A summability condition on the gradient ensuring $BMO$.

Alberto FIORENZA*

Abstract

It is well-known that if $u \in W^{1,1}(\Omega)$, $\Omega \subset \mathbb{R}^N$ satisfies $|Du| \in L^N(\Omega)$, then $u$ belongs to $BMO(\Omega)$, the John-Nirenberg Space. We prove that this is no more true if $|Du|$ belongs to an Orlicz space $L_A(\Omega)$ when the N-function $A(t)$ increases less than $t^N$. In order to obtain $u \in BMO(\Omega)$, we impose a suitable uniform $L_A$ condition for $|Du|$.

1 Introduction

In a recent paper Fusco-Lions-Sbordone ([FLS]) gave imbeddings of Orlicz-Sobolev spaces $W^{1,A}(\Omega)$, $\Omega$ a cube in $\mathbb{R}^N$, in Orlicz spaces with exponential growth, when the Young function $A$ is of type $A(t) = t^N \log^\sigma (e + t)$. If $\sigma = 0$, the space $W^{1,A}(\Omega)$ reduces to $W^{1,N}(\Omega)$ and it is well-known that such space is imbedded in $BMO(\Omega)$. If $\sigma = 1$ there are some counterexamples (see [GISS]) showing that $W^{1,A}(\Omega)$ is not imbedded in $BMO(\Omega)$.

In this paper first we show, adapting an example appeared in [GISS], that for any Young function $A(t)$ which grows essentially less than $t^N$, the space $W^{1,A}(\Omega)$ is not imbedded in $BMO(\Omega)$. Such a result has been recently proved, in a different way, in a paper by Cianchi-Pick [CP]. Moreover, if we require that, in some sense, the gradient of a function $u$ is in $L_A(\Omega, \mathbb{R}^N)$ uniformly with respect to the cubes contained in $\Omega$, then

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we get the imbedding in $BMO(\Omega)$, even if the Young function $A(t)$ has a growth essentially less than $t^N$. Namely, let us introduce the uniform Orlicz spaces

$$f \in \mathcal{U}_A(\Omega, \mathbb{R}^N) \iff \sup_{Q \subset \Omega} |Q|^{1/H} \|f\|_{L^A(Q)} < +\infty$$

where the supremum is extended to all cubes $Q$ contained in $\Omega$ with sides parallel to the coordinate axis. If $A(t) = t^N$, then $\mathcal{U}_A(\Omega, \mathbb{R}^N)$ reduces to $L^N(\Omega, \mathbb{R}^N)$; if $A(t) = \frac{t^N}{\log^2(e+t)}$, $\sigma > 0$, then $\mathcal{U}_A(\Omega, \mathbb{R}^N)$ contains $L^N(\Omega, \mathbb{R}^N)$. We show that for such $A$ if $\nabla u \in \mathcal{U}_A(\Omega, \mathbb{R}^N)$ then $u \in BMO(\Omega)$ (see Corollary 3.4) and, more generally, following [IS], if we introduce the space

$$f \in \mathcal{U}_0^N(\Omega, \mathbb{R}^N) \iff \sup_{Q \subset \Omega} |Q|^{1/H} \sup_{0 < \epsilon \leq 1} \left( e^{\sigma} \frac{\int_Q |f|^{N-\epsilon} \, dx}{\int_Q |f| \, dx} \right)^{\frac{1}{N-\epsilon}} < +\infty$$

we have that $\mathcal{U}_A(\Omega, \mathbb{R}^N) \subset \mathcal{U}_0^N(\Omega, \mathbb{R}^N)$ (see Proposition 3.2) and, if $\nabla u \in \mathcal{U}_0^N(\Omega, \mathbb{R}^N)$, then $u \in BMO(\Omega)$ (see Theorem 3.3).

Finally, following [FLS], we will prove also some imbedding results in Orlicz spaces for the Riesz Potential Operator in the critical case (see Theorem 3.5).

2 Notation and Preliminary results

Let us fix notation and recall basic concepts. For our purposes, a Young function will be any nonnegative, even, convex function $\Phi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\Phi$ is (strictly) increasing on $[0, \infty)$, and $\lim_{t \to 0} \Phi(t)/t = 0$, $\lim_{t \to \infty} \Phi(t)/t = \infty$.

