The relation between transport provision and accessibility: a mathematical perspective

Alan HAY *

It is generally recognised that accessibility from an area to transport facilities is inversely related to the density of provision, a point which can be demonstrated mathematically for regular networks (Hay, 1973, pp. 36-37; Melut and O'Sullivan, 1974; Evans, 1985), and had long before been noted empirically for irregular networks (e. g. Jefferson 1929). This relationship holds true for linear fixed networks (e. g. roads) linear service networks (e. g. bus services) and for point facilities (e. g. air ports).

It is less widely recongnised that if, for a given area the costs of provision are linearly related to the density of provision the resulting equation

$$C \ = \ \frac{a}{L} \ + \ bL \qquad\qquad 1.$$

(where C = total social cost of the system
  L = level of provision
  a and b are constants)
can be differentiated to yield a least total social cost level of provision where

$$L \ = \ \frac{a}{b} \qquad\qquad 2.$$

The coefficient a in this equation reflects the number of trips made and the costs of moving to the network in a given time period while b reflects the costs of providing network over the same period (including a sum for servicing capital costs and maintenance). This derivation was used by

* (University of Sheffield) United Kingdom.

Howe (1971) and subsequently by Rayner (1980) and Hay (1982) to specify regression relationships between network density and population variables in developing countries and can be seen as a *post hoc* rationalisation of the results reported by Taaffe, Morril and Gould (1963). Even if the simplifying assumptions are relaxed the same basic forms emerge from the differentiation.

A second consequence of the density/accessibility relationship refers to the provision of scheduled services (3. g. bus networks). If an operator has a fixed capacity (vehicle/km/day) there is an inverse relation between the density of services in space and the frequency of services in time (Bly and Oldfield, 1974). For the user of those services this becomes an inserve relationship between access costs and waiting times (because just as access is inversely related to density so waiting times are inversely related to frequency). If total user cost (U) is seen as related to these two components

$$U = \frac{x}{L} + yL \qquad\qquad 3.$$

where x is a coefficient for the cost of access (per unit of distance) and y is a coefficient for costs of waiting (per unit of time), it too can be differentiated to given an optimal network length where

$$L = \sqrt{\frac{x}{y}} \qquad\qquad 4.$$

It can be shown that this is achieved when access costs equal waiting costs. Workman (1985) has shown that the same relation is true for a variety of regular networks. Empirical studies have shown that these relations too hold in practice, though the variances in irregular networks are naturally higher than those for regular networks.

The two arguments put forward above can be combined to establish the optimal allocation of public service capacity to an area assuming that the capacity so allocated will be used with the optimal trade off between network density and service frequency for a uniformly distributed population. The first step is to derive the solution for optimal network length (given capacity, T) as

$$L = \sqrt{\frac{aT}{b}} \qquad\qquad 5.$$

If this is substituted in the equation

$$C = \frac{Pa}{L} + PbL \qquad\qquad 6.$$

then $\qquad\qquad C = \sqrt{\frac{4ab}{T}} \cdot P \qquad\qquad 7.$

where P is a measure of trip making (at its simplest Population, but it could be a weighted value to take into account trip making propensities). The total cost to users and providers would then be given by;

$$M = P \sqrt{\frac{4ab}{T}} + cT \qquad 8.$$

which can be differentiated to give minimum total cost when

$$T = \sqrt[3]{\frac{4abP^2}{c^2}} \qquad 9.$$

The consequences of these highly simplified mathematical relationships for geographical studies of accessibility are two-fold. First, if it is intented to look for statistical associations between network provision and other variables (population, costs of access etc.) these equations suggest a specification of variables which is not intuitively obvious: for example equation 2 above would suggest an association between network provision and the square root of population (see Howe, 1971) while equation 9 suggets an association between sheduled service provision and the cube root of population. The second consequence refers to the assessment of equity or fairness in network provision. It will be evident that these optimal networks will not meet either of the two criteria commonly adopted for equity in provision. On the one hand the optimal networks will in general show much greater variations in provision than a policy of equal access for all, on the other hand they will in general show less variation than would be expected under a policy of making provision simply proportional to population.

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