A comment on modelling the paleogeography of confined and unconfined marine Gilbert-type deltas

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ABSTRACT

Geological records of marine Gilbert-type deltas, if poorly preserved, permit multiple reconstructions of the paleogeography. Erosional unconformities at the set bases, related to lateral confinement of the deltas, are one of the keys to the problem. The numerical model dealing with constant conditions indicates that (1) ancient lateral confinement cannot be reconstructed only from the pattern/shape of the delta-front curves, unless other basinal conditions have been estimated; (2) the delta front turns landwards at the early stage of the evolution even if any external factor would not change; (3) steady rises of relative sea-level unfavour vertical stacking of multiple sets of the deltas; and (4) the basal unconformities result from quick falls of relative sea-level, or from negative, high-gradient seaward delta-front vectors. Multiply-stacked Gilbert-type systems associated with the unconformities and alluvial-fan deposits can be interpreted to have prograded within fan-dissecting valleys which elongated along the maximum-slope direction of the basin-margin area.

Key words: Numerical model, Gilbert-type delta, lateral confinement, erosional unconformity, fan-dissecting valley

RESUMEN

Cuando los registros geológicos de deltas marinos de tipo Gilbert están mal conservados, permiten múltiples reconstrucciones paleogeográficas. Una de las claves del problema son las superficies erosivas en la base de los sets, relacionadas con el confinamiento lateral de los deltas. El modelo numérico que trata condiciones constantes indica que: (1) el antiguo confinamiento lateral no puede reconstruirse sólo a partir de las curvas de repartición/morfología, a menos que se hayan estimado otras condiciones de la cuenca; (2) el frente deltaico se vuelve hacia tierra en una fase temprana de la evolución, incluso si no cambian los factores externos; (3) las subidas continuadas del nivel relativo del mar no favorecen el apilamiento vertical de múltiples sets deltaicos; y (4) las discordancias basales se deben a descensos rápidos del nivel relativo del mar o a vectores negativos de alto gradiente hacia el mar del frente deltaico. Los sistemas de apilamientos múltiples de sistemas de tipo Gilbert asociados a las discordancias y a los depósitos de abanico aluvial se pueden interpretar como el resultado de progradaciones dentro de valles encajados en el abanico, alargados según la dirección de máxima pendiente del margen de cuenca.

Palabras clave: Modelo numérico, delta tipo Gilbert, confinamiento lateral, discordancia erosiva, valle disector de abanico.

INTRODUCTION

Poorly-preserved geological records make possible multiple reconstructions of paleogeography and make it difficult to choose the most reasonable one. The present paper offers a comment on this problem for the case of marine Gilbert-type systems.

The Gilbert-type delta ideally shows a distinct tripartite organization consisting of topset, foreset and bottomset and develops where the sediment supply has been sufficient to construct the composite depositional prism. Gilbert-type deltas were formerly assumed to develop uniquely in lacustrine basins, as supported by the theory of homopycnal flow (Bates, 1953). The occurrence in marine environments has, however, been recently documented by many authors (e.g., Prior, Wiseman & Bryant, 1981; Prior et al., 1982; Postma & Roep, 1985; Postma, 1984; Postma & Cruickshank, 1988; Ori & Roveri, 1987; Colella, De Boer & Nio, 1987; Muto & Blum, 1989; García-Mondéjar, 1990; Gawthorpe & Colella, 1990). Marine Gilbert-type systems tend to occur as «Gilbert-type fan deltas» (Ethridge & Wescott, 1984; Massari & Colella, 1988) and thus to be very coarse-grained. The density contrast between the sea water and the inflowing river water causes the separation of
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the muddy fraction from the coarse-grained sediments and permits the
development of steep mud-poor coarse-grained foresets (Colella et al., 1987).

Undoubtedly, laterally confined or protected low-energy settings favour
regular progradation of the Gilbert-type systems because of minimal sediment
dispersal by basin processes (Colella et al., 1987; Muto & Blum, 1989).
Reconstruction of ancient confinement from the poorly-preserved records is
a hard work, however, because the confinement can operate in various
degrees and on different scales. One of the basic questions is whether the
progradation took place in open or laterally confined situations; another is
what was responsible for the confinement.

