

**MODELING MAGMATIC INTRUSION'S EFFECTS ON THE
GEOID AND VERTICAL DEFLECTION. APPLICATION TO
LANZAROTE, CANARY ISLANDS, AND LONG VALLEY
CALDERA, CALIFORNIA.**

**M. Charco⁽¹⁾, J. Fernández⁽¹⁾,
M. J. Sevilla⁽¹⁾ and J. B. Rundle⁽²⁾**

(1) Instituto de Astronomía y Geodesia (CSIC-UCM), Fac. C. Matemáticas.
Ciudad Universitaria. 28040-Madrid, Spain.

(2) Center for Computational Science and Engineering, University of California.
One Shields Avenue, Davis, CA 95616, USA.

ABSTRACT

Volcanic activity produces ground deformation and gravity changes in response to geodynamic processes within the crust. Many of these precursors are measurable with present-day technology like precise surveying techniques or “high-technology” as those use in satellite-based geodesy (e.g. Global Positioning System). It is usually assume that vertical deflection and geoid height needed for comparing such as techniques, are not significantly affected by the intrusion process. In this work, we have tested theoretically this assumption and applied to active zones with different crustal structures that resemble layered media, namely Lanzarote (Canary Islands, Spain) and Long Valley Caldera (California). Considering the geoid as an equipotential surface of the gravitational field we have used the elastic-gravitational deformation model, proposed by Rundle and Fernández, to compute geoid and vertical deflection changes produced by a magmatic intrusion in the crust. This technique represents the geoid and vertical deflection due to a point source, which therefore can be used as Green’s function with which to convolve an arbitrary distribution of subsurface mass or pressure change. The results show that the magma intrusion radius should be of approximately 1 km for the effects on both geoid undulations and vertical deflection not to be negligible.

This radius would decrease for shallow intrusions. The pressure effects computed with the model, if we considered realistic pressure changes values, would be always negligible.

1. INTRODUCTION.

The effects associated to a crustal intrusion that can be used in monitoring and predicting possible volcanic activity are those that can be detected on the ground's surface before an eruption. These effects generally reflect the adjustments of the volcano's surface in response to subsurface movements of magma into or out of the volcanic edifice and may also be related to variations in the pressure and/or flow fluids in the geothermal system of the volcano.

Many of these precursor phenomena, like gravity changes and ground deformation, are measurable with present-day technology that provides essential data to interpret past or current volcanic behavior by using deformation models (e.g., Newhall and Dzurisin, 1988; Rymer and Brown, 1989; Sevilla and Romero, 1991; Dvorak and Dzurisin, 1997; Fernández *et al.*, 1999; Fernandez *et al.*, 2001a). Monitoring active zones by applying geodetic techniques involves terrestrial triangulation and trilateration techniques to detect horizontal crustal movements. Spirit or trigonometric leveling is used to detect vertical displacements. These types of control networks are generally supplemented by gravity, tilt and extensometer measurements, that allow continuous monitoring in order to recognize the anomalous behavior that might predict an eruption. GPS methods are being increasingly used to measure both vertical and horizontal ground displacements. Researchers have been attracted by a number of advantages of GPS, including its all-weather operational capability, portability, the fact that benchmarks do not have to be intervisible and the accuracy of the system in comparison to other terrestrial and space-based measurement systems (Dixon, 1991).

Prior to the widespread use of GPS, height above the geoid (the one gravity equipotential surface that approximates the mean sea level over the whole Earth, e.g., Vanicek and Krakiwisky (1992)) was the only height that could be measured accurately. The GPS delivers a measurement of height above the reference ellipsoid (geometric reference surface which approximates the geoid). The ellipsoidal height is reckoned from the surface of any reference ellipsoid to the point of interest on the highly irregular topographic surface, along the ellipsoidal normal. The height above the geoid, namely orthometric height, is reckoned from the surface of the geoid to the point of interest along the plumb line (lines of force of the Earth's gravity field). Converting orthometric heights to ellipsoidal heights or vice versa

involves knowing the vertical deflection (angle at ground level between the actual plumb line and the ellipsoidal normal) and the geoid undulation (geoid height from the surface of any reference ellipsoid to the geoid, reckoned along the ellipsoidal normal) to refer the measurements to same reference surface (e.g., Vanicek and Krakiwsky, 1992; Milbert and Smith, 1996; Strang and Borre, 1997; Featherston, 2001). In the case of the ground geodetic networks used to measure displacement, vertical deflection and geoid undulation must be known to refer the measurements to the same surface (e.g. an ellipsoid). If one considers that the area studied in volcano monitoring tends to be relatively small, the reference surface problem can be solved locally. In this case, we would only have to consider the local anomalies of the vertical deflection and the geoid undulation caused mainly by the distortion of the gravity field due to the distribution of the surrounding masses (e.g., Baldi and Unguendoli, 1987).

