of different types of elastic waves.

The technique has also been extended to 3D geometries (Niwa and Hirose, 1985) for different forms of sedimentary basins under the incidence of $P$, $S$ and Rayleigh waves (Horike et al., 1990; Horike and Takeuchi, 1992; Kuribayashi et al., 1992; Ohori et al., 1992; Uebayashi et al., 1992; Jiang et al., 1993).

This technique is efficient in terms of computational requirements and it correctly expresses the displacement field. However, it is limited to the study of irregularities with a soft slope and not very high frequencies.

**Solutions with integral equations**

These methods are based on the fact that an integral representation of the wave-field is possible in terms of boundary elements of the considered space. The discretization of these surfaces and the boundary conditions together generate a linear system of integral equations, which can be managed in several different ways. One of them is direct formulation, so-called because the unknowns of the problem are the displacement and traction values. Wong and Jennings (1975) and Zhang and Chopra (1991) used this formulation to perform 2D and 3D studies, respectively, of the seismic response of canyons. In the indirect version, the unknowns are the force densities in each element-source.

This technique has been combined with the discrete wave number method in order to calculate the Green’s functions that appear in the integral representation. The direct (Campillo and Bouchon, 1985; Kawase, 1988; Kawase and Aki 1989; Bouchon et al., 1989; Campillo et al., 1990) and
indirect formulation (Hisada et al., 1992, 1994) have been used in this combination to study some elastic wave diffraction problems. In this procedure, integrating the Green’s functions along the boundaries avoids the singularities that arise when the position of the source and the point of observation coincide. However, calculating these Green’s functions is very costly in terms of computing resources.

On the other hand, when the Green’s functions are analytical, the singularities at the source position can be integrated (Brebbia, 1978). Thus, the sources can be located on the surface (simple layer sources) and their effects be considered more appropriately. At the same time, the uncertainty about the location of the sources is eliminated (see e.g. Sánchez-Sesma and Rosenblueth, 1979; Sánchez-Sesma and Esquivel, 1979; Dravinski, 1982; Wong, 1982; Dravinski and Mossessian, 1987) as occurred in other works where the singularities were located outside the region of interest. This method has been used to obtain the seismic response of irregular sedimentary basins with 2D geometry (Ramos-Martínez and Sánchez-Sesma, 1991; Sánchez-Sesma et al., 1992, 1993a; Luzón et al., 1995), and also to compute the case of 3D alluvial valleys (Sánchez-Sesma and Luzón, 1995; Janod and Coutant, 2000). The technique has been used recently to compute the 3D seismic response of the Granada basin (Southern Spain) for low frequencies (Gil-Zepeda et al., 2002).

These methods are formulated in terms of the Green’s function of the problem, which represents the solution for a unit force applied at a point within a certain domain of interest. The availability of these functions is a severe limitation of the boundary element methods. In fact, the Green’s functions can only be obtained easily for homogeneous unbounded medium. This has constrained the study of the seismic response of sedimentary basins to homogeneous models. However, there is a wide class of problems for which it is reasonable to assume an increase of wave propagation velocities with depth (see e.g. Bard and Gariel, 1986; Vrettos, 1990). Recently, Sánchez-Sesma et al. (2001) presented and tested new approximate analytic Green’s functions for media with wave velocities varying linearly. On the other hand, Luzón et al. (2002) used these Green’s functions with the boundary element method and pointed out the principal differences between the seismic response of homogeneous alluvial valleys and those where wave velocity grows linearly with depth.