We propose some principles of paleogeographic reconstructions for
ancient marine Gilbert-type deltas. Some of the deltas show clear erosional
unconformities at their bases (Kondo, 1985; Yano, 1986; Colella et al., 1987;
Colella, 1988a; Muto & Blum, 1989; García-Mondéjar, 1990; Gawthorpe &
Colella, 1990; see also Ori & Roveri, 1987). In our opinion the erosional
unconformity is related to the lateral confinement and must be one of the keys
to the paleogeography.

MULTIPLE POSSIBILITIES

For a clearer understanding of the problem, a couple of excellent,
published examples of ancient marine Gilbert-type deltas are introduced from
the upper Pleistocene strata exposed on the Pacific coast of central Japan
(Figs. 1, 2). One is from the Kunosan Formation in the Udo Hills (Tsuchi,
1960; Sugiyama et al., 1982; Kondo, 1985); the other from the Kouzu and
overlying Fudousan Conglomerates in the Oiso Hills (Yano, 1986). The ages
of the were estimated to be 0.5-0.1 Myr BP for the Negoya Formation
underlying the Kunosan Formation, 0.5-0.3 Myr BP for the Kouzu
Conglomerate, and 0.3-0.1 Myr BP for the Fudousan Conglomerate (Okada,
1989). These hills are located close to the convergent boundary between the
Philippines Sea and Eurasian plates. Active neotectonics related to the plate
movements are implied from landward dipping of the Pleistocene beds
(northwestwards in the Udo Hills, northwards in the Oiso Hills) (e.g. Tsuchi,
1984). However, paleomagnetic evidence indicates that the Udo and Oiso
areas have not undergone significant horizontal rotation since 0.6 Myr BP
(Kitazato et al., 1981; Koyama & Kitazato, 1989).

The Kunosan Formation as well as the Negoya Formation and the Kouzu
and Fudousan Conglomerates consist of (i) a single or multiple sets of steeply
inclined (over 20°) pebble to boulder beds with a very coarse sandy matrix and
subhorizontal beds (or erosional surfaces; see below) at the top and/or base,
and (ii) gently inclined mudstone or alternating beds of gravel and silt, which
yield molluscs and benthic foraminifers indicative of shallow marine
environments and contain slump deposits. The maximum thickness of
Fig. 1.—Two examples of marine Gilbert-type systems of upper Pleistocene age. A. The Udo Hills and the Recent Abegawa fan-delta (simplified from Kondo, 1985). B. The Oiso Hills and the Recent Sakawagawa fan-delta (adapted from Yano, 1986). Note that the measured foreset-orientations significantly differ from the general trends of the Recent fan deltas.

Fig. 1.—Dos ejemplos de sistemas marinos de tipo Gilbert de edad Pleistoceno Superior. A. Las Udo Hills y el fan delta reciente de Abegawa (simplificado de Kondo, 1985). B. El Oiso Hills y el fan delta reciente de Sakawagawa (adaptado de Yano, 1986). Obsérvese que las medidas de orientaciones de foreset difieren considerablemente de las tendencias generales de los fan deltas recientes.

Individual sets is 100 m for the Kunosan Formation, 35-65 m for the Negoya Formation, 350 m for the Kouzu Conglomerate, and 260 m for the Fudousan Conglomerate. The dimensions, geometry and facies strongly suggest a marine Gilbert-type system and associated prodelta (see Kondo, 1985; Yano, 1986 for detailed descriptions and interpretations). Those deltas, except the Negoya system, differ sharply from «classical» Gilbert-type deltas first described by Gilbert (1885), as to the character of the set bases (Fig. 2). Tsuchi (1960) and Kondo (1985) noted that the boundary between the Kunosan and Negoya Formations is an erosional unconformity descending eastwards or
northeastwards. Yano (1986) emphasized the presence of erosional unconformities between the Fudousan and Kouzu Conglomerates and between the Kouzu Conglomerate and the underlying rocks including Miocene tuffaceous deposits. However, those authors gave no explanation to the origin of the unconformities and their paleogeographic significances.

The ancestral Abegawa River was responsible to the construction of the Kunosan Gilbert-type system, as supported by petrographic evidence (Tsuchi, 1960; Sugiyama et al., 1982). The measured orientations of the foresets are uniquely the northeastward (Fig. 1 A). Tsuchi (1960) and Kondo (1985) interpreted that the ancient system had an open geometry which expanded southeasterly as the Recent Abegawa fan-delta did, and that only a small part in the east has been preserved.