Let $\Omega$ be a bounded open set in $\mathbb{R}^N$. The Orlicz space $L_\Phi(\Omega)$ is defined to be the smallest vector space containing the set of all measurable functions $f$ defined on $\Omega$ such that $\Phi(|f|) \in L^1(\Omega)$. It may be checked that $L_\Phi(\Omega)$ is a Banach space with respect to the norm

$$\|f\|_\Phi = \inf \left\{ \lambda > 0 : \int_\Omega \Phi \left( \frac{|f|}{\lambda} \right) \, dx \leq 1 \right\}$$
where the symbol \( \int \limits _\Omega \frac{1}{|e|} \int \) stands for \( \frac{1}{|e|} \int \). A special case is \( \Phi(t) = \frac{p^t}{p} \) (\( p \geq 1 \)), in which \( L_\Phi(\Omega) \) reduces to \( L^p(\Omega) \). If \( \Phi(t) = \frac{t^p}{\log^\sigma(e + t)} \) (\( p > 1, \sigma \geq 0 \)) then the corresponding Orlicz space will be denoted by \( L^p \log^{-\sigma} L(\Omega) \). Following [15], we will consider also a space larger than \( L^p \log^{-\sigma} L(\Omega) \), namely \( L^p_\sigma(\Omega) \) (\( p > 1, \sigma \geq 0 \)), defined as the Banach space of all measurable functions on \( \Omega \) such that

\[
\|f\|_{L^p_\sigma} = \sup_{0 < \epsilon \leq 1} \left( \epsilon^\sigma \int \limits _\Omega |f|^{p-\epsilon} \, dx \right)^{\frac{1}{p-\epsilon}} < +\infty.
\]

Following [G], the closure of \( L^{\infty}(\Omega) \) in \( L^p_\sigma(\Omega) \) will be denoted by \( \Sigma^p_\sigma(\Omega) \) (by \( \Sigma^p(\Omega) \) if \( \sigma = 1 \)), and it is characterized as the space of all measurable functions on \( \Omega \) such that

\[
\lim_{\epsilon \to 0} \left( \epsilon^\sigma \int \limits _\Omega |f|^{p-\epsilon} \, dx \right)^{\frac{1}{p-\epsilon}} = 0.
\]

In [FLS] it is proved the following extension of Trudinger's imbedding theorem ([T]) for \( W_0^{1,N}(\Omega) \) functions:

**Theorem 2.1.** If \( u \in W_0^{1,1}(\Omega) \) is such that \( |Du| \in L^N(\Omega) \) for some \( \sigma \geq 0 \), then there exist \( c_1 = c_1(N,\sigma) \), \( c_2 = c_2(N,\sigma) \) such that

\[
\int \limits _\Omega \exp \left( \frac{|u|}{c_1 \|Du\|_{L^N(\Omega)}} \right)^{\frac{N}{N-1+\sigma}} \, dx \leq c_2.
\]

We remark that if \( \Omega \) is convex, then an inequality of the same type is true also for functions \( u \in W^{1,1}(\Omega) \), provided \( |u| \) is replaced by \( |u - \int \limits _\Omega u \, dx| \). In fact, giving a closer look to the proof of Theorem 2.1, the assumption \( u \in W_0^{1,1}(\Omega) \) has been used only to write the inequality

\[
|u(x)| \leq C(N) \int \limits _\Omega |Du| |x - y|^{1-N} \, dy
\]
If \( u \in W^{1,1}(\Omega) \) and \( \Omega \) is convex, replacing \( |u| \) by \( |u - \int_{\Omega} u\,dx| \), this inequality is true with the constant in the right hand side depending only on \( N \) and the shape of \( \Omega \), but independently on the measure of \( \Omega \) ([GT]). In the proof of Theorem 3.3 we will use such inequality with \( \Omega \) replaced by a cube, therefore the constants will depend only on \( N \).

In [FLS] it is proved also that if \( u \in W_0^{1,1}(\Omega) \) and \( |Du| \in \Sigma^N(\Omega) \) then \( u \in \exp(\Omega) \), that is the closure of \( L^\infty(\Omega) \) in the Banach space

\[
\text{EXP}(\Omega) = \left\{ f \in L^1(\Omega) : \exists \lambda > 0 \text{ such that } \int_{\Omega} \exp \left( \frac{|f|}{\lambda} \right) \, dx < \infty \right\}.
\]

More generally, we will denote by \( \exp_\alpha(\Omega) \), \( \alpha > 0 \), the closure of \( L^\infty(\Omega) \) in \( \text{EXP}_\alpha(\Omega) \), the Orlicz space generated by the function \( \Phi(t) = \exp(t^\alpha) - 1 \).