In this work, we show how vertical deflection and geoid undulations are affected by the presence of a magmatic intrusion in the crust. Knowing the magnitude of these effects will let us distinguish between the effect of the intrusion and the previously existing background field caused by differences between the real Earth and reference body potential field. We have developed a theoretical model to compute vertical deflection and geoid undulation changes produced by a magmatic intrusion in an elastic medium considering the existence of the ambient gravity field. Because we are interested in near-field effects we can consider flat half-space models that do not take account of the Earth's curvature. Furthermore, by modeling the effects produced by an intrusion in stages immediately prior to the eruption, we do not consider anelastic effects. Instead of deformation models that consider homogeneous media (e.g., Mogi, 1958), our model considers a layered medium, although we do use a point source to permit analytical solutions very attractive for solving the inverse problem.

2. VERTICAL DEFLECTION AND GEOID CHANGE.

Volcanic activity commonly caused ground deformation as response to the geodynamic processes that are taking place in Earth's interior. Whereas elevation changes resulting from land-based geodetic techniques are referred to the geoid, elevation variations resulting from GPS are referred to the geometric reference ellipsoid. Therefore, geoid height and vertical deflection must be known to compare the leveling and GPS surveys. From Heiskanen and Moritz (1985) the geoid undulation is defined at each point as:

$$N \approx h - H \quad (1)$$

where N is the geoid height, h is the ellipsoidal height and H is the orthometric height. The approximate equality results from neglecting the vertical deflection (Helmert projection).

The deformation model considers a layered half-space that includes the ambient gravity field (Rundle, 1980, 1982; Fernández and Rundle, 1994a, b). The disturbing source is located at depth c in the layered half-space. The solutions are expressed as integrals using the general propagator matrix formula (Thomson, 1950; Haskell, 1953). The displacement vector, disturbing potential and gravity change are given, respectively, by the following expressions (Rundle, 1982; Fernández et al, 1997):

$$\underline{\mathbf{u}} = M \int_0^\infty \{x_0^1(0)\underline{\mathbf{P}}_0 + y_0^1(0)\underline{\mathbf{B}}_0\} k dk, \quad (2)$$

$$\phi = M \int_0^\infty \omega_0^1(0) J_0(kr) k dk, \quad (3)$$

$$\delta g = -M \int_0^\infty q_0^1(0) J_0(kr) k dk + \beta_0 u_z, \quad (4)$$

where $x_0^1(0)$, $y_0^1(0)$, $\omega_0^1(0)$ and $q_0^1(0)$ are the kernel functions. $\underline{\mathbf{P}}_0$ and $\underline{\mathbf{B}}_0$ are functions depending on the Bessel functions of first kind order zero, $J_0(kr)$, and are given in Rundle (1982) by expression (6), $\beta_0 = 4\pi G \rho_0$ with G being the universal gravitational constant and ρ_0 the unperturbed density. The kernel functions depend on the gravitational wave number k and on the characteristics of the medium (e.g., Rundle, 1980, 1982; Fernández and Rundle, 1994a,b).

Vertical deflection is a spatial angle and is usually split into two orthogonal components, a North-South component ξ and an East-West component η . If we denote by θ the component of the vertical deflection in a given plane, this is expressed as (Heiskanen and Moritz, 1985)

$$\theta = -\frac{dN}{dS} \quad (5)$$

thus the component in that plane is given by the derivative of geoid undulation, N , with respect to the arc element, S , in the considered direction.

Rundle (1982) pointed out that the additional change in apparent height h^* which is due solely to the change in potential ϕ caused by the intrusion process is:

$$h^* = \frac{\phi}{g} \quad (6)$$

where g is the unperturbed surface gravitational acceleration, considered constant in our deformation model (Rundle, 1980, 1982). The expression (6) is the Bruns's formula (see equation 2-144 by Heiskanen and Moritz, (1985)), which provides geoid undulation from the disturbing potential. h^* can be identified with a geoid undulation change, dN , produced by the magmatic intrusion, taking surface $z = 0$ to be the reference equipotential surface. Figure 1 displays a diagram illustrating the effects of an intrusion on the vertical deflection and geoid undulation.