A similar reconstruction was adopted for the Kouzu and Fudousan Gilbert-type systems (Yano, 1986). The observed foresets dip eastwards (Fudousan) and generally northeastwards (Kouzu), whereas the Recent fan-delta mainly fed by the Sakawagawa River elongates southeastwards (Fig. 1 B). Yano (1986) interpreted that the geological records represent eastern small parts of the open fan-deltas spreading southerly.

The above reconstructions look certainly possible. However, the poor preservation does not exclude alternative paleogeography, and there is no
logical necessity of adopting the existing interpretation. The presence of basal erosional unconformities implies that the Gilbert-type systems prograded along a morphologic depression caused by valley entrenchment, i.e. under some lateral confinement. Actually, Muto & Blum (1989) reported those occurring within dissecting-valleys of coastal fans (sensu Muto, 1989; cf. Nemec & Steel, 1988). Colella et al., (1987) also recognized an erosional unconformity between two superimposed Gilbert-type systems, although it can hardly be imagined from Colella's (1988 b) drawing for the inferred depositional setting (see her Fig. 14).

Consequently, multiple reconstructions are possible for the above cases. At least three possibilities can be imagined (Fig. 3). One possibility is of «open fan-deltas», where no or insignificant lateral confinement operates upon the deltaic sedimentation and the delta grows quasi-isotropically. The river streams emerging from the feeder-canyon mouth can have a radiation angle of 180°. The preserved foresets do not represent the general trend of the progradation. The paleogeography proposed by Tsuchi (1960), Kondo (1985) and Yano (1986) falls under this category.

A second possibility is of «basin-confining deltas (or fan-deltas)», where the basin configuration strongly restricts the lateral or radial expansion of Gilbert-type systems. The foreset orientations indicate the trends of the elongate basins. This possibility is best illustrated by fjord deltas (e.g. Prior & Bornhold, 1988). However, small deltas occurring in relatively wide fjords may be laterally unconfined (e.g. Postma & Cruickshank, 1988); such deltas will rather illustrate the first possibility. There exists a spectrum of numerous intermediates between the first and second possibilities.

A third possibility is of «channel-confining deltas (or fan-deltas)», where the deltas are confined to channels entrenched into the coastal-fan and/or related deposits. If the system occurs in fan-delta settings, the channel is most likely to be the «fan valley» of Muto (1987), a fan-dissecting channel being connected with the feeder canyon of the coastal-fan system. The early Pleistocene Ogasayama Formation in central Japan illustrates this possibility (Muto & Blum, 1989). Measured orientations of the foresets represent the trend of the channel. Because the dissection of the fan takes place by the feeding river with relative sea-level lowering, the fan valley trends in the maximum-slope direction of the coastal region (Muto, 1989).

The geological records preserved in the Udo and Oiso Hills do not deny the second and third possibilities, which rather are convenient for interpreting the erosional unconformities. We would state that the ancient systems completely differ from the Recent ones in terms of the topographic conditions. The ancient channels or basement walls must have trended northeastwards (Kunosan, Kouzu) or eastwards (Fudousan), suggesting that the two coastal regions were affected by westward-tilting events after the deposition (cf. Muto, 1989). Stacked deltas in the Negoya system may unfavour the third possibility because no erosional unconformity has been recognized at the set bases (Fig. 2).
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One of the reasons why the multiple reconstructions are allowed is that there has been no established framework for the paleogeography. Here we propose a numerical model of marine Gilbert-type systems and, based on it, discuss preferable reconstructions later.

The Gilbert-type delta has a clear slope-break at the «delta front». This term usually designates the zone which includes the shoreline and the seaward-dipping profile which extends offshore (e.g. Elliot, 1986). However, the present paper defines it as the line (point in a cross section) of the topset/foreset or subaerial/subaqueous boundary (Fig. 4). The characteristic geometry makes it easy to create a numerical model of Gilbert-type deltas. The bottomset beds usually occupy only a small part of the entire delta; in fact, some are completely absent in the proximal area (e.g. Colella et al., 1987; Muto & Blum, 1989; Gawthorpe & Colella, 1990). Ignoring the bottomset for simplification, an approximate volume of the delta can be described using the
Fig. 4.—Geometrical parameters of Gilbert-type deltas assumed in the present paper. A. Plan view. B. Longitudinal profile.