Finally, let us recall that \( BMO(\Omega) \) is defined (see [S] for instance) as the space of the measurable functions \( u \) such that

\[
||u||_{BMO} = \sup_{Q \subset \Omega} \int_{Q} |u - u_Q| \, dx < +\infty
\]

where the supremum is taken over all cubes \( Q \) with sides parallel to the coordinate axes, and \( u_Q \) stands for \( \int_{Q} u\,dx \). We would get an equivalent definition of \( BMO(\Omega) \) if we replace the family of all cubes by the family of all balls. It is possible to prove (see [KJF] for instance) that if \( \Omega \) is a cube then \( BMO(\Omega) \) functions can be characterized by the following property:

\[
\exists \lambda > 0 : \sup_{Q \subset \Omega} \int_{Q} \exp \left( \frac{|u - u_Q|}{\lambda} \right) \, dx < +\infty.
\]

3 The main results

Let us recall that by Moser's inequality ([M]) \( W^{1,N}(\Omega) \) functions are \( \exp_{\frac{N}{N-1}}(\Omega) \) functions, and if \( |Du| \in L^N \log^{-1/q} L(\Omega) \) then \( u \in \exp_{\frac{N}{N-1} + \frac{N}{q}}(\Omega) \).

We now study imbeddings in \( BMO(\Omega) \). While \( W^{1,N}(\Omega) \) functions are \( BMO(\Omega) \) functions, if \( A(t) \) is a Young function with a growth essen-
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Finally less than $t^N$, then the Orlicz-Sobolev space $W^{1,A}(\Omega)$ is not imbedded in $BMO(\Omega)$. In fact we have the following example (see [GISS] for the case $A(t) = t^N \log^{-\sigma}(e+t)$):

**Example 3.1.** Let $\Omega$ be a bounded open set in $\mathbb{R}^N$, and let $A$ be a Young function of the type $A(t) = t^N \varphi(t)$, $\varphi(+\infty) = 0$. Then there exists a measurable function $u$ such that $|Du| \in L_A(\Omega)$ and $u \notin BMO(\Omega)$.

Let $\{a_j\}_{j \in \mathbb{N}}$ be such that

$$\sum_j a_j^N j^{-2} < +\infty \quad (3.1)$$

and let $\{r_j\}_{j \in \mathbb{N}}$ be such that

$$\sum_j r_j < +\infty \quad (3.3)$$

Let us note that by (3.3) we can find a sequence of points $x_j \in \Omega$ such that the balls $B(x_j, r_j)$ are pairwise disjoint and contained in $\Omega$ (at least for $j$ large enough). Let us define

$$h_j(x) = a_j h \left( \frac{x - x_j}{r_j} \right) \quad \forall x \in \Omega, \forall j \in \mathbb{N}$$

where

$$h(z) = \begin{cases} 
0 & \text{if } |z| \geq 1 \\
- \log |z| & \text{if } \frac{1}{2} \leq |z| \leq 1 \\
\log 2 & \text{if } |z| \leq \frac{1}{2}
\end{cases} \quad \forall z \in \mathbb{R}^n$$

and let $u = \sum_j h_j$. Notice that $u(x) = h_j(x)$ if $|x - x_j| < r_j$.

Hence, we have

$$\|u\|_{BMO} \geq \int_{B_j} |h_j - (h)_B| \, dx = a_j \int_B |h - (h)_B| \, dx \quad \forall j \in \mathbb{N}$$

where $B$ is the unit ball of $\mathbb{R}^n$, and therefore, by (3.2), $u \notin BMO(\Omega)$. 