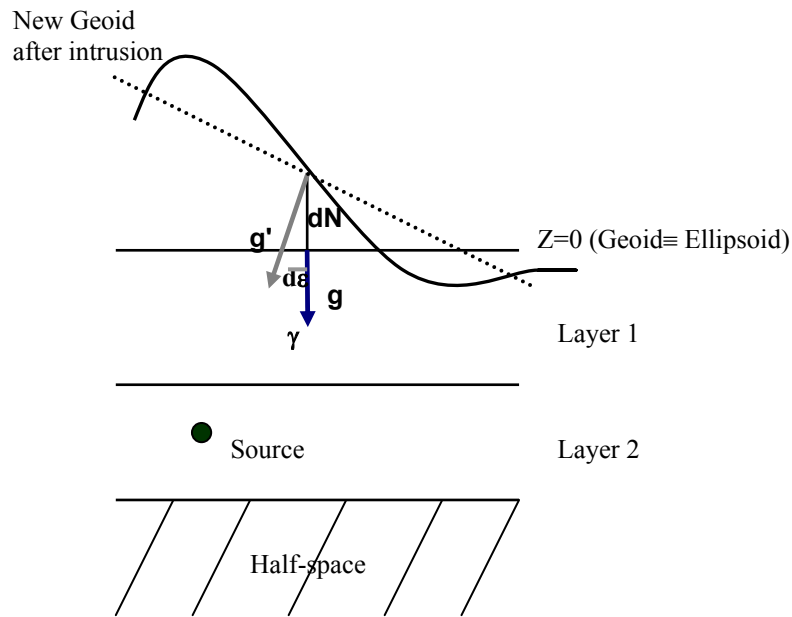


Figure 1. Effects of a magmatic intrusion on the vertical deflection component and the geoid undulation. The media is composed by two layers over halfspace and the source is located on the second one. Vertical distance between the reference surfaces is exaggerated. \mathbf{g} is unperturbed gravity vector, γ denotes the gravity vector produced by the reference body, ellipsoid. Geoid and ellipsoid are considered identical before the intrusion to simplify the diagram. Therefore geoid undulation prior to the intrusion is null. \mathbf{g}' is the perturbed gravity vector, $d\epsilon$ is the vertical deflection and dN is geoid height after the intrusion process.

Thus, we can use the analytical expression of the disturbing potential given by (3) to obtain the effect on the vertical deflection components caused by the presence of a disturbing source in the layered half-space. From expressions (5) and (6),

$$d\xi = -\frac{\partial h^*}{\partial x} = -\frac{1}{g} \cdot \frac{\partial \phi}{\partial x}, \quad (7)$$

$$d\eta = -\frac{\partial h^*}{\partial y} = -\frac{1}{g} \cdot \frac{\partial \phi}{\partial y}, \quad (8)$$

where x and y are the directions of the meridian and the parallel respectively. Thus we can calculate the intrusion's effect on both components as follows:

$$d\xi = \frac{M}{2g} \int_0^\infty x \omega_0^1(0) \cdot (J_0(kr) + J_2(kr)) \cdot k^3 dk, \quad (9)$$

$$d\eta = \frac{M}{2g} \int_0^\infty y \omega_0^1(0) \cdot (J_0(kr) + J_2(kr)) \cdot k^3 dk, \quad (10)$$

Equations (9) and (10) can be expressed as the sum of the effects calculated for a center of dilatation and a point mass due to their linearity with regard to the source functions (e.g., Fernández and Rundle, 1994b).

To compare the results obtained with (9) and (10) against a recognized methodology, we have used the Vening Meinesz formulae for near zones (e.g., expression 2-235 on Heiskanen and Moritz, 1985) since we are interested in near-field effects. The numerical integration of this problem is not applied to the whole Earth. Instead of that it is applied to a given spherical distance. The error that results from neglecting the zones beyond this spherical distance is known as the truncation error (e.g., Sjöberg, 1984). Figure 2 shows the differences between both methodologies in the North-South component of the vertical deflection produced by a point mass of 1 MU (10^{15} gr = 10^{12} kg), located at different depths in a medium comprised by a layer overlying the half-space. Subindex 1 corresponds to the values obtained using equation (9), while the components with subindex 2 correspond to the values obtained using Vening-Meinesz formulae. It can be observed that the difference between both methods decreases in line with the radial distance to the source. For the East-West component it is obtained the same result. The

maximum differences for both vertical deflection components at each depth are shown in Table 1. Both Table 1 and Figure 2 show that the differences between the two sets of results are very small for intrusion depth below 1.5 km, the ones typical used in Volcanology. We have not taken account of the truncation error. Furthermore, it should be remembered that the model described does not take account of topographic relief. The magnitude order of the effect caused by a center of expansion at the same depth is smaller than the point mass effect.

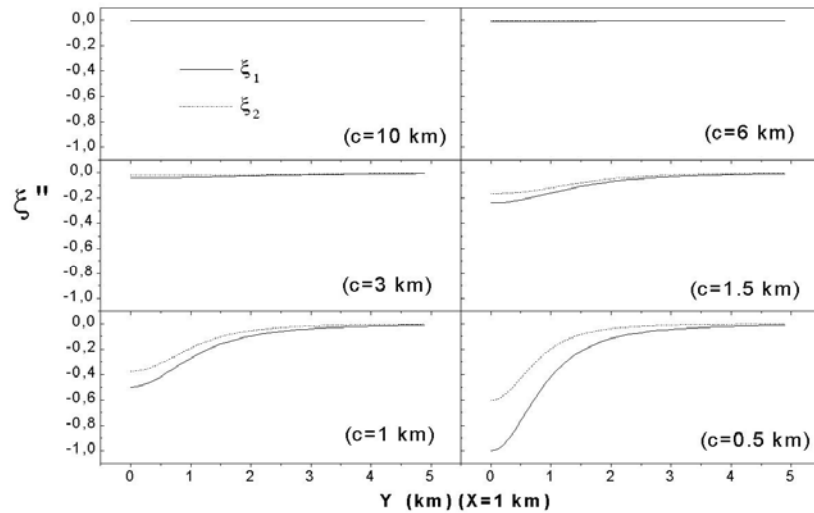


Figure 2. Effect on the N-S component of the vertical deflection produced by a point mass of 1 UM located at 0.5, 1.5, 3, 6 and 10 Km in a homogeneous medium with elastic parameters λ and μ equal to $3 \cdot 10^{10}$ Pa and density equal to 3300 Kg/m^3 . Subindex 1 corresponds to the values obtained using equation (9) and subindex 2 corresponds to the values obtained using Vening-Meinesz formulae. The effect is plotted versus horizontal coordinate Y and is given in arc seconds.