One of the problems that is currently being researched is the reduction of computing time and memory requirements for large models, when boundary elements techniques are used to deal with realistic 3D problems. In these calculations, to maintain a reasonable ratio between the minimum wavelength...
and element size, the dimension of the coefficient’s matrix grows approximately with the square of frequency, and therefore the computing requirements can be too big for some problems. In a recent paper, Ortiz-Alemán et al. (1998) overcame the problem by employing threshold criteria to convert the full matrix into a sparse one and then using the biconjugate gradient method together with an iterative scheme to solve the linear system of equations that appear when the boundary conditions are imposed. To test their approach, they satisfactorily reproduced the results of Luzón et al. (1997) who studied the diffraction of $P$, $S$ and Rayleigh waves by 3D topographic surfaces. In another paper, Gil-Zepeda et al. (2002) simulated the wave propagation in a 3D model of the Granada basin (Southern Spain) using a partitioned matrix instead of using the full matrix of the linear system. These authors reordered rows and columns to obtain block matrices and found that the system had a particular structure and could be easily constructed by using a static matrix condensation scheme, solving it by parts. Some of the blocks inside the matrix were full of zeros and were completely eliminated in the numerical computations, making it much faster than the classical LU decomposition used by Sánchez-Sesma and Luzón (1995). With this procedure Gil-Zepeda et al. (2002) found that it was possible to compute, in the same computer and with the same 3D problem, the solution corresponding to approximately twice the frequency calculated with the classical form using the full matrix of the linear system of equations. Anyway, we think that future work in this field will necessarily take into account the advantages and possibilities that parallel computing offers for solving large problems, such as the simulation of wave propagation in realistic models.

**Example and short description of the Indirect Boundary Element Method**

Consider a halfspace with an elastic inclusion $R$ under the incidence of elastic waves as shown in Figure 3.

The motion that takes place on the surface of this irregular configuration is formed by the interferences of the incident waves with the reflected, diffracted and refracted waves. Assuming harmonic excitation, the refracted displacement inside the basin can be expressed as:

$$u_j^r(x) = \int_{SR} \phi^R_j(\xi)G_{ij}^R(x,\xi)dS_\xi$$  \hspace{1cm} (6)
where \( G^R_\theta(x,\xi) \) = Green’s function for the elastic region \( R \) (the Green’s function is the displacement produced in direction \( i \) at \( x \) due to the application of a harmonic unit line force at point \( \xi \) in direction \( j \)), \( \phi^R_\theta(\xi) \) = force density in direction \( j \) at \( \xi \), and \( \Sigma_R \) = boundary of region \( R \). Equation (6) can be obtained from Somigliana’s identity (see Sánchez-Sesma and Campillo, 1991).

Moreover, the total wavefield in the halfspace is the sum of the so-called free-field and diffracted wave

\[
\mathbf{u}_i(x) = \mathbf{u}^{(0)}_i(x) + \mathbf{u}^{(d)}_i(x) = \mathbf{u}^{(0)}_i(x) + \int_{\Sigma_R} \phi^E_j(\xi) G^E_\theta(x,\xi) d\Sigma_\xi
\]

Figure 3. Halfspace \( E \) and inclusion \( R \) under incident elastic waves with angle \( \gamma \).
\[ t_{ij}(x) = k \phi_i(x) + \int_{S} \phi_j(\xi) T_{ij}(x,\xi) dS_{\xi} \]  
\[ \text{(8)} \]

where \( T_{ij}(x,\xi) \) = traction Green’s function; \( k \) is equal to zero if \( x \) is not at the boundary of the elastic space, and equal to 1/2 or -1/2 if \( x \) tends to the boundary from inside or outside the space, respectively. Equation (8) is used to compute the tractions of both media, the half-space \( E \) (plus the free-field traction), and the alluvial valley \( R \).

Let us now consider the boundary conditions of the problem. In the common interface of both media, \( E \) and \( R \), the continuity of displacements and tractions can be expressed by the following equations:

\[ u_{ij}^e(x) = u_{ij}^{(0)}(x) + u_{ij}^{(d)}(x) \]  
\[ \text{(9)} \]

that is

\[ \int_{S} \phi_i^E(\xi) G_{ij}^E(x,\xi) dS_{\xi} = \int_{S} \phi_j^E(\xi) G_{ji}^E(x,\xi) dS_{\xi} \]  
\[ \text{(10)} \]

for displacements, and

\[ t_{ij}^e(x) = t_{ij}^{(0)}(x) + t_{ij}^{(d)}(x) \]  
\[ \text{(11)} \]

that is

\[ k \phi_i^E(x) + \int_{S} \phi_j^E(\xi) T_{ij}^E(x,\xi) dS_{\xi} = \int_{S} \phi_j^E(\xi) T_{ji}^E(x,\xi) dS_{\xi} \]  
\[ \text{(12)} \]

for tractions.