On the other hand
\[ |Dh_j| \leq \begin{cases} \frac{a_j}{|x - x_j|} & \text{if } \frac{r_j}{2} \leq |x - x_j| \leq r_j \\ 0 & \text{if } |x - x_j| \leq \frac{r_j}{2} \end{cases} \]
and therefore, by (3.4),
\[
\int_{|x-x_j| \leq r_j} A(|Dh_j|) \, dx \leq \int_{\frac{r_j}{2} \leq |x-x_j| \leq r_j} A \left( \frac{a_j}{|x - x_j|} \right) \, dx
\]
\[
= N \omega_N \int_{\frac{r_j}{2}}^{r_j} A \left( \frac{a_j}{\rho} \right) \rho^{N-1} \, d\rho
\]
\[
= N \omega_N a_j^N \int_{\frac{r_j}{2}}^{r_j} \frac{1}{\rho} \varphi \left( \frac{a_j}{\rho} \right) \, d\rho
\]
\[
\leq N \omega_N a_j^N \int_{\frac{r_j}{2}}^{r_j} \frac{1}{\rho} \cdot \frac{1}{j^2 \log 2} \, d\rho
\]
\[
= N \omega_N a_j^N \frac{1}{j^2}
\]
where \( \omega_N \) denotes the measure of the unit ball in \( \mathbb{R}^n \), from which, summing over \( j \) and using (3.1), we get \( |Du| \in L_A(\Omega) \).

We remark that if \( |Du| \) belongs to some suitable spaces containing \( L^N(\Omega) \) (for instance, weak-\( L^N(\Omega) \)) then it is known that \( u \in \text{BMO}(\Omega) \). Now we introduce some new spaces having this property, which represent a variant of the classical Orlicz spaces. Namely, we consider the functions \( f \in L_A(\Omega) \) such that
\[
|f|_{p,A,\Omega} = \sup_{Q \subset \Omega} |Q|^{\frac{1}{p}} \left\| f \right\|_{L_A(Q)} < \infty
\]
If \( A(t) = t^p \), then \( |f|_{p,A,\Omega} \) reduces to the classical norm in \( L^p \) spaces. If \( p = N \) and \( A(t) = \frac{t^N}{\log^\sigma (e + t)} \) \( (N > 1, \sigma > 0) \) then \( |f|_{p,A,\Omega} \) is a norm.
defining a Banach space and it is different from $\|f\|_{L_A(\Omega)}$. The following result hold:

**Proposition 3.2.** Let $A(t) = \frac{t^N}{\log^\sigma(e + t)}$ ($N > 1, \sigma > 0$). If

$$\sup_{Q \subset \Omega} |Q| \frac{1}{t} \|f\|_{L_A(Q)} < +\infty$$

then

$$\sup_{Q \subset \Omega} |Q| \frac{1}{t} \left( \frac{\int_Q |f|^{N-\epsilon} \, dx}{\log^\sigma(e + t)} \right)^{\frac{1}{N-\epsilon}} < +\infty$$

**Proof.** Let $f \in L_A(\Omega)$, $f \geq T_\sigma$ where $A(T_\sigma) = 1$. By using the elementary inequality

$$(e + t)^\epsilon < e + t^\epsilon \quad (0 < \epsilon < 1, t \geq 0)$$

we obtain

$$e^\sigma f^{N-\epsilon} = \frac{\log^\sigma[(e + f)^\epsilon]}{f^\epsilon} \frac{f^N}{\log^\sigma(e + f)} \leq \frac{\log^\sigma(e + f)}{f^\epsilon} \frac{f^N}{\log^\sigma(e + f)} \leq C_\sigma \log^\sigma(e + f)$$

for some $C_\sigma > 0$, therefore

$$\sup_{0 < t \leq 1} \left( \frac{\int_Q |f|^{N-\epsilon} \, dx}{\log^\sigma(e + t)} \right)^{\frac{1}{N-\epsilon}} \leq C_\sigma \frac{f^N}{\log^\sigma(e + f)} \int_Q dx$$

If we drop the condition $f \geq T_\sigma$, applying the previous estimate to $\max(|f|, T_\sigma)$ we get

$$\sup_{0 < t \leq 1} \left( \frac{\int_Q |f|^{N-\epsilon} \, dx}{\log^\sigma(e + t)} \right)^{\frac{1}{N-\epsilon}} \leq C_\sigma \frac{\max(|f|, T_\sigma)^N}{\log^\sigma(e + \max(|f|, T_\sigma))} \int_Q dx$$

$$\leq C_\sigma \frac{f^N}{\log^\sigma(e + f)} dx + D_\sigma$$
for some $D_\sigma \geq 0$.