3. COMPUTATIONS.

We have perform an analysis to obtain the mass and pressure increments in the magma chamber that could produce vertical deflection and geoid heights values within the precision levels attainable, i.e., values that can be detected over the existing background. These values are needed for comparing spirit or

trigonometric leveling with GPS surveys or for reducing classical terrestrial geodetic observations to any reference surface.

Table 1. Maximum differences between vertical deflection components, calculated by the method provided by the expressions given in (11)-(12) and the approximate method of Vening-Meinesz. See text for details.

c (km)	$d\xi''$	$d\eta''$
0.5	-0.400	-0.220
1	-0.120	-0.090
1.5	-0.070	-0.044
3	-0.020	-0.017
6	-0.005	-0.005
10	-0.001	-0.002

Geoid models like EGM96 (Lemoine, et al, 1997) usually provides geoid undulations for solving the problem of the reference surface. The usual levels of accuracy of the geoid models after applying topographic effect corrections are around 10 cm (e.g., Nahavandchi and Sjöberg, 2001; Kuroishi, 2001). Therefore we have considered an effect on N of 15 cm as a detectable value. The calculations have been made by using an elastic-gravitational homogeneous half space. The reference density of the medium is $\rho = 3 \cdot 10^3 \text{ kg m}^{-3}$ and the Lamé constants are $\lambda = \mu = 30 \text{ GPa}$.

Figure 3 displays the geoid undulation change produced by a point mass of 1 MU located at 5 km depth and the effect caused by a center of expansion of strength (product of the pressure change by the cubic potency of its radius) 10^3 MPa km^{-3} at the same depth. The maximum magnitude of the effect in the geoid undulation is 1.5 mm. This critical point is located over the intrusion projection on the free surface. Expression (6) provides the geoid height caused by a perturbation source, thus for an intrusive mass M , the geoid height change is given by:

$$dN = \frac{GM}{Rg}, \quad (11)$$

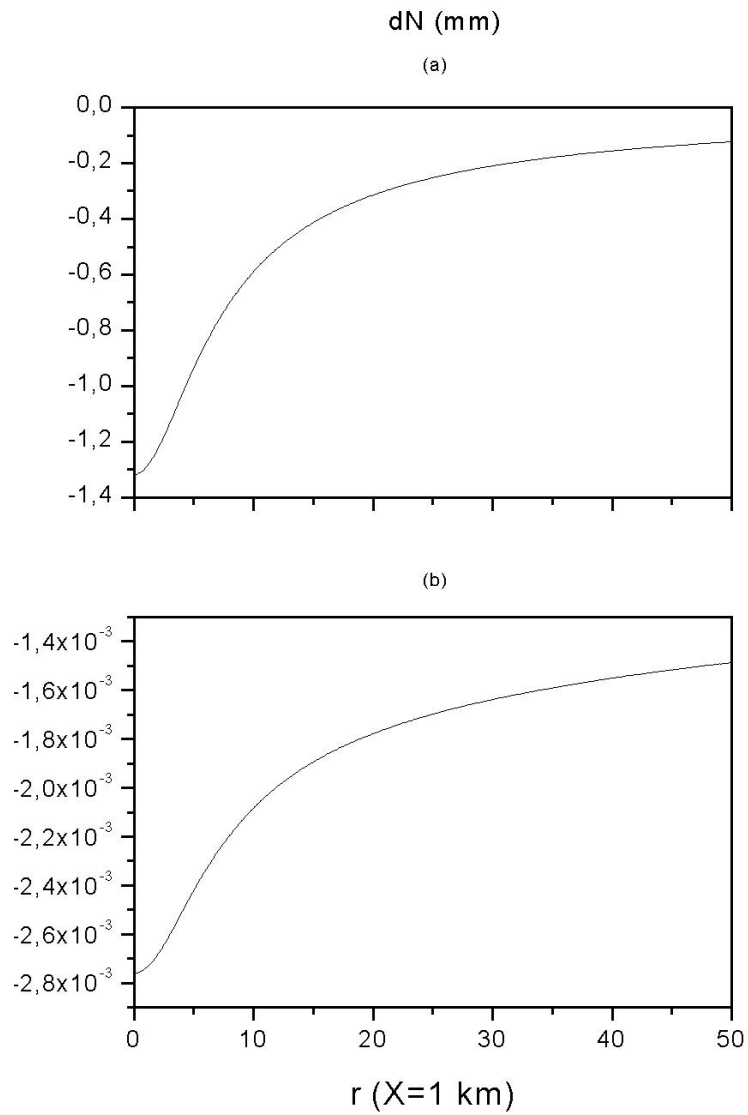


Figure 3. Effect on the geoid undulation (mm) caused by (a) a point mass of 1 MU, and (b) a center of expansion of strength equal to 10^3 MPa km^3 located in a homogeneous media. See text for the characteristic of the media.