Fig. 4.—Parámetros geométricos de los deltas de tipo Gilbert, asumidos en este trabajo. A. Planta. B. Perfil longitudinal.

delta-front’s coordinates (X, Y) in a vertical x-y plane parallel to a direction of the progradation (Fig. 4).

Assuming a Gilbert-type delta fed through a single narrow canyon, the solid S shown by Fig. 5, a part of cone (Cone A - D D'D''), approximates the delta body. Its volume (S) can be calculated with the formula \[ S = C_2 - C_1 - C_4 + C_3 \] defined in Fig. 5. C_1, C_2, C_3 and C_4 are given by:

\[
C_1 = \frac{\pi}{3} X^3 \tan \beta \times \frac{\vartheta}{360}
\]

(1)

\[
C_2 = \frac{\pi}{3} X'^2 (X \tan \beta + Y + X' \tan \Phi) \times \frac{\vartheta}{360}
\]

(2)

\[
C_3 = \frac{\pi}{3} X^3 \tan \alpha \times \frac{\vartheta}{360}
\]

(3)

\[
C_4 = \frac{\pi}{3} X'^3 \tan \Phi \times \frac{\vartheta}{360}
\]

(4)

respectively; where \( \vartheta (\circ) \) is radiation angle of the delta at the feeder-canyon mouth \( (0 < \vartheta \leq 180\circ) \), \( \alpha (\circ) \) is topset inclination, \( \beta (\circ) \) is foreset inclination and
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Volume of Solid $[BC'C''-OD'D''] = S$

- Cone $[A-CC'C''] = C_1$
- Cone $[A-DD'D''] = C_2$
- Cone $[B-CC'C''] = C_3$
- Cone $[O-DD'D''] = C_4$

$S = C_2 - C_1 - C_4 + C_3$

Fig. 5.—Estimation of an approximate volume of a Gilbert-type delta fed through a single narrow canyon. $P(X,Y)$ and $P'(X',Y')$ indicate the spatial coordinates of the delta front and toe, respectively. The solid $S$ constitutes a part of a cone (Cone A-DD'D'') the volume of which is given by the formula shown in the figure.

Fig. 5.—Estimación del volumen aproximado de un fan delta de tipo Gilbert alimentado por un cañón estrecho simple. $P(X,Y)$ y $P'(X',Y')$ indican las coordenadas espaciales del frente deltaico y el pie respectivamente. El sólido $S$ constituye una parte de un cono (Cono A-DD'D'') cuyo volumen lo da la fórmula mostrada en la figura.

Φ (°) is basal-slope inclination (Fig. 4). Φ can be a useful index of the lateral confinement; Φ for the open fan-deltas ranges up to 180°.

$X'$ represents the x-coordinate of the delta's toe, i.e. the base of the foreset slope, and can be expressed with $X$ and $Y$:

$$X' = \frac{X \tan \beta + Y}{\tan \beta - \tan \Phi}$$

Therefore,

$$S = C_2 - C_1 - C_4 + C_3$$

$$= \frac{\pi \Phi}{1080} X'^2 \left( (X \tan \beta + Y) - X^3 (\tan \beta - \tan \alpha) \right)$$
Assuming that sediment input $D$ per unit time is constant and that all the sediments supplied are to contribute to the growth of the delta, the total sediment supply, or the sediment volume ($S$), at the time $t$ after the initiation of the progradation is given by:

$$S = D t$$

(7)

If the basin is subsiding at a uniform rate of $v$,

$$Y = v t \quad (Y = 0 \text{ at } t = 0)$$

(8)

because the $Y$-coordinate equals to the relative sea-level. Equations (6) and (7) can be combined without using $t$:

$$\frac{DY}{v} = \frac{\pi \Phi}{1080} \left\{ \frac{(Y + X \tan \beta)^3}{(\tan \beta - \tan \Phi)^2} - X^3 (\tan \beta - \tan \alpha) \right\}$$

(9)

This is rewritten as:

$$aX^3 + 3b^2X^2Y + 3bXY^2 + Y^3 - cY = 0$$

(10)

where

$$a = (2\tan\beta - \tan\Phi) \tan\beta \tan\Phi + \tan\alpha (\tan\beta - \tan\Phi)^2$$

(11)

$$b = \tan\beta$$

(12)

$$c = 1080 D (\tan\beta - \tan\Phi)^2 / (\pi \Phi v) \quad (v \neq 0)$$

(13)

Equation (10), the delta-front's coordinate equation, provides the logical base of discussion in the present paper. This equation does not explain the origin of individual lobes constituting a delta but the general dimensions and geometry of the whole body of the delta.