On the other hand, zero tractions on the free surface of the basin and of the half-space provide the next equations:

\[ t_{ij}^{(0)}(x) + t_{ij}^{(d)}(x) = 0 \]  
\[ \text{(13)} \]

that is

\[ t_{ij}^{(0)}(x) + k \phi_i^E(x) + \int_{S} \phi_j^E(\xi) T_{ij}^E(x,\xi) dS_{\xi} = 0 \]  
\[ \text{(14)} \]

in the free surface of \( E \), and
\[ t_i^{(s)}(x) = 0 \]  

that is

\[ k \phi_i^s(x) + \int_{S_g} \phi_j^s(\xi) T^s_{ij}(x, \xi) dS_\xi = 0 \]  

valid on the free surface of \( R \).

Equations (10), (12), (14) and (16) constitute a linear system of integral equations for boundary sources, \textit{i.e.} those producing the diffracted and refracted fields. These expressions have to be discretized along the surfaces involved in the problem in order to solve the linear system. For further details on this method and the discretization process, please refer to Sánchez-Sesma and Luzón (1995). These authors presented the seismic response of an irregular basin using circles to construct the surfaces of the medium (see figure 4) under incident plane waves.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Discretization of the 3D alluvial basin considered in Sánchez-Sesma and Luzón (1995) using circles.}
\end{figure}

As an example, figure 5 displays the displacements \( u, v \) and \( w \) (in the directions \( x, y \) and \( z \), respectively) produced by the vertical incidence of \( SH \) waves. 51 equidistant receivers are located along the \( x \) axis from \( x = -4 \) km to \( x = 4 \) km which are represented by \((x,0)\). Another 51 equidistant receivers are
located along the $y$ axis from $y = -4$ km to $y = 4$ km, and their displacements are represented by the seismograms shown by $(0,y)$ in the figure. The incident source function was a Ricker wavelet (Ricker, 1977).

![Figure 5](image)

**Figure 5.** Displacements produced at 51 receivers located along the $x$ axis $(x,0)$, and other 51 along the $y$ axis $(0,y)$ in the three directions of the space for the vertical incidence of $SH$ waves. 3D alluvial basin considered in Sánchez-Sesma and Luzón (1995).

This figure shows that the displacements produced inside the basin are very important both in the amplification level and in the duration, with respect
to the amplitudes produced by the incident wave-field, as occurs with the motion inside sedimentary inclusions produced by real earthquakes.

**CONCLUDING REMARKS**

Throughout History, many of the earthquakes known as “destructive” have been influenced by local effects, and the absence of such effects might have caused other kinds of results with less injuries on the population. The 1985 Michoacán earthquake is perhaps the most dramatic example, where the local conditions in Mexico City played a fundamental role in the final result.

In this review, various empirical, semi-empirical and numerical methods developed to study site effects in sedimentary basins have been described. This paper offers interested readers guidelines for pursuing independent study. This brief review of some common methodologies for estimating local site effects on strong ground motion has left out many relevant techniques. Coverage is far from exhaustive and, for example, one obvious absence is the effect of non-linear behavior of soils, including liquefaction.

Domain methods, such as finite difference and finite element procedures, are well-established techniques that can be used if the geometry and mechanical properties are well known and represented. On the other hand, boundary element methods seem to be powerful tools for simulating wave propagation in homogeneous media with lateral irregularities. If detailed data of media properties are not available, reasonable estimates of ground motion can be obtained with BEM using simplified models. Indirect BEM is being applied to 3D realistic configurations considering the advantages of present computing resources.

The various techniques have advantages and limitations and the desired precision and availability of data should dictate the choice of method for a given problem.

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