where R is the radial distance from the point mass to the point at the surface where the effect is computed. 1 MU produces a geoid height change magnitude of around 1mm in a point located at 5 km from the intrusion projection in the free surface. This is in agreement with Figure 3a. Considering an intrusion of density equal to 3300 kg m^{-3} we would need an intrusion of 0.9 km radius, or $\Delta V = 3 \text{ km}^3$ to modify the geoid undulation by a detectable amount. The radius would vary by 0.95 km if we considered an intrusion of density 2800 kg m^{-3} . Thus, we would need intrusions with a radius of about 1 km to obtain measurable effects on geoid height caused by sources located at not very shallow depths.

Similarly, we can obtain the strength of a center of expansion that would be necessary to obtain the detection value in the geoid. Considering the values of Figure 3, the strength value we would need to detect the effect on the geoid height is $5.5 \cdot 10^6 \text{ Mpa km}^{-3}$, i.e., with a radius of 1 km the pressure increment would be $5.5 \cdot 10^6 \text{ MPa}$ that is an unrealistic pressure change. In the same way, if we set a pressure change of 10 MPa, the radius would be 81 km, which is an unrealistic magma chamber radius. It must be note that the variations on geoid height are directly proportional to the intrusive mass and therefore the bigger masses, the larger effects. Similar results are obtained if we calculate the effect on the vertical deflection components.

4. APPLICATION

We apply our model to illustrate that the effect on vertical deflection components and on geoid undulation caused by a magmatic intrusion located beneath the crust is not inside the precision limits we can obtain with geodetic techniques. In the first hypothetical example we compute the effect caused mainly by the pressure change, while in the second case the inflation is mainly produced by magma intrusion in the crust.

Figure 4 shows the results of the first case. The magmatic intrusion is very small and the main effect is caused by the pressurization of the chamber. Table 2 lists the characteristics of the crust, the elastic parameters and the density values. We consider a spherical intrusion of 0.3 km radius, located at 4 km depth and suppose a pressure increase of 150 MPa and a density of 2800 kg/m^3 , resulting in a mass of $3.2 \cdot 10^{11} \text{ kg}$. This case is similar to the 1730-1736 episode in Lanzarote (Canary Islands) (Fernández and Rundle, 1994a). For further information on volcanic activity on the Lanzarote island and in the Canary Islands in general, see e.g., Schminke (1982), Ortiz et al., (1986), Ancochea et al. (1990), Araña (1991), Araña and Ortiz (1991).

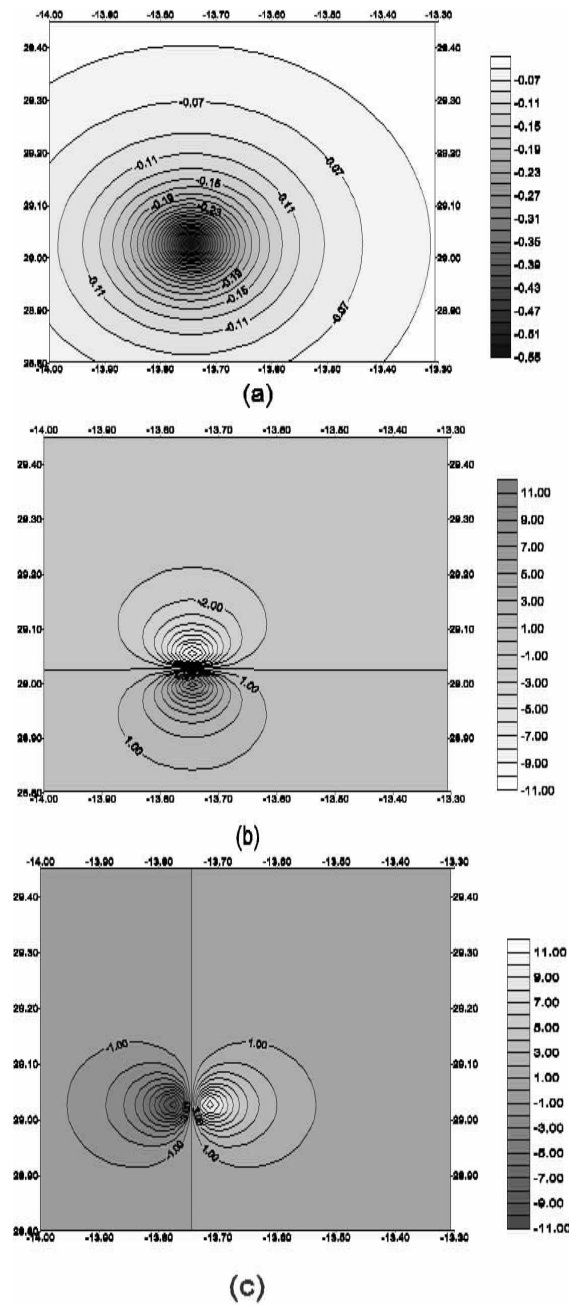


Figure 4. Effect on (a) geoid undulation (mm), (b) N-S component and (c) E-W component of the vertical deflection (msec) produced by an intrusion in the island of Lanzarote of the characteristics described in the text.