Replacing $f$ by $\frac{f}{\|f\|_{L^A(Q)}}$, the right hand side is majorized by a constant depending only on $\sigma$, and independent of $Q$, therefore the assertion follows multiplying by $\|Q\|^{\frac{1}{N}} \|f\|_{L^A(Q)}$ and taking the supremum over all cubes $Q$ contained in $\Omega$.

\[\blacksquare\]

**Theorem 3.3.** If $u \in W^{1,1}(\Omega)$, $\Omega$ cube in $\mathbb{R}^N$ ($N > 1$), is such that $|Du|$ verifies the condition

\[ |Du| \in U^N_\sigma(\Omega, \mathbb{R}^N) \iff \sup_{Q \subset \Omega} |Q|^{\frac{1}{N}} \sup_{0 < c \leq 1} \left( c^\sigma \frac{\int_Q |Du|^N \, dx}{|Q|^N} \right) = M_{u, \sigma} < +\infty, \quad (3.5) \]

for some $\sigma > 0$, then $u \in \text{BMO}(\Omega)$.

**Proof.** Without loss of generality we can assume $0 < \sigma \leq 1$. Let us fix $Q \subset \Omega$ and let us apply Theorem 2.1 with $\Omega$ replaced by $Q$, and $u$ replaced by $u - u_Q$. We have

\[ \int_Q \exp \left( \left( \frac{|u - u_Q|}{c_1 M_{u, \sigma}} \right)^{\frac{N}{N-1+\sigma}} \right) \, dx \leq \int_Q \exp \left( \left( \frac{|u - u_Q|}{c_1 \|Du\|_{L^\infty} |Q|^N} \right)^{\frac{N}{N-1+\sigma}} \right) \, dx \leq c_2(N, \sigma), \]

from which

\[ \int_Q \exp \left( \frac{|u - u_Q|}{c_1 M_{u, \sigma}} \right) \, dx = \int_{\frac{|u - u_Q|}{c_1 M_{u, \sigma}} < 1} \exp \left( \frac{|u - u_Q|}{c_1 M_{u, \sigma}} \right) \, dx + \int_{\frac{|u - u_Q|}{c_1 M_{u, \sigma}} \leq 1} \exp \left( \frac{|u - u_Q|}{c_1 M_{u, \sigma}} \right) \, dx \]
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\[ \leq \int_Q \exp \left( \frac{|u - u_Q|}{c_1 M_{u,\sigma}} \right) \frac{N}{N-1+\sigma} \, dx + \int_Q \exp(1) \, dx \]

\[ \leq c_2(N, \sigma) |Q| + e^{|Q|} \]

and therefore

\[ \sup_{Q \subset \Omega} \int_Q \exp \left( \frac{|u - u_Q|}{c_1 M_{u,\sigma}} \right) \, dx < +\infty. \]

Since \( \Omega \) is a cube, then \( u \in BMO(\Omega) \).

We prove now the following

**Corollary 3.4.** Let \( A(t) = \frac{t^N}{\log^\sigma (e + t)} \) \( (N > 1, \sigma > 0) \). If \( |Du|_{N,\Lambda,\Omega} < +\infty \), then \( u \in BMO(\Omega) \).

**Proof.** For any \( Q \subset \Omega \) we have \( \|f\|_{L_A(Q)} < +\infty \) and therefore (see [BFS] lemma 3; see also [G])

\[ \lim_{\epsilon \to 0^+} \left( e^{\epsilon} \int_Q |Du|^{N-\epsilon} \, dx \right)^{\frac{1}{N-\epsilon}} = 0 \]

from which

\[ \sup_{0 < \epsilon \leq 1} \left( e^{\epsilon} \int_Q |Du|^{N-\epsilon} \, dx \right)^{\frac{1}{N-\epsilon}} < +\infty \quad \forall Q \subset \Omega. \]

We have

\[ \sup_{Q \subset \Omega} \left( e^{\epsilon} \int_Q |Du|^{N-\epsilon} \, dx \right)^{\frac{1}{N-\epsilon}} \]

\[ \leq \sup_{Q \subset \Omega} \left( e^{\epsilon} \int_Q |Du|^{N-\epsilon} \, dx \right)^{\frac{1}{N-\epsilon}} \]

\[ \leq c(N, \sigma) \sup_{Q \subset \Omega} |Q| \|Du\|_{L_A(Q)} \]

\[ = c(N, \sigma) |Du|_{A,\Omega} < +\infty \]
and therefore by Theorem 3.3 the assertion follows.