Substituting zero into $X$ in the coordinate equation, we find that there exist two solutions for $Y$; i.e. $Y = 0$ and $Y = \sqrt{c}$. This implies that $X$ has a maximal value, or that the delta front turns landwards at some time and finally attaches with the feeder-canyon mouth. We can confirm these as follows:
Differentiating the coordinate equation with respect to $X$,

$$3aX^2 + 6b^2 XY + 3b^2X^2 (\frac{dY}{dX}) + 3bY^2 + 6bXY (\frac{dY}{dX}) + 3Y^2 (\frac{dY}{dX}) - c (\frac{dY}{dX}) = 0$$

(14)

$$\frac{dY}{dX} = \frac{3(aX^2 + 2b^2 XY + bY^2)}{c - 3 (bX + Y)^2}$$

(15)

$X$ has the maximal value when $dY/dX = \infty$, or when the denominator of the right becomes zero. Assuming

$$c - 3 (bX + Y)^2 = 0$$

(16)

the coordinate equation is rewritten as:

$$aX^3 - 3bXY^2 - 2Y^3 = 0$$

(17)

$$a - 3b\left(\frac{Y}{X}\right)^2 - 2 \left(\frac{Y}{X}\right)^3 = 0$$

(18)

we finally get:

$$2k^3 + 3bk^2 - a = 0$$

(19)

where $k = Y/X$ ($X$ becomes the maximum when $Y = kX$). This equation always has a single positive solution for $k$, which is less than $b/2$ (Fig. 6).

The maximum of $X$ and corresponding $Y_{X_{\text{max}}}$ and $t_{X_{\text{max}}}$ are given by:

$$X_{\text{max}} = \frac{\sqrt{c/3}}{k + b}$$

(20)

$$Y_{X_{\text{max}}} = \frac{k \sqrt{c/3}}{k + b}$$

(21)

$$t_{X_{\text{max}}} = \frac{k \sqrt{c/3}}{\nu (k + b)}$$

(22)

These values do not depend upon $D, \nu$ and $\theta$. The time $t_{\text{at}}$ at which the attachment takes place is given by:

$$t_{\text{at}} = \frac{Y_{\text{max}}}{\nu} = \frac{\sqrt{c}}{\nu}$$

(23)
The curve of \( f(k) = 2k^3 + 3bk^2 - a \). Note that there always exists a single positive solution \( k \) in eq. (19) and it is less than \( b/2 \).

When \( t_{at} \), the delta loses its topset part; such deltas are no longer regarded as "Gilbert-type". Period of the delta-front’s progression relative to the life period of the delta, \( T_{pro} \), is given by:

\[
T_{pro} = \frac{t_{max}}{t_{at}} = \frac{1}{\sqrt{3}} \left( \frac{k}{k+b} \right)
\]  

(24)

\( T_{pro} \) is a monotone increasing function of \( k \) (\( k > 0 \)). Substituting \( b/2 \) for \( k \) in eq. (24),

\[
T_{pro} = \frac{1}{3\sqrt{3}} = 0.19245...
\]  

(24)

Therefore, the delta-front’s progression is possible only during the early short time (less than 20%) of the delta’s evolution (Fig. 7 A). The “turning” of the delta front takes place even if the steady conditions would be maintained, or no external factor would change; while the delta toe only makes progression (Fig. 7 B).

The prediction that the delta front begins to retrograde at some time does not coincide with our empirical “fact”. Gilbert-type deltas with longitudinal profiles such as drawn in Fig. 7 B have never been reported from the stratigraphic records (see Colella et al., 1987; Postma & Cruickshank, 1988; Muto & Blum, 1989; Bardají et al., 1990; García-Mondéjar, 1990), implying that the natural delta-fronts do not retrograde but only prograde or are reworked. However, the discordance between the prediction and the “fact” is
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\[ X_{\text{max}} = \frac{\sqrt{c/3}}{k+b} \]

\[ Y_{\text{max}} = \sqrt{c} \]

\[ Y_{X_{\text{max}}} = \frac{k\sqrt{c/3}}{k+b} \]