The geoid undulation change has a maximum with an absolute value of 0.5 mm. When this value is compared with the EGM96 model (Lemoine et al., 1997), shown in Figure 5, we see that the considered intrusion has a negligible effect on the geoid. The effect on the vertical deflection components has a maximum value of the order of $0.01''$, resulting in no considerable change in the island component map. Furthermore, both kinds of modeled effects are so small that they do not have to be taken into account when we compare the leveling and GPS surveys or reduce classical terrestrial geodetic observations to a reference surface. However, they could be detected with measurements performed with high-precision tiltmeters (e.g., water tubes) because they exceed the levels of precision achievable (e.g., Fernández et al., 1999). However, a correct interpretation should involve correcting other types of disturbing effects (e.g., Agnew, 1986, 1987), and the measurement of other possible precursory effects, such as displacements and gravity changes.

Table 2. Densities and Lamé constants for the crustal model of Lanzarote Island (after Fernández and Rundle, 1994a).

Layer	Thickness (km)	ρ (10^3 Kg/m^3)	μ 10^{10} Pa	λ 10^{10} Pa
1	4	2.2	1.065	1.390
2	7-8	2.7	2.940	3.519
Mantle		3.1	5.211	6.554

We use two spherical point sources in a layered crust to consider the second hypothetical case in which the inflation is caused mostly by magma recharge. Table 3 shows the densities and the elastic parameters considered for the crustal model. The larger spherical source located at 9.9 km has a volume increment (ΔV) of 0.036 km^3 while the second at only 0.008 km^3 is located at depth 7.3 km. This model can be used to interpret the 1989 inflation event in Long Valley Caldera (California) (Tiampo et al., 2000). There have been three episodes of rapid inflation of the central resurgent dome, accompanied by seismicity inside and around the caldera, without any eruptions or deflations between these episodes over the last 20 years in Long Valley. The first episode began in 1980 and continued in the Eighties. The second episode started in October 1989 after several years of relative calm. The most recent episode of inflation and uplift of the resurgent dome within the caldera occurred in 1997 – 1998. For more details see the very extensive

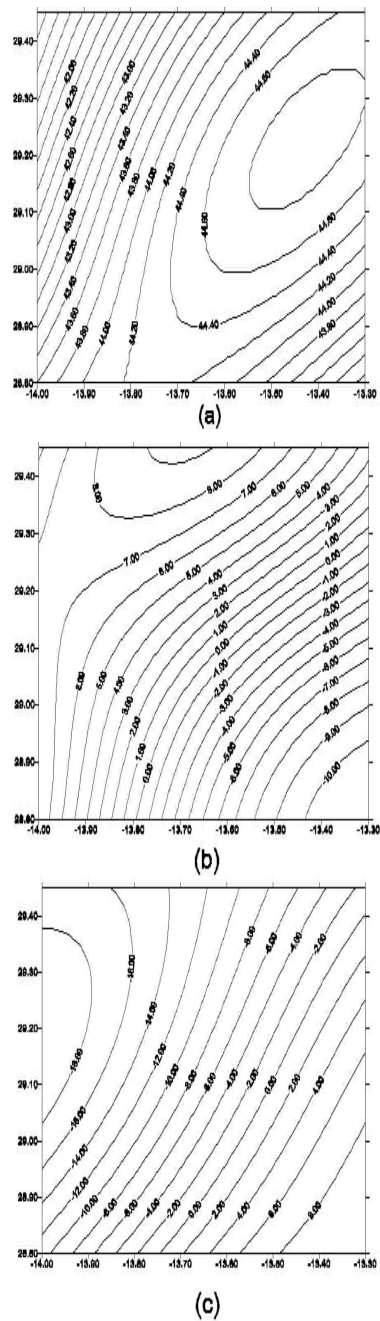


Figure 5. (a) geoid undulation (m), (b) N-S component and (c) E-W component of the vertical deflection (sec) on the island of Lanzarote obtained with model EGM96.

existing literature about Long Valley Caldera (e.g., Ewert and Harpel, 2000 and other references in this paper).

Table 3. Densities and Lamé constants for the crustal model of Long Valley Caldera.