By Corollary 3.4, the function \( f \) of Example 3.1 is such that \( |Df|_{N,A,N} = +\infty \). This fact could be also verified directly, by proving that
\[
|B_j|^{\frac{1}{N}} \sup_{0<\epsilon \leq 1} \left( \epsilon \int_{B_j} |Dh_j|^{N-\epsilon} \, dx \right)^{\frac{1}{N-\epsilon}} = c(N) a_j \quad \forall j \in \mathbb{N}.
\]

Let us note also that the BMO function \( u(x) = \log |x| \cdot (|x| \leq 1) \) verifies the condition (3.5), and is such that \( u \not\in L^\infty, \sum |D u| \not\in L^N \).

We remark that by using the same arguments to prove Theorem 2.1 it is possible to give an alternative proof of a well-known result by Adams [A] (see Corollary 4.2) about the Riesz Potential Operator defined by
\[
I_{\frac{p}{p-1}} f = \int_{\Omega} \frac{|x-y|^{N-p}}{|x-y|^{p-1}} f(y) \, dy.
\]

**Theorem 3.5.** Let \( 1 < p < +\infty, \sigma > 0 \) if \( f \in L^p_0(\Omega) \), then \( I_{\frac{p}{p-1}} f \in EXP_{\frac{p}{p-1}+\sigma}(\Omega) \).

**Proof.** Let us start again from the inequality
\[
||I_{\frac{p}{p-1}} f||_q \leq q^{\frac{1}{q} - 1} \cdot q^{\frac{1}{q} - 1} \cdot \omega_N^{\frac{1}{q} - 1} \cdot |\Omega|^{\frac{1}{q} - 1} \cdot ||f||_{p-\epsilon} \quad \forall q \geq p, \quad \forall 0 < \epsilon \leq 1.
\]

We have
\[
e^{\sigma \epsilon^{\frac{p}{p-1}}} ||I_{\frac{p}{p-1}} f||_q \leq q^{\frac{1}{q} - 1} \cdot q^{\frac{1}{q} - 1} \cdot \omega_N^{\frac{1}{q} - 1} \cdot |\Omega|^{\frac{1}{q} - 1} \cdot \epsilon^{\sigma \epsilon^{\frac{p}{p-1}}} ||f||_{p-\epsilon} \leq q^{\frac{1}{q} - 1} \cdot q^{\frac{1}{q} - 1} \cdot \omega_N^{\frac{1}{q} - 1} \cdot |\Omega|^{\frac{1}{q} - 1} \cdot ||f||_{L^p}^{\frac{1}{p}}
\]

and therefore
\[
\sup_{0<\epsilon \leq 1} \left( \epsilon \int_{\Omega} \left( \frac{I_{\frac{p}{p-1}} f}{||f||_p} \right)^{\frac{1}{q}} \, dx \right)^{\frac{1}{q}} < c(n)
\]

from which the assertion follows.
Corollary 3.6. Let \( 1 < p < +\infty \). There exist constant \( c_0 = c_0(N) \), \( c_1 = c_1(N,p) \) such that for any \( f \in L^p(\Omega) \) the following inequality holds:

\[
\int_{\Omega} \exp \left( \frac{|I_p f|}{c_0 \|f\|_p} \right) dx \leq c_1
\]

Applying to the Theorem 3.5 the same density argument as in [CS], if a function \( f \) is in the closure of \( L^\infty(\Omega) \) of \( L^p(\Omega) \) then the image of \( f \) by the linear continuous operator \( I_p \) must be in the closure of \( L^\infty(\Omega) \) of \( EXP_{p-\sigma}^\infty(\Omega) \), therefore we have also the following

Corollary 3.7. Let \( 1 < p < +\infty \), \( \sigma > 0 \). If \( f \in \Sigma^p(\Omega) \), then \( I_p f \in EXP_{p-\sigma}^\infty(\Omega) \)

We remark that, in the same way, as a corollary of Theorem 3.5, we get that if \( f \in L^p(\Omega) \), then \( I_p f \in EXP_{p-\sigma}^\infty(\Omega) \).

References


Alberto Fiorenza


Alberto Fiorenza,
Dipartimento di Matematica e Applicazioni,
"R. Caccioppoli",
Via Cintia,
80126 Napoli,
Italy
e-mail: fiorenza@matna2.dma.unima.it

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