Fig. 7. A.—Delta-front curve for a Gilbert-type delta evolving under steady/constant conditions \((v > 0)\). B. Longitudinal profiles (outlines) of the delta drawn at constant time intervals. Note that the progression (seaward migration) of the delta front is replaced by the retrogression (landward migration) in the early stage \((t_{\text{xmax}})\) of the entire life period of the delta \((t_{\text{xmax}})\).

quite probable because of our highly idealized assumptions that \(v, D, \alpha, \beta, \Phi, \) and \(\delta\) do not change during the entire life period of the delta. The
discordance suggests that (i) the initial, steady conditions are unlikely to be maintained in natural Gilbert-type deltas; (ii) the modified conditions intercept the «ideal» evolution; and (iii), even if the delta has accomplished it, the subsequent external/erosional processes do not permit the deposits to be preserved in the stratigraphic records.

One of the most probable origins of the discordance is temporal changes of \( v \). Increases in \( v \) are equivalent with decreases in \( D \), and vice-versa (see eq. 13). The coordinate equation is still useful even if \( v \) is a function of time, because:

\[
Y = \int_0^1 v(t) \, dt
\]  

(26)

We below discuss how the lateral confinement, when combined with \( v(t) \), controls the evolution of the deltas, and how the erosional unconformities are generated.

### PALEOGEOGRAPHIC IMPLICATIONS

If \( v(t) \) is kept constant, the vertical and longitudinal dimensions of the deltas are inversely proportional to the square root of \( \theta \) (eqs. 20, 21 and 22). For example, \( X_{\text{max}} \) at \( \theta = 10^\circ \) are twice as large as at \( \theta = 40^\circ \) (Fig. 8). The delta-front curves for different \( \theta \) show a geometrical similarity. This means that, unless other variables including \( v(t) \) and \( D \) have been estimated, \( \theta \) cannot be reconstructed only from the pattern or shape of observed delta-front’s curves.

The basin water at \( t = t_{\text{at}} \) is much deeper than at \( t = 0 \) to \( t_{\text{xmax}} \) (Fig. 7 B). The second cycle of the delta growth can never begin as far as the steady conditions are being kept and would require a substantial decrease of \( v \) and/or increase of \( D \). In fact, multiply-stacked Gilbert-type deltas usually present erosional unconformities at their set boundaries which clearly indicate some episodic events disturbing the steady state.

Figure 9 shows an ideal example of a series of delta-front curves calculated for \( \theta \) and \( v(t) \) which is decreasing and getting close to zero. Three patterns are distinguished: (i) for large \( \theta \) (\( \leq 180^\circ \)), the initial progression is followed by the retrogression and finally causes the attachment with the feeder-canyon mouth; (ii) for intermediate \( \theta \), the initial progression is followed by the retrogression which, in turn, is replaced by the second progression; (iii) for small \( \theta \), no retrogression takes place. The first pattern indicates that the lateral dispersion of the sediments forbids the delta to develop until the relative sea-level rise is sufficiently decelerated. In the second and third patterns the poor dispersion helps the delta to bear the fast rise of relative sea-level in the early stage, so that the delta front can move seawards as far as the system exists. However, rate of the progradation gradually decreases because of the spatial deepening of the
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basin due to given \( \Phi \). Certainly the lateral confinement is advantageous to regular and steady progradation of the delta.

Figure 10 shows another ideal example of a series of delta-front curves calculated for \( \vartheta \) and \( v(t) \) which changes periodically (\( v > 0 \)). The resulting curves oscillate horizontally. Because of the temporal and spatial deepening
Fig. 9. A.—Curve of given $v(t)$, which gradually decreases and gets close to zero. Other conditions are kept constant. B. Computer-drawing of consequent delta-front curves. $\theta$, where not indicated, is given every 10 degrees. Note that small $\theta$ favours regular and steady progradation of the delta.

Fig. 9. A.—Curva de un $v(t)$, dado que disminuye progresivamente y se hace próximo a cero. Las demás condiciones se mantienen constantes. B. Dibujo de ordenador de las curvas consecuentes de frente deltaico. Cuando no se indica nada en contra, los valores de $\theta$ son cada 10°. Obsérvese que los valores pequeños de $\theta$ favorecen la progradación regular y constante del delta.
of the basin; (i) the delta-fronts' progression is limited for every $\theta$; (ii) the attachment with the feeder-canyon mouth occurs finally; (iii) the oscillations gradually become less apparent; (iv) variations due to different $\theta$ become

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**Fig. 10.** A.—Curve of given $v(t)$, which oscillates periodically ($v \geq 0$). Other conditions are kept constant. B. Computer-drawing of consequent delta-front curves calculated for every 10 degrees of $\theta$.

**Fig. 10.** A.—Curva de un $v(t)$, dado que oscila periodicamente ($v \geq 0$). Las demás condiciones se mantienen constantes. B. Dibujo de ordenador de las curvas consecuentes de frente deltaico calculadas para cada 10$^\circ$ de $\theta$. 
unclear; and (v) the general pattern is analogous with the curves for perfectly steady conditions (Fig. 8). In this example the Gilbert-type systems basically consist of single sets and will have no erosional unconformities, because the given dimension of the oscillations is far smaller than $X_{\text{max}}$.

*Multiply-stacked sets would need such sudden, substantial changes of $v$ as to turn delta-front vectors quickly. Magnitude and direction of the vector are defined as:*

$$
\frac{\sqrt{(dx/dt)^2 + (dy/dt)^2}}{(dx/dt, dy/dt)}
$$

(27)

(28)

respectively. When gradient of the delta-front vector ($dy/dx$) becomes smaller than $-\tan \alpha$ and larger than $-\tan \beta$ ($\alpha < \beta$, for Gilbert-type systems), i.e.

$$
\frac{dy}{dx} < \frac{\tan \alpha}{\tan \beta} < -\tan \alpha
$$

(29)

the Gilbert-type system undergoes erosion or modification of the geometry (Fig. 11). If the delta front is moving landwards ($dx/dt < 0, dy/dt > 0$), the young part of the foreset beds must be reworked and subsequently redeposited over the topset slope (Fig. 11 A). However, this is absolutely impossible and ascribed to the assumption that the two-slope geometry represented by $\alpha$ and $\beta$ is always maintained, or that grain-size distribution, delta-basin dynamics and river discharge remain constant. In other words, the delta-front movements predicted by the coordinate equation do not coincide with what happens actually. Probably, the present delta-front is abandoned while the new delta-front occurs upslope. The system will begin to develop the four-slope geometry consisting of active topset-foreset slopes on the upslope side and inactive ones on the downslope side. When the marine transgression is followed by substantial deceleration of $v$, a new set of Gilbert-type delta will prograde just above the older one. The transgression may cause marine erosion of the older deltas; the boundary of the two superimposed sets may be unconformable. The unconformity would not have large dimensions and not be associated with significant/deep denudation, because the erosional processes are limited to the shoreface zones.

The marine transgression followed by subsequent decreasing of $v$ and the resulting sedimentary events are actually possible (i) with eustatic sea-level fluctuations, and/or (ii) when initial high-magnitude increment of dip-slip fault motion is followed by creep in small increments (Fig. 9 A for a single cycle). The second scenario is nearly the same as Colella's (1988 a) Castovillari Model which aims to explain some Gilbert-type fan-deltas in the Plio-
Fig. 11.—Delta-front vectors and corresponding geometrical modifications suggested by the numerical model. A. Delta-front vector due to a quick rise of relative sea-level. The vector orientates the landward and has a negative, small gradient. Assuming that the two-slope geometry represented by a and b, is maintained, the sediments are required to remove from the foresets ($S_f$), the topographically low place, to the topsets ($S_t$), the topographically high place. B. Delta-front vector due to a quick fall of relative sea-level. The vector orientates the seaward and has a negative, large gradient. The sediments are required to remove from the topsets ($S_t$) to the bottomsets ($S_b$). For the two-slope geometry to be maintained, however, the denudation must take place uniformly over the topset slope at any moment.

Fig. 11.— Vectores de frente deltaico y modificaciones geométricas correspondientes sugeridas por el modelo matemático. A. Vector de frente deltaico debido a una subida rápida del nivel relativo del mar. El vector se orienta hacia tierra y tiene un gradiente negativo y pequeño. Asumiendo que se mantiene la geometría de dos pendientes representada por a y b, se requiere que se remueva sedimento de los foresets ($S_f$), el lugar geométricamente más bajo hacia los topsets ($S_t$), que son las partes más altas topográficamente. B. Vector del frente deltaico debido a una bajada rápida del nivel relativo del mar. El vector se orienta hacia el mar y tiene un valor negativo y grande. Se requiere que los sedimentos se remuevan de los topsets ($S_t$) a los bottomsets ($S_b$). Sin embargo, para que se mantenga la geometría de dos pendientes, se debe producir una denudación uniforme en la pendiente del topset.
Pleistocene Crati Basin, southern Italy. However, the temporal deepening of the basin restricts the seaward migration of the delta fronts; the younger deltas can hardly prograde beyond the $X_{\text{max}}$ of the older deltas. To settle this problem, Colella (1988 a) assumed that the bottomsets and/or non-deltaic fine-grained sediments were thickly accumulated as well as the topsets and foresets (see her Fig. 4).

Gawthorpe & Colella (1990) found multiply-stacked delta sequences with erosional truncations, from the Plio-Pleistocene Kerinitis fan-delta system exposed in the footwall of the active fault bounding the south side of the Gulf of Corinth. They noted that the progradation onto the hanging wall is limited; and, referring the parallel studies on alluvial architectures (e. g. Bridge & Leeder, 1979), ascribed it to the tectonic tilt vectors (see also Leeder & Alexander, 1987). Even if such vectors do not operate, however, we can expect the limitation of the deltas’ progradation, as discussed above.

As far as $\frac{dY}{dt} > 0$ ($v > 0$), no major unconformity can occur. Conversely, when $\frac{dX}{dt} > 0$ and $\frac{dY}{dt} < 0$ ($v < 0$), the coordinate equation predicts that the upper part of the topsets is eroded out by the feeding-river’s processes and redeposited on the foreset slope (Fig. 11 B). This event seems possible when the relative rise of sea level is replaced by the fall, and will appear as an unconformity in the stratigraphic records if preserved. However, it is difficult to imagine that the erosional processes prevail uniformly over the topset slope at any moment. Instead, they are likely to be restricted to some narrow place(s) and thus cut valley(s) into the topset slope (Muto, 1987, 1988). The present topset slope is surely abandoned because the supplied sediments just pass through the major dissecting valley (fan valley) to be accumulated on the foreset slope. The new delta body begins to develop from the valley’s mouth; thus the system will have the four-slope geometry consisting of inactive topset-foreset slopes on the upslope side and active ones on the downslope side. This also represents the discordance between the predictions by the model and the real systems. When the relative sea-level fall is followed by the rise, a new set of Gilbert-type delta will prograde within the valley (Muto & Blum, 1989).

The basal unconformities of the Kunosan, Kouzu and Fudousan Gilbert-type systems imply quick sea-level falls and subsequent rises perhaps due to Quaternary eustatic changes combined with regional tectonic movements. It is uncertain if the Negoya system experienced relative sea-level falls, because geometry of the set boundaries within the system is unclear yet.

CONCLUSIONS

1. When marine Gilbert-type deltas are recognized from the poorly-preserved geological records, at least three possibilities should be examined
for their paleogeography: i.e., open systems, basin-confining systems and channel-confining systems.

2. The radiation angle, or the lateral-confinement angle, of the delta cannot be inferred only from the pattern/shape of observed delta-front curves, unless other factors including relative sea-level changes and sediment supply have been estimated.

3. There exists a limitation of seaward migration of the delta front as far as the relative sea-level continues to rise. The delta front tends to turn landwards at some time even if any external factor would not change. However, natural deltas look not to retrograde. Modifications of the initial conditions due to external factors may account for the discordance between the numerical model and the empirical «fact».

4. Constant conditions can never contribute to multiple stacking of marine Gilbert-type deltas, which requires sudden and substantial changes of the rate of relative sea-level fluctuations. The basal erosional unconformity can result from a quick fall of relative sea-level, or a negative, high-gradient delta-front vector orientating the seaward. If the sea-level is lowering slowly, the unconformity will not be generated. Multiply-stacked Gilbert-type systems associated with such unconformities and alluvial-fan deposits probably represent the «channel-confining systems». Those channels are the major fan-dissecting valleys elongating along the maximum-slope directions of the basin-margin areas.

5. The numerical model, which assumes highly idealized conditions, provides a logical base of reconstruction of the paleogeography. External factors controlling natural Gilbert-type deltas will be better understood when compared with the «ideal» deltas.

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