Layer	Thickness (km)	ρ (10^3 Kg/m^3)	μ 10^{10} Pa	λ 10^{10} Pa
1	2.0	2.56	1.5	3.82
Half-space		2.65	5.0	2.7

Taking into account the chamber volume increases given above and the initial radii of the sources, $a_1 = 4 \text{ Km}$ (Elbring and Rundle, 1985) for the deepest magmatic chamber located beneath the resurgent dome and $a_1 = 1.5 \text{ Km}$ for the shallowest chamber, located beneath the south moat (Sanders, 1984) we can obtain the pressure increase and the final masses of the intrusions. In the equation (McTigue, 1987):

$$\Delta p = 4\mu \frac{\Delta a}{a_1}, \quad (12)$$

Δp is the chamber pressure increase, μ is the rigidity constant, Δa is the intrusion radius increase caused by the expansion volume, and a_1 is the initial radius. If we consider the chambers as a spherical source, in terms of the volume increase we get,

$$\Delta p = 4\mu \left[\left(\frac{3\Delta V}{4\pi a_1^3} + 1 \right)^{1/3} - 1 \right]. \quad (13)$$

Then the final mass is given in the following expression:

$$M_f = \rho \left(\frac{4}{3} \pi a_1^3 + \Delta V \right), \quad (14)$$

where ρ_m is the chamber magma density.

Battaglia et al., (1999) note that the gravity change observed during 1982-1998 period in Long Valley Caldera requires an intrusion of silicate magma and excludes in situ thermal expansion or pressurization of the hydrothermal system as the cause of uplift and seismicity. The mass increment (ΔM) is given by $\rho_m \Delta V$. Using the values described above we can get the pressure and mass changes in the magma chamber. In particular, a density of 3300 kg m^{-3} , corresponding to an intrusion of the type described by Battaglia et al. (1999), and the volume increments obtained by Tiampo et al (2000), result in a $\Delta p = 9 \text{ MPa}$ for the source beneath the resurgent dome (deepest source), with a mass increment of 0.1188 UM and a $\Delta p = 37.7 \text{ MPa}$ for the shallowest source, with a mass increment of 0.026 UM.

The results obtained are shown in Figure 6. The geoid undulation change ranges between -0.025 and -0.1 mm . The maximum value (in absolute value) of change on the vertical deflection components is $0.001''$. That value is again too small to be taken into account when reducing geodetic data. Geoid undulations changes are below the mm. If we compare the results obtained for Long Valley with the EGM96 model (Figure 7) we see that they are totally negligible in magnitude for solving the reference surface problem. At first sight, it may be due to the fact that mass increments we use are small. As regards Lanzarote, the masses considered are not very important and therefore the effects mainly associated to them are small in magnitude. It must be borne in mind that the effects are directly proportional to the mass of the intrusions and therefore the bigger the masses of the intrusion, the bigger the effects that are computed.

4. SUMMARY AND CONCLUSIONS

Terrestrial and satellite-based measurements of crustal deformation have become a routine part of volcano monitoring. The estimation of the effects produced by a magmatic intrusion on the vertical deflection and on the geoid undulation are important since it is usually assume that the reference vertical direction and the geoid height are not significantly affected by the process that is causing the ground deformation. We have extended the deformation elastic-gravitational model developed and described by Rundle, Fernández and co-workers, to calculate the effect on the vertical deflection components caused by the presence of a disturbing point source in an elastic-gravitational layered medium.

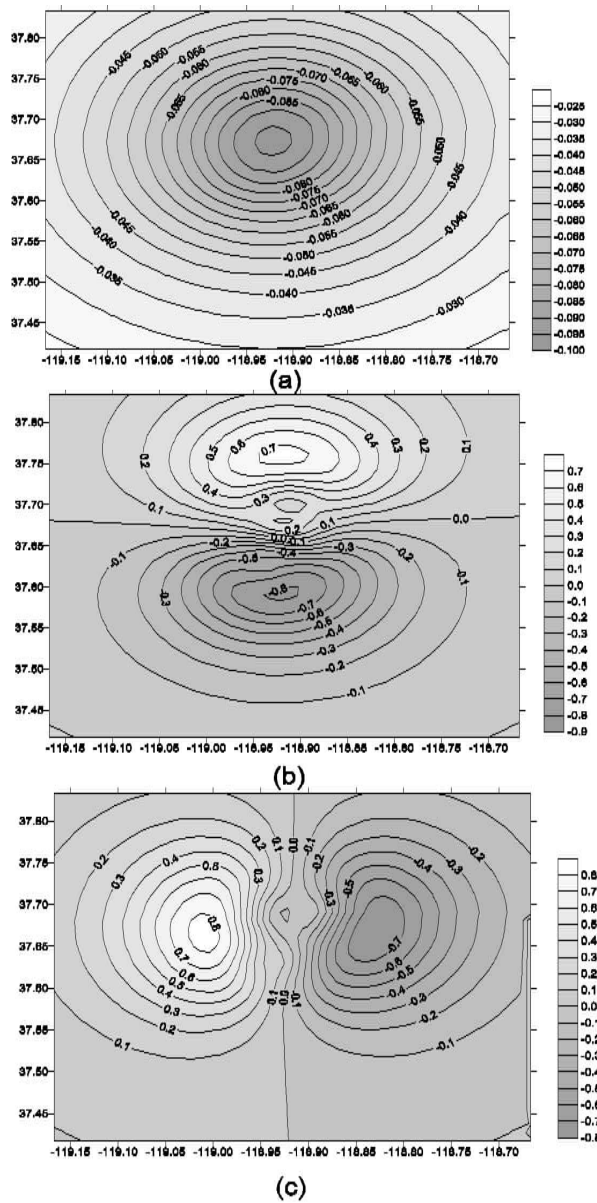


Figure 6. Effect on (a) geoid undulation (mm), (b) N-S component and (c) E-W component of the vertical deflection (msec) produced in Long Valley Caldera, by two intrusions, considering the masses and pressures values given in the text, and the layered crustal model described in Table 3.

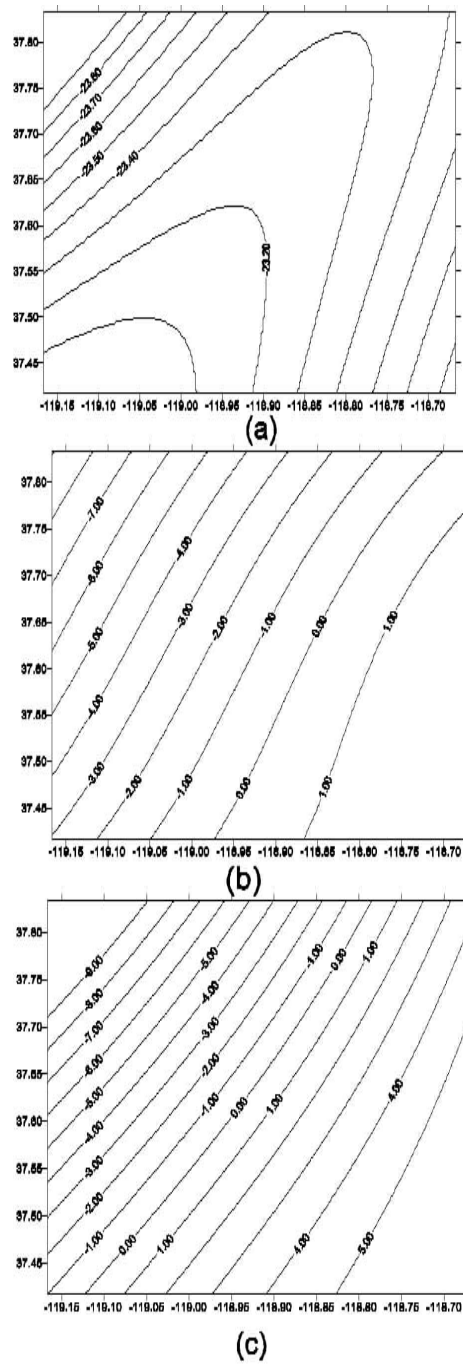


Figure 7. (a) geoid undulation (m), (b) N-S component and (c) E-W component of the vertical deflection (sec) in Long Valley Caldera obtained with model EGM96.

The values obtained with this model have been compared to those calculated using the approximate formulae of Vening-Meinesz. The theoretical cases show that the difference is negligible except for very shallow intrusions.

We have tested the characteristics that the intrusion should need to have, for obtaining detectable effects in volcano monitoring. It has been shown that it is necessary unrealistic pressure and radius increments in the magma chamber to get vertical deflection and geoid height changes that could be detected over the existing background since the computed quantities are related to the intrusive mass. However we have to take in mind this technique represents the geoid and vertical deflection due to a point source, which therefore can be used as a Green's function with which to convolve an arbitrary distribution of subsurface mass or pressure change. A magma chamber radius of around 1 km should be needed to detect effects on both geoid undulations and vertical deflection. This radius is valid for magma density range from 2800 to 3300 kg m⁻³ and the considered depth. Lower radii or density limits would be need for intrusions located at shallow depths. The effects related with the pressurization of the magma chamber would be negligible if we considered realistic pressure changes.

We have also studied the possible effects on active zones with different crustal structures. In applying the model we have considered two cases. In the first case we compute the effect caused mainly by the pressure change in the magma chamber, while in the second case the inflation is mainly produced by magma recharge. The first example approximates to the 1730-36 volcanic episode on Lanzarote Island, The Canaries, Spain. The second one approximates to the 1989 inflation episode in Long Valley Caldera, California, USA. In both cases we observe that existing undulation and vertical deflection background, as provided by model EGM96, would not be affected by the presence of magma intrusions of the types considered.

Therefore, our model can be used to theoretically establish whether the assumption of neglecting vertical deflection and geoid height caused by a magma intrusion is valid or not. Nevertheless, volcanic areas, like Long Valley Caldera and Lanzarote, are usually characterized by large geoid undulation which should be accurately determined before comparing measured displacements using different geodetic techniques. It must be noted that this model is elastic-gravitational and we should use models such as Bonafede et al, (1986), Folch et al, (2000) or Fernández et al, (2001b) when the involved period of time or circumstances imply taking into account of the viscoelastic properties of the medium